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Apr. 1909.

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PRACTICAL CALCULATIONS FOR ENGINEERS

FOR THE USE OF

ENGINEERING STUDENTS, APPRENTICES, DRAUGHTSMEN,
MECHANICS, FOREMEN, AND OTHERS PRACTI-
CALLY ENGAGED IN ENGINEERING WORK.

BY

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Engineering Library

PREFACE.

THE Authors have been induced to undertake the labour of writing a manual dealing with calculations and graphical methods applied to practical engineering with the object of satisfying a want experienced by engineering students and practical men, who alike require a sound and ready knowledge of modern time-saving methods in their everyday work. They have at the same time endeavoured to give, *in one volume*, information, some of it largely neglected, with which all young engineers should be familiar. The many mathematical works for engineers which have in recent years been published deal almost entirely with calculations from the point of view of the student's requirements at a University or Technical College. For this purpose they have answered admirably, though the Authors claim that the present volume will form a useful supplement to them. Unfortunately, however, mathematical works, as such, fail to grip the large body of *present-day* practical engineers, who have to deal with calculations in a ready and businesslike way. It is partly to satisfy a want which obviously exists in this direction that the Authors venture to launch the present work.

The table of contents indicates the scope of this work, but we would indicate briefly the general principles we have kept steadily in view in the preparation of the MSS. and diagrams. These are, to familiarise the student with:—

(1) *Modern methods of making rapid calculations by the calculating slide rule.* A description of the log-log scale recently introduced by Professor Perry is also included, and numerous examples are given illustrating its value in engineering work. There is undoubtedly a great field for the use of these mechanical calculating devices, for almost all the ordinary cal-

culations with which the engineer has to deal can be worked out in a very small fraction of the time that would otherwise be required by the usual arithmetical methods, or even of the time required by the use of common logarithms. That time is valuable, or, to use a metaphor, "time is money," cannot, in these days of strenuous competition, be overlooked in our training of the young engineer. Yet it is surprising to find how few, comparatively, even of professional men are able to take full advantage of the uses to which the slide rule can be applied.

(2) *Some of the more important engineering problems which have to be solved in experimental work, in the workshops, drawing offices, or by the management.*

(3) *Graphical methods (illustrated by engineering problems) for the ready interpretation of the results of experimental and commercial work.* The uses of squared paper to the young engineering student, to practical engineers in the works, drawing office, and estimating department, as well as to the general management of an engineering concern, are fully dealt with. The use of squared paper is invaluable to the business man, for by its means economy may be effected in all departments, errors and irregular working may be detected, costs and dimensions of manufactures may be reduced to formulæ, and a clear light thrown on the entire working of an engineering establishment. Further, by its aid results are readily interpreted which would have a formidable aspect when presented in tabular or mathematical form.

(4) *The importance of making a study of the business side of his profession.* To this end chapters are included dealing with the calculation of weights, the preparation of estimates, and the remuneration of labour—on the time, bonus, and premium systems. In this section of the work a special chapter is included enlarging on the great value of squared paper for the graphical representation of the variations and fluctuations which are common to an engineering establishment. These sections are first introduced by a chapter giving a general survey over the whole field of the commercial side of the business. It is hoped that the section on the calculation of weights may prove of practical assistance in the drawing office and estimating department, as well as to the student pursuing his studies at some

technical college. It is surprising how many educated men there are engaged in engineering work, who, with mathematics at their fingers' ends, are unable to apply their knowledge in a quick and workmanlike manner to such simple problems as the calculation of weights for simple pieces of machinery. In the rush and turmoil of business, the man who can do his work quickly in a businesslike way, even if his results are not correct to two, three, or more places of decimals, is the one who will get on.

In conclusion, the Authors desire to express their indebtedness to C. Larard, Esq., Consulting Engineer, for valuable assistance generously given; to the Council of the Institution of Mechanical Engineers, for permission to reproduce a number of diagrams dealing with graphical methods; to the Institution of Civil Engineers, for permission to include their diagram of heat losses for boiler and steam engine plant; and finally they have to thank Messrs P. W. Smith, A. C. Travis, and A. F. Thornton for valuable assistance given in the preparation of the diagrams for this work, and to Mr P. W. Smith they are further indebted for working through the majority of the exercises given.

CHARLES E. LARARD.
HENRY A. GOLDING.

LONDON, 1907.

CONTENTS.

SECTION I.

CHAPTER I.

PRELIMINARY CALCULATIONS, ABBREVIATIONS, APPROXIMATIONS, ETC.

	PAGE
Contracted forms of arithmetic, and approximations, etc.—Contracted multiplication—Contracted division—Square root—Approximations—Multiple—Submultiple, ratio, compounding ratios—Proportion—Power and roots—Variation—Engineering exercises.	1-23

CHAPTER II.

TECHNICAL MENSURATION.

Forms with a right angle—Similar triangles, and similar plane forms generally—Intersecting chords in a circle—Perimeters—The Opisometer—Areas—The Planimeter—Areas determined by weighing—Area of figure under curve, $xy^n = \text{constant}$ —Parabolic segment—Surface of volume forms—Volumes—Cuboids, prisms, and pyramids, cone, sphere, and cylinder, etc.—Spherical and conical shells—Segments of solids—Rings—Solids of revolution—Conic sections—Parabola, ellipse, hyperbola—Metric measure and conversion numbers—Engineering exercises.	24-69
---	-------

CHAPTER III.

CALCULATION BY COMMON LOGARITHMS.

System of common logarithms—Logs of products, quotients, powers and roots of numbers—Logarithm tables—Practical calculations by the aid of logarithms—Napierian logarithms.	70-86
---	-------

CHAPTER IV.

MECHANICAL CALCULATION DEVICES.

	PAGES
The slide rule—Principles of, and calculations by, 10-inch Gravet rule—Watch or pocket calculator—Fuller's spiral slide rule—Prof. Perry's log-log scale—Engineering exercises	87-139

CHAPTER V.

THE VALUE OF SQUARED PAPER.

Squared paper and its uses—Paper with inches and tenths-of-an-inch divisions, and with centimetre and millimetre divisions—Squared paper with logarithmic divisions in two directions at right angles—Paper with logarithmic divisions in one direction and ordinary unit divisions in the other—Engineering exercises	140-160
--	---------

SECTION II.

CHAPTER VI.

PULLEYS, WHEELS IN TRAIN, CHANGE WHEELS, TOOLS, ETC.

Pulleys and wheels in train ; simple and compound trains ; speed ratios and revolutions—Change wheels for screw-cutting : right and left hand screws—Speed cones and step pulleys—Change wheel, feed gear, and calculations for screw-cutting—Double and treble lathe gearing : gear wheels, cutting speeds—Exercises	161-180
---	---------

CHAPTER VII.

PRINCIPLES OF MOMENTS.

Principle of Moments applied to practical calculations—Safety valve—Beams and girders—Levers—Cranes and derricks—Engineering exercises	181-189
--	---------

CHAPTER VIII.

WORK, POWER, AND ENERGY.

Work and power units : British and C.G.S. graphical representation—Horse-power—Work done during hyperbolic expansion—Work during adiabatic expansion—Mean pressure—Graphical method for finding expansion curve—Work determined from indicator diagrams—Energy, kinetic and potential—Fly-wheel : determination of weight for given fluctuation of speed—Engineering exercises	190-215
--	---------

CHAPTER IX.

TRANSMISSION OF WORK THROUGH MACHINES.

	PAGES
Machines as agents for doing work—Mechanical advantage. Velocity ratio—Mechanical efficiency—Principle of work—Various machines—Experiments to determine advantage, friction and efficiency—Graphical representations and interpretations of the results—Engineering exercises	216-238

CHAPTER X.

TRANSMISSION OF POWER.

Methods of power transmission—Ropes, belts, toothed gearing, shafting, etc.	239-248
---	---------

CHAPTER XI.

CENTRIFUGAL FORCE AND BALANCING.

Motion in a circle—Effect of unbalanced weights—Stress in the fly-wheel rims caused by centrifugal force—Safe working speed of rim—Train rounding a curve—Engineering exercises	249-255
---	---------

CHAPTER XII.

ACCELERATION, MOMENTUM, FORCE-ACTION.

Momentum and force-action—Acceleration—Distinction between mass and weight—Force or pressure exerted upon momentum reduced or destroyed—Impact—Laws of motion—Engineering exercises	256-273
---	---------

SECTION III.

CHAPTER XIII.

PRELIMINARY CALCULATIONS.

Fahrenheit and Centigrade scales—Units of heat : British and C.G.S.—Specific heat—Heat carried away by flue gases from boilers—Heat and work—Joule's mechanical equivalent of heat—Calorimetry and heat values of fuels—Condensation of steam—Engineering exercises	274-288
---	---------

CHAPTER XIV.

THE STEAM BOILER AND ITS CALCULATIONS.

	PAGES
Steam tables and curves—Formation of steam at constant pressure —Total and latent heat of evaporation—Moisture in suspension, and dryness fraction—Dryness fraction calorimeters— Determination of dryness fraction—Superheated steam—Boiler efficiency—Evaporative power of fuel—Calculation of size of boiler for given output of steam—Fuel and flue-gas calculations —Engineering exercises	289-311

CHAPTER XV.

THE ENGINE AND ITS CALCULATIONS.

Engine power—Cycle of operations for double-acting steam engine —Compound and triple-expansion engines—Otto cycle—Power calculations: indicated, brake, and mechanical efficiency— Mean effective pressure from diagrams—Determination of probable mean effective pressure—Diagram factor—Calculation of size of cylinder to develop a given H.P.—Willans' law and steam consumption—Thermal efficiency of steam and other heat engines—Carnot heat engine of comparison— Rankine standard steam engine of comparison—Efficiency ratio—Heat units per I.H.P. and per B.H.P. minute—Heat losses—Electrical horse-power	312-342
--	---------

CHAPTER XVI.

COMPARATIVE COSTS OF POWER PRODUCTION.

Various methods of power production for steam, gas, oil engines, etc. Commercial efficiency—Comparative cost of various power plants—Engineering exercises on Chapters XIII., XIV., XV., and XVI.	343-352
--	---------

SECTION IV.

CHAPTER XVII.

THE COMMERCIAL SIDE OF ENGINEERING.

CHAPTER XVIII.

CALCULATION OF WEIGHTS.

	PAGES
Weight of materials—Weight of plates, covers and sections, pipes and cylinders—Shafting: solid or hollow, crank-shafts, crank discs, etc.—Forgings: calculated weights and actual weights—Tanks and boilers—Castings: simple and complicated—Weight conversion tables	362-395

CHAPTER XIX.

PREPARATION OF ESTIMATES, DIVISION OF COSTS.

Material and labour: shop charges and establishment charges—Cost of raw material: cost of labour in different departments—Approximate estimates of weights—Selling prices—Profit—Specimen quantities lists for engine and boiler	396-406
--	---------

CHAPTER XX.

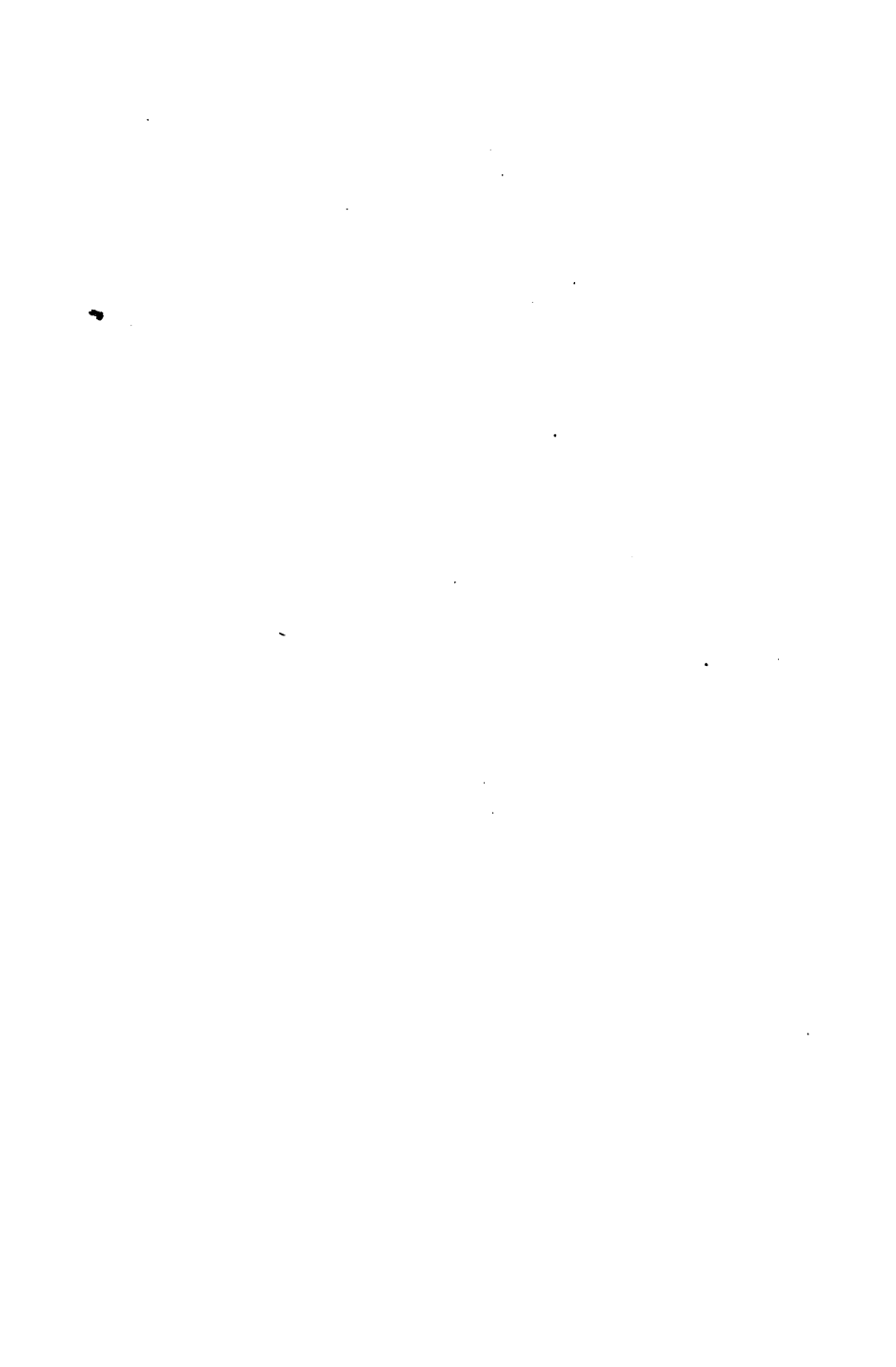
REMUNERATION OF LABOUR.

Time, piece, and premium systems—Weir, Halsey, and Rowan systems—Diagrammatic representation of the premium systems	407-415
---	---------

CHAPTER XXI.

USE OF SQUARED PAPER IN AN ENGINEERING ESTABLISHMENT.

Numerous diagrams from actual practice—Value of squared paper to the foreman, in the estimating department and drawing office, and to works manager—Engineering exercises on Section IV.	416-446
INDEX	447-455



PRACTICAL CALCULATIONS FOR ENGINEERS.

SECTION I.

CHAPTER I.

PRELIMINARY CALCULATIONS, ABBREVIATIONS, APPROXIMATIONS, Etc.

IN the following pages it will be assumed that the reader is familiar with simple arithmetical calculations. There are, however, methods of shortening calculations which reduce the labour and time involved, where only approximate results are desired. Several of these methods will first of all receive attention before passing on to more important chapters.

1. Contracted Forms of Arithmetic and Approximations.—

The accuracy of any result is entirely dependent upon the degree of accuracy to which each of the observations or quantities involved can be measured, and does not depend upon the number of decimal places used in expressing the result. Suppose, for example, we time the speed of a moving object, say a motor car, with an ordinary watch, and observe that the car passes over a distance of 220 yards in 13 seconds. To express this by saying that the car is travelling at a speed of 34·6153 miles per hour is entirely misleading, seeing that the degree of accuracy with which the time can be noted is dependent upon the reading of the seconds hand of an ordinary watch, and may be incorrect to $\frac{1}{2}$ or 1 second on either side of the observed time. The error involved in reading the time, if it amounts to only 1 second, is equal to $\frac{1}{13}$ or 7 per cent. of the observed quantity, and the result therefore can be relied upon to be correct within the limits of this error, even assuming that the distance of 220 yards has been measured correctly. To express the result as 34·6153 miles per hour is therefore ridiculous, seeing that the actual speed may be anything between $33\frac{1}{2}$ and $35\frac{1}{2}$ miles per hour.

Some time ago we read in the press that the engines of a certain vessel on its trial trip developed a maximum power of 13,546 I.H.P. under the given conditions. Knowing that the calculation of the indicated horse-power of an engine involves the use of various observed quantities, some of which may not be accurate to 2, 3, 4, or 5 per cent., then, to work out the horse-power to five significant figures is waste of time, and misleading. In the first place, the diameters of the cylinders have probably never been accurately gauged (more often they are assumed to be the nominal diameters given by the contractors' drawings), and again the springs used in the indicators have in all probability never been tested under the required conditions of temperature and pressure, and cannot be relied upon to within 4 or even 5 per cent.; consequently it is absolutely useless to express the result as 13,546 I.H.P., seeing that the actual power developed may be anything between 13,400 and 13,700. Generally speaking, therefore, it is sufficiently accurate for most mechanical purposes to express results dependent upon ordinary measurements or observation to three or (at the outside) four significant figures, as being well within the errors of observation usually entailed. For this purpose the contracted forms of multiplication and division are much to be preferred to the ordinary methods given in text-books on arithmetic, and we will now proceed to consider these forms.

Contracted Multiplication.—When any two or more numbers have to be multiplied together, the contracted form saves very considerably the time and labour involved. Take, for example, the two numbers 3781 and 5369. If the product is required correct to four significant figures, proceed as shown in the following example, by reversing the multiplier and omitting one figure at the right of the multiplicand in each succeeding line of the process, thus :—

Ordinary Method.	Contracted Method.
3781	3781
5369	9635
34029	18905
22686	1134
11343	227
18905	34
20300189	20300000

If the result be required correct to five significant figures, it will be seen that all the work by the ordinary method to the right of

the vertical line is quite unnecessary, and can be omitted. Thus instead of multiplying by the units figure of the multiplier first (the least important figure of the multiplier), by reversing the multiplier and writing it backwards, thus,

9635 instead of 5369,

we use the most important figure of the multiplier first (the 5), and in each succeeding line of the process we omit one figure at the right of the multiplicand, so that instead of multiplying 3781 by 3 we only multiply 378 by 3, and put a dot over the 1 in the multiplicand to show that it has been omitted. Similarly, in the third line of the partial product, we put a dot over the 8, to show that it is omitted, and multiply 37 by 6. At the same time we must not reject the 8 following 37, but multiply it mentally, thus, $6 \times 8 = 48$; so we carry over 5 (the nearest number that 10 will go into 48) and add it to the next figure, thus, $6 \times 7 = 42 + 5 = 47$. Similarly in multiplying the last figure $9 \times 3 = 27 + 7$ (carried over) = 34. When the process is completed, and the product obtained, add a cipher or zero for every figure in the multiplicand that has been omitted, *i.e.* count the number of dots over the multiplicand and add a cipher to the product for each dot. In the example we are considering there are three dots, and we therefore add three 0's, making the approximate final product 20,300,000.

The same method can be applied to decimal quantities; thus, to multiply 4.567 by 87.06 correct to five significant figures, proceed by ignoring the decimal point, and reversing the order of the figures in the multiplier, thus:—

$$\begin{array}{r}
 4\cdot\dot{5}\dot{6}\dot{7} \\
 60\cdot78 \\
 \hline
 36536 \\
 3197 \\
 27 \\
 \hline
 397\cdot6 \text{ 0000}
 \end{array}$$

If there be any liability of mistaking the significance of the dot placed over the various figures in the multiplicand, the figures may be crossed out one at a time, in the following manner:—

When multiplying by the second figure in the multiplier (7), omit the last figure in the multiplicand (7), crossing it out, but

carry on its value in multiples of 10 as explained; thus, $7 \times 7 = 49 = 5 \times 10$ (nearest multiple of 10). Then $7 \times 6 = 42 + 5 = 47$. In multiplying the next figure (0) in the multiplier, we cross out the 6 in the multiplicand and pass on to the next and last figure in the multiplier (6). Finally, adding three ciphers for the figures in the multiplicand crossed out, and placing the decimal point according to the sum of the figures following the decimal point in both multiplicand and multiplier, we get the answer 397.6.

Contracted Division.—A similar method can be followed in division, when the result is only required correct to four significant figures, by using only five figures of the dividend and contracting the working by cutting out one figure of the division at each stage of the operation, commencing at the unit or least important end, as shown by the following two examples.

Example 1.—To divide 71854 by 4256, correct to four successive figures :—

Ordinary Method.	Contracted Method.
4256)71854(16.8829...	~~4256~~)71854(16.88
4256	4256
-----	-----
29294	2929
25536	2554
-----	-----
. 37580	375
34048	340
-----	-----
. 35320	35
34048	34
-----	-----
. 12720	
8512	

. 42080	
38304	

By reference to the ordinary method it will be seen that, when the result is only required correct to four significant figures, all the work to the right of the dotted line is quite unnecessary, and only takes up time and space. The same method applied to decimal quantities is shown in the following example.

Example 2.—Divide 58.73542 by 0.3065, correct to four significant figures :—

Ordinary Method.	Contracted Method.
0.3065)58.73542(191.632...	0.3065)58.73542(191.6
30 65	30 65
28 085	28 08
27 585	27 58
.. 5004	50
3065	31
19392	19
18390	18
. 10020	
9195	
.. 8250	
6130	
2120	

Square Root.—The method of calculating the square root can be shortened very considerably, if only an approximation is required, by omitting one figure in each division after, say, three or four figures of the root have been obtained. For instance, to find the square root of 321.735, correct to five significant figures.

Here we proceed in the usual way until three figures of the root have been obtained, and then we cancel one figure in the division each time, as explained in Contracted Division, and find the root correct to five significant figures to be 17.937

$$\begin{array}{r}
 13'21'73'50'/17.9369 \\
 \begin{array}{r}
 1 \\
 27 \overline{)221} \\
 \underline{189} \\
 349 \overline{)3273} \\
 \underline{3141} \\
 358 \overline{)13250} \\
 \underline{1075} \\
 358 \overline{)250000} \\
 \underline{2152..} \\
 358 \overline{)3480000} \\
 \underline{3228...}
 \end{array}
 \end{array}$$

There are many other shortened forms of arithmetic with which the student should make himself familiar, and examples are given in the following pages.

(1) *To multiply by 5*, add a cipher to the multiplier or move the decimal point one place to the right, and divide by 2, thus :—

$$\begin{aligned} &67\cdot294 \times 5 \\ &= 672\cdot94 \div 2 \\ &= \mathbf{336\cdot47.} \text{ Ans.} \end{aligned}$$

(2) *To multiply by 25*, add two ciphers, or move the decimal point *two* places to the right, and divide by 4, thus :—

$$\begin{aligned} &156\cdot2 \times 25 \\ &= 15620 \div 4 \\ &= \mathbf{3905.} \text{ Ans.} \end{aligned}$$

(3) *To multiply by 125*, add three ciphers, or move the decimal point three places to the right, and divide by 8, thus :—

$$\begin{aligned} &6\cdot852 \times 125 \\ &= 6852 \div 8 \\ &= \mathbf{856\cdot5.} \text{ Ans.} \end{aligned}$$

(4) *To divide by 25*, move the decimal point two places to the *left*, and multiply by 4, thus :—

$$\begin{aligned} &15625 \div 25 \\ &= 156\cdot25 \times 4 \\ &= \mathbf{625.} \text{ Ans.} \end{aligned}$$

(5) *To divide by 125*, move the decimal point three places to the left, and multiply by 8, thus :—

$$\begin{aligned} &468\cdot75 \div 125 \\ &= 0\cdot46875 \times 8 \\ &= \mathbf{3\cdot75.} \text{ Ans.} \end{aligned}$$

(6) *To multiply by 101*, add two ciphers, or move the decimal point two places to the right, and add the number, thus :—

$$\begin{aligned} &56\cdot8 \times 101 \\ &= 5680 + 56\cdot8 \\ &= \mathbf{5736\cdot8.} \text{ Ans.} \end{aligned}$$

(7) *To multiply by 99*, add two ciphers, or move the decimal point two places to the right, and subtract the number, thus :—

$$\begin{aligned} &144 \times 99 \\ &= 14400 - 144 \\ &= \mathbf{14256.} \text{ Ans.} \end{aligned}$$

The same principle involved in the last two cases can be applied to such cases as the multiplication of any number by 102, 98, 1001, 999, 1002, 998, etc. Sometimes the multiplier consists of an arrangement of figures such that one portion is a multiple or sub-multiple of another portion; for example:—

(8) *To multiply by 12111* we notice that

$$12111 = 12100 + 11,$$

and that

$$12100 = 1100 \times 11.$$

We can therefore obtain the product by two lines, first by multiplying by 11, and then multiplying this partial product by 11 again, thus:—

To multiply 567·43 by 12111:

$$\begin{array}{r} 567\cdot43 \\ 12111 \\ \hline 624173 \quad \dots (i) \\ 6865903 \quad \dots (ii) \\ \hline 6872144\cdot73 \end{array}$$

The first line (i) of the partial product is 11 times 567·43 and the second line is 11 times line (i) removed two places to the left, and is therefore 1100 times (i) and therefore $1100 \times 11 = 12,100$ times 567·43. The final product is therefore $11 + 12,100$ times the multiplicand.

The same method can be used for such multipliers as

- (a) $14412 = (12 + 12 \times 1200)$
- (b) $11011 = (11 + 10 \times 1100)$
- (c) $1089 = (9 + 12 \times 90)$
- (d) $648 = (8 + 8 \times 80)$
- (e) $864729 = (9 + 8 \times 90 + 12 \times 72000)$
- (f) $3125625125 = (125 + 5 \times 125000 + 5 \times 625000000).$

The last two examples require three lines each in the partial product.

Approximations.—It sometimes happens that the multiplier and multiplicand are very nearly equal to some whole number, in which case an approximation to the product can be found by squaring the number which is a mean of the two; for example:—

(1) *To multiply 99 by 101:*

Here the multiplicand is 1 less, and the multiplier 1 more, than 100, so that by squaring the mean of the two numbers

(100), a result is obtained very near to the correct answer, thus :—

$$\begin{aligned} & 99 \times 101 \\ & \text{(Approx.) } 100^2 = 10000. \\ & \text{Correct answer} = 9999. \end{aligned}$$

Here the difference is only 1 in 10,000 or $\frac{1}{1000}$ of 1 per cent.

(2) *To multiply 14.5 by 15.5 :*

$$\begin{aligned} & \text{The square of 15} = 225. \\ & \text{Correct answer} = 224.75 \text{ (error} = \frac{1}{2} \text{ of 1 per cent.)}. \end{aligned}$$

(3) *To multiply 197 by 2.03 :*

$$\begin{aligned} & = \text{(Approx.) } 200 \times 2 = 400. \\ & \text{Correct answer} = 399.91 \text{ (error} = \frac{1}{4} \text{ of 1 per cent.)}. \end{aligned}$$

Here it will be noticed that the error gets greater as the difference between the actual numbers and the assumed numbers increases, and the method can only be employed where this difference is large when an approximate answer is required. As the difference gets larger, or when a correct answer is required, we may obtain the same by subtracting from the result the square of the difference, thus :—

$$\begin{aligned} & 145 \times 155 \\ & = (150 - 5) \times (150 + 5) \\ & = 150^2 - 5^2 \\ & = 22500 - 25 = 22475. \text{ Ans.} \end{aligned}$$

Again,

$$\begin{aligned} & 91 \times 109 \\ & = 100^2 - 9^2 = 10000 - 81 = 9919. \end{aligned}$$

The student who has read a little algebra will readily recognise the above as $(a+b)(a-b) = a^2 - b^2$ and remember the method more easily by the formula.

Further, in squaring numbers which differ but slightly from some even number, it is frequently preferable to square the even number, and either add or subtract twice the product of the number into the difference involved—the comparative value of the square of the difference being so small that it can be neglected.

Example 1 :—

$$\begin{aligned} & 149^2 \\ & = (150 - 1)^2 = 150^2 - 2(150 \times 1) + 1^2 \\ & = 22500 - 300 + 1 \\ & = 22201. \end{aligned}$$

In this case the square of the difference is only 1, and had this been neglected, it would only have introduced an error in the result of 1 in 22,000 or $\frac{1}{22000}$ of 1 per cent.

The same method applies in finding the square of a number slightly greater than an even number, only in this case the product of twice the number into the difference must be *added*, and (if accuracy demands it) the square of the difference.

Example 2.—Find the square of 20·3 :

$$\begin{aligned} &= (20 + 0\cdot3)^2 \\ &= 20^2 + 2(20 \times 0\cdot3) + (0\cdot3)^2 \\ &= 400 + 12 + 0\cdot09 \\ &= 412\cdot09. \end{aligned}$$

Example 3.—Find the square of 1·23 :

$$\begin{aligned} &(1\cdot23)^2 \\ &= (1\cdot2 + 0\cdot03)^2 \\ &= 1\cdot2^2 + 2(1\cdot2 \times 0\cdot03) + (0\cdot03)^2 \\ &= 1\cdot44 + 0\cdot072 + 0\cdot0009 \\ &= 1\cdot5129. \end{aligned}$$

The same method may be extended to find the approximate value of the cube of a number which differs only slightly from some even number, only in this case we must add or subtract three times the product of the square of the number into the difference involved.

Thus,

$$\begin{aligned} &(100\cdot1)^3 \\ &= (100 + 0\cdot1)^3 \\ &= (\text{approx.}) 100^3 + 3(100^2 \times 0\cdot1) \\ &= 1,000,000 + 3000 \\ &= 1,003,000. \\ &\text{Correct answer} = 1,003,003\cdot001. \end{aligned}$$

The student who has some knowledge of algebra will again remember this method better by its algebraic equivalent of

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3,$$

and where b is small compared with a , the comparative value of b^2 is very small, and that of b^3 much smaller still, and can be neglected without introducing any appreciable error.

Again, $(1\cdot001)^3 = (\text{approx.}) 1^3 + 3(1^2 \times 0\cdot001) = (\text{approx.}) 1\cdot003$.

Further, $98^3 = (100 - 2)^3 = (\text{approx.}) 100^3 - 3(100^2 \times 2)$
 $= (\text{approx.}) 1,000,000 - 60,000$
 $= (\text{approx.}) 940,000$.

Correct answer = 941,192.
 (Error = $\frac{1}{3}$ of 1 per cent.)

Multiple.—One quantity is said to be an exact multiple of another when it contains the other by division an exact number of times. If in comparing two quantities there be a remainder after division, the relation between the two is expressed as far as two, three, four, or more places of decimals.

Sub-multiple.—One quantity is a sub-multiple of another when it divides into the other an exact integral number of times. Thus, for example :

10	is a multiple of the numbers	2 and 5
15	"	" " 3 " 5
20	"	" " 2, 4, 5, 10
100	"	" " 2, 4, 5, 10, 20, 25, 50

and 2 and 5 are sub-multiples of 10

3	"	5	"	"	15
2, 4, 5,	{		"	"	100
10, 20, etc.	}				

Ratio.—Ratio is the relation between quantities of the same kind ; and we compare two numbers by finding what multiple or sub-multiple one is of the other.

Thus the ratio of 75 to 100, written $75 : 100$, may be expressed by finding what fraction 75 is of 100, thus $\frac{75}{100} = \frac{3}{4}$; i.e. 75 is $\frac{3}{4}$ of

100. The ratio of 5 to 4, written $5 : 4$, may be expressed as $\frac{5}{4}$ or $1\frac{1}{4}$ times.

Generally, taking any two quantities A and B, the ratio of A to B is written $A : B$, or $\frac{A}{B}$; and the ratio of B to A is written $B : A$,

or $\frac{B}{A}$. The quantity expressed first is called the *antecedent* ; the second term being called the *consequent*.

When the two terms in a ratio can be expressed by the ratio of two integers, they are said to be **commensurable** or **commensurate**. If they cannot be so expressed they are said to be **incommensurable** or **incommensurate**.

Although two integers cannot be found to express the ratio of incommensurate quantities, yet a close approximation quite near enough for any practical purposes can be taken.

Thus, for example, the incommensurate ratio $\frac{\sqrt{2}}{2} = 0.707 : 1$ approximately. If a close approximation be required, the $\sqrt{2}$

may be taken to two, three, four, or more places according to the degree of accuracy required.

The value of a ratio is not altered by multiplying or dividing the antecedent and consequent by the same quantity.

Thus, $12 : 25 = \frac{12}{25}$ is the same in value as $12 \times 3 : 25 \times 3$, i.e., as $\frac{12 \times 3}{25 \times 3}$; or $a : b = ac : bc = \frac{ac}{bc}$.

Compounding Ratios.—Ratios are very often *compounded* with one another, and this is done by multiplying together the fractions which denote them; or, which is the same thing, by multiplying the antecedents together for a new antecedent and the consequents for a new consequent, and then comparing the resulting numbers.

Thus, compounding $\frac{25}{4}$ with $\frac{5}{7}$ we get $\frac{25 \times 5}{4 \times 7} = \frac{125}{28}$.

Symbolically, $\frac{a}{b}$ compounded with $\frac{c}{d} = \frac{ac}{bd}$ or $ac : bd$.

Compounding $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$, we get $\frac{ace}{bdf}$.

If a ratio be compounded with itself, the resulting ratio is spoken of as the "*duplicate ratio*" of the two quantities forming the first ratio.

Thus the duplicate ratio of $3 : 4$ is $9 : 16$.

Symbolically, the duplicate ratio of $a : b$ is $a^2 : b^2 = \frac{a^2}{b^2}$.

Also *triplicate ratio* of $a : b$ is $a^3 : b^3 = \frac{a^3}{b^3}$.

If the square roots of both terms in a ratio be compared, the result is called the *sub-duplicate* of the first ratio.

If the cube roots are compared we get the *sub-triplicate*.

Thus sub-duplicate ratio of $a : b = \sqrt{a} : \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, and sub-triplicate ratio of $a : b = \sqrt[3]{a} : \sqrt[3]{b} = \sqrt[3]{\frac{a}{b}}$.

Example 1.—A steam engine is employed to drive a dynamo, and the mechanical efficiency of the steam engine or the ratio of B.H.P. to I.H.P. is 85 per cent., and the efficiency of the dynamo or ratio of work applied to work potential of current from dynamo terminals is 90 per cent. What is the combined efficiency of the plant, measured from the I.H.P. to the current generated?

Combining the two ratios } $= \frac{85}{100} \times \frac{90}{100} = 76\frac{1}{2}$ per cent.
for combined efficiency

Example 2.—Taking a generating plant consisting of boiler, engine, and dynamo, and assuming—

- (i) Efficiency between engine stop valve and furnace
= 55 per cent.
- (ii) „ „ I.H.P. and engine stop valve
= 15 per cent.
- (iii) „ „ B.H.P. and I.H.P. = 85 „
- (iv) „ „ E.H.P. and B.H.P. = 90 „

Calculate the combined efficiency of the plant between the output and the furnace.

$$\text{Combined efficiency} = \frac{55}{100} \times \frac{15}{100} \times \frac{85}{100} \times \frac{90}{100} = 6.3 \text{ per cent.}$$

A ratio whose antecedent is less than the consequent has its value increased by adding the same quantity to both terms, and diminished by subtracting the same quantity; and vice versa, a ratio whose antecedent is greater than the consequent is diminished by addition and increased by subtraction.

Example.—In an engine the **apparent expansion ratio** is found by dividing the total volume swept out by the piston by the volume swept out by the piston from the commencement of its stroke to its position at cut-off. The **real expansion ratio** is found by adding to each of these quantities, before division, the **clearance volume** or space occupied by the steam when the piston is just beginning its stroke. Accepting the statement made above, it follows that *the real expansion ratio is less than the apparent expansion ratio*, and the student should show that the difference between these two ratios increases for higher grades of expansion.

$$\left. \begin{array}{l} \text{Apparent} \\ \text{expansion} \\ \text{ratio } r \end{array} \right\} = \frac{\text{Vol. swept out by piston}}{\text{Vol. by piston to cut-off}} = \frac{A \times L}{A \times l} = \frac{L}{l},$$

where L = stroke of piston and l = distance moved by piston from commencement of stroke to cut-off.

Now let c denote clearance volume, as an equivalent length of stroke, then

$$\text{Real expansion ratio } R = \frac{L+c}{l+c},$$

and it can easily be shown that, since $L > l$, then

$$\frac{L+c}{l+c} < \frac{L}{l}.$$

The real expansion ratio (R) can be expressed in terms of the apparent expansion ratio (r) and the ratio (r_1) of the clearance volume to the volume (V) swept out by the piston, for

$$R = \frac{V + r_1 V}{\frac{V}{r} + r_1 V} = \frac{V(1 + r_1)}{V(\frac{1}{r} + r_1)} = \frac{r(1 + r_1)}{1 + r r_1}$$

when the apparent expansion ratio = 5
and the clearance ratio $r_1 = \frac{1}{10}$,

then the real expansion ratio $R = \frac{5(1 + \frac{1}{10})}{1 + 5 \times \frac{1}{10}} = 3\frac{3}{4}$.

If $r = 10$ and $r_1 = \frac{1}{10}$,

then $R = \frac{10(1 + \frac{1}{10})}{1 + 10 \times \frac{1}{10}} = 5\frac{1}{2}$.

If we have a number of equal ratios $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$,

then each of these ratios is equal to $\sqrt[n]{\frac{pa^n + qc^n + re^n + \dots}{pb^n + qd^n + rf^n + \dots}}$

where p, q, r, n are any quantities.

If in the above result $p = q = r = n = 1$,

then $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$ or $\frac{a}{b} = \frac{c}{d} = \frac{a+d}{b+c}$.

Proportion.—Four numbers are said to form a proportion when the first bears to the second the same ratio as the third to the fourth. Proportion is thus made up of a pair of equal ratios.

For example, since $25 : 100$ and $3 : 12$ are two ratios of equal value, the antecedent in each case being $\frac{1}{4}$ of the consequent, we may state the equality thus:—as $25 : 100 = 3 : 12$, i.e. $\frac{25}{100} = \frac{3}{12}$.

The proportion is, however, generally stated thus,—as $25 : 100 :: 3 : 12$, and reads—as 25 is to 100 so is 3 to 12.

The two middle terms in a proportion (in this example 100 and 3) are called the “means,” and the two outside terms (in this example 25 and 12) the “extremes.”

If the student will multiply the means together in any proportion, and then the extremes, he will find he obtains two quantities which are numerically equal. This is expressed by the usual statement that—

In any proportion the product of the extremes is equal to the product of the means.

Symbolically, four quantities, a, b, c, d , are in proportion when a is the same multiple or sub-multiple of b that c is of d , i.e.,

$$\text{when } a : b :: c : d \text{ or } \frac{a}{b} = \frac{c}{d}.$$

Cross multiplying, we obtain $ad = bc$.

This relationship enables us to find any term in a proportion when the other three are given.

Quantities are said to be in **continued proportion** when the first is to the second as the second is to the third, and as the third is to the fourth, and so on.

3, 9, 27, 81 are in continued proportion, for $3 : 9 = 9 : 27 = 27 : 81$

Symbolically, a, b, c are in continued proportion when $a : b :: b : c$, and here $b \times b = a \times c$, i.e., $b^2 = ac$, $\therefore b = \sqrt{ac}$. b is called a "**mean proportion**" or "**geometric mean**" between a and c . Thus the mean proportion between two quantities is found by taking the square root of their product.

If four quantities a, b, c, d form a proportion, the relationships between the terms may be variously stated:—

$$(1) \quad a : b :: c : d \quad \text{or} \quad \frac{a}{b} = \frac{c}{d}.$$

$$(2) \quad b : a :: d : c \quad \text{or} \quad \frac{b}{a} = \frac{d}{c}.$$

$$(3) \quad a : c :: b : d \quad \text{or} \quad \frac{a}{c} = \frac{b}{d}.$$

$$(4) \quad c : a :: d : b \quad \text{or} \quad \frac{c}{a} = \frac{d}{b}.$$

Also, the following important results hold, and since these have practical applications, they should be remembered:—

$$(5) \quad \frac{a+b}{b} = \frac{c+d}{d}.$$

$$(6) \quad \frac{a-b}{b} = \frac{c-d}{d}.$$

$$(7) \quad \frac{a+b}{a-b} = \frac{c+d}{c-d} \quad \text{or} \quad \frac{a-b}{a+b} = \frac{c-d}{c+d}.$$

$$\text{If} \quad \frac{a}{b} = \frac{c}{d} \quad \text{and} \quad \frac{e}{f} = \frac{g}{h},$$

$$\text{then} \quad (8) \quad \frac{ae}{bf} = \frac{cg}{dh}.$$

Also if $\frac{a}{b} = \frac{c}{d}$ and $\frac{b}{x} = \frac{d}{y}$,

then (9) $\frac{a}{x} = \frac{c}{y}$.

Powers and Roots (Indices and Surds).—The process of finding the values of the powers of numbers is called **involution**. The number or symbol denoting the power is called the **index** (plural, indices) or exponent. The converse of this, or the determination of roots, is called **evolution**. Any expression under the radical sign $\sqrt[n]{}$ where n has any value is called a **surd**.

The square root of a given number cannot always be expressed exactly, and for most practical purposes it is sufficient to take the root to two, three, or more places of decimals. Students are recommended to commit the following roots to memory:—

$$\sqrt{2} = 1.4142 \qquad \sqrt{3} = 1.7321$$

$$\sqrt{5} = 2.2361 \qquad \sqrt{6} = 2.4495$$

$$\sqrt{7} = 2.6458 \qquad \sqrt{10} = 3.1623$$

$$\sqrt{11} = 3.3166.$$

The process of extracting the root of a number can often be simplified where the number whose root is required can be split into factors, the roots of one or more of which can be found exactly.

$$\text{Thus, } \sqrt{200} = \sqrt{100 \times 2} = \sqrt{100} \times \sqrt{2} = 10 \sqrt{2} = 14.142$$

$$\sqrt{135} = \sqrt{9 \times 15} = \sqrt{9} \times \sqrt{15} = 3 \sqrt{15} = 11.619$$

$$\sqrt[3]{192} = \sqrt[3]{64 \times 3} = 4 \sqrt[3]{3} = 5.768.$$

Surd forms may sometimes be simplified for calculation by *rationalising the denominator*.

$$\text{Example 1. } \sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{7} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2} \sqrt{14}.$$

$$\text{Example 2. } \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{1}{6} \sqrt{3} = 0.28868.$$

$$\text{Example 3. } \frac{a}{b\sqrt{c}} = \frac{a\sqrt{c}}{bc}.$$

Example 4. Simplify $\frac{5}{3 - \sqrt{5}}$.

Multiply numerator and denominator by $3 + \sqrt{5}$

thus,
$$\frac{5(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})} = \frac{15 + 5\sqrt{5}}{9 - 5} = 6.545.$$

Example 5.
$$\frac{2}{2\sqrt{3} - \sqrt{2}} = \frac{2(2\sqrt{3} + \sqrt{2})}{12 - 2} = .97568.$$

Example 6.
$$\frac{\sqrt{3} + 2\sqrt{2}}{5 + 2\sqrt{6}} = \frac{(5 - 2\sqrt{6})(\sqrt{3} + 2\sqrt{2})}{25 - 24}$$

$$= 5\sqrt{3} + 10\sqrt{2} - 2\sqrt{6} \times \sqrt{3} - 4\sqrt{6} \times \sqrt{2}$$

$$= 5\sqrt{3} + 10\sqrt{2} - 2\sqrt{18} - 4\sqrt{12}$$

$$= 5\sqrt{3} + 10\sqrt{2} - 2\sqrt{9 \times 2} - 4\sqrt{4 \times 3}$$

$$= 5\sqrt{3} + 10\sqrt{2} - 6\sqrt{2} - 8\sqrt{3}$$

$$= 4\sqrt{2} - 3\sqrt{3} = (4 \times 1.4142) - (3 \times 1.7321)$$

$$= 5.6568 - 5.1963 = 0.4605.$$

Example 7.
$$\frac{\sqrt{a} \pm b\sqrt{c}}{d \pm e\sqrt{f}} = \frac{(\sqrt{a} \pm b\sqrt{c})(d \mp e\sqrt{f})}{d^2 - e^2f}.$$

The powers of 10 are frequently used in expressing large numbers ; first note :

$$\begin{aligned} 10^1 &= 10 \\ 10^2 &= 100 \\ 10^3 &= 1000 \\ 10^4 &= 10000, \text{ etc.,} \end{aligned}$$

i.e. the index of the power equals the number of ciphers which follow the 1 ; and the student should make himself familiar with this short and ready method of writing large numbers, whether they be exact multiples of 10 or not. For example :—

$$\begin{aligned} 29,000,000 &= 29 \times 10^6 \\ 17,540,000 &= 1754 \times 10^4 \\ 1,250,000 &= 125 \times 10^4. \end{aligned}$$

Numbers less than unity can be expressed in a similar way
Note that

$$\begin{aligned} 0.1 &= \left(\frac{1}{10}\right) = 10^{-1} \\ 0.01 &= \left(\frac{1}{10}\right)^2 = 10^{-2} \\ 0.001 &= \left(\frac{1}{10}\right)^3 = 10^{-3} \\ 0.0001 &= \left(\frac{1}{10}\right)^4 = 10^{-4} \\ 0.00001 &= \left(\frac{1}{10}\right)^5 = 10^{-5} \\ 0.000001 &= \left(\frac{1}{10}\right)^6 = 10^{-6} \\ 0.0000001 &= \left(\frac{1}{10}\right)^7 = 10^{-7}; \end{aligned}$$

then

$$\begin{aligned} 0.00000075 &= 75 \times 10^{-8} \\ 0.0000015 &= 15 \times 10^{-7} \\ 0.00684 &= 68.4 \times 10^{-4}. \end{aligned}$$

Variation.—One quantity A is said to vary directly as another B when, as B successively takes different values, the corresponding values taken by A are proportional to the values of B.

The word “directly” is generally omitted, and we say briefly A varies as B; symbolically, $A \propto B$, where the symbol $\propto$ denotes *varies as*. Thus, for example, the strength of a rectangular beam of given material measured by the load W varies directly as, or is directly proportional to, the breadth, when other quantities—depth, span, fixing—are the same (or remain constant); symbolically, $W \propto b$.

One quantity A *varies inversely* as another B when A varies directly as the reciprocal of B, that is, when $A \propto \frac{1}{B}$.

Thus, for example, the strength of a rectangular beam of given material varies inversely as the span (L) or directly as $\frac{1}{L}$, when the other quantities—breadth, depth, fixing—remain constant. Symbolically, $W \propto \frac{1}{L}$.

If one quantity A varies as another B,

then $A = B \times \text{some constant quantity};$

that is, $A = B \times p$, when p is some constant.

When one quantity A varies as another B when a third C is constant, and when A varies as C when B is constant,

then $A \propto B \times C,$

and $A = B \times C \times \text{some constant}.$

Thus, for example, the strength W of rectangular beams of a given span supported and loaded in a like manner varies as the

square of the depth (d^2) when the breadth b is constant, and as b when d^2 (and $\therefore d$) is constant; hence strength varies as $d^2 \times b$; or strength $= d^2 \times b \times \text{constant}$.

Symbolically, $W \propto d^2$ when b is constant.

$W \propto b$ „ d^2 „

$\therefore W = d^2 b \times \text{constant}$.

Taking the span into account, we know the strength to vary inversely as L , or directly as $\frac{1}{L}$, when $b \times d^2$ are constant.

Hence strength $W \propto \frac{bd^2}{L}$,

or $W = \frac{bd^2}{L} \times \text{constant}$.

Numerous illustrations of variation are constantly occurring in practical calculations. We summarise a few; and give a number of practical exercises to be worked by the student, at the end of this chapter.

The area of a triangle or parallelogram varies as base $\times$ altitude.

The area of similar plane forms varies as the square of lines similarly drawn across the figures.

The area of a circle of expanding radius varies as the square of the radius, and is equal to the (radius) $^2 \times \text{constant} = r^2 \times \pi$.

The volume of similar solids varies as the cube of similar dimensions. Thus, for example, the volume of a sphere of expanding diameter D varies as D^3 and $= D^3 \times \text{constant} = D^3 \times \frac{\pi}{6}$.

The volume of cone or cylinder $\propto D^2 \times h$, and $= D^2 \times h \times \text{constant}$, where D represents diameter of base, and h the altitude.

The strength of rectangular beams for any given material $\propto b \times d^2 \times \frac{1}{L}$, and $= \frac{bd^2}{L} \times \text{constant}$.

The strength of solid cylindrical shafts of given material, as measured by load W on crank pin, $\propto D^3$ when the length of the crank is constant, or as $D^3 \times \frac{1}{L}$ when both dimensions vary.

The strength of hollow cylindrical shafts varies as $\frac{D^4 - d^4}{D}$ when the length of crank L is constant. D represents the outside,

d the inside diameter. When diameters and crank vary, the strength varies as $\frac{D^4 - d^4}{D} \times \frac{1}{L}$, and $= \frac{D^4 - d^4}{DL} \times \text{constant}$.

To compare the strength of a solid cylindrical shaft with a hollow shaft of the same kind of material, we compare d^3 with $\frac{D^4 - d^4}{D}$, when both have the same length of crank; or by comparing $\frac{d^3}{L}$ with $\frac{D^4 - d^4}{DL}$, when the cranks vary as well.

The deflection δ of rectangular beams varies inversely as the breadth, inversely as the cube of the depth, and directly as the cube of length of the span. Hence δ varies as $\frac{L^3}{bd^3}$, and $\delta = \frac{L^3}{bd^3} \times \text{constant}$.

The twist or angle of torsion, n° , of a solid cylindrical shaft varies inversely as the fourth power of the diameter D , directly as the length, and directly as the twisting moment T .

For the solid shaft, $n^\circ \propto \frac{T \cdot L}{d^4}$ or $n^\circ = \frac{T \cdot L}{d^4} \times \text{constant}$.

For the hollow shaft, $n^\circ \propto \frac{T \cdot L}{D^4 - d^4}$ or $n^\circ = \frac{T \cdot L}{D^4 - d^4} \times \text{constant}$.

The temperature remaining the same, the pressure P of a perfect gas varies inversely as the volume V .

Hence $P \propto \frac{1}{V}$, or $P = \frac{1}{V} \times \text{constant}$, or $P \times V = \text{constant}$.

EXERCISES.

1. Find the value of the following products correct to four significant figures:—

$$5432 \times 678, \quad 54 \cdot 32 \times 26 \cdot 78, \quad 5 \cdot 76 \times 0 \cdot 192, \quad 78 \cdot 324 \times 9 \cdot 56.$$

Ans. 3683000, 1455, 1·106, 748·8.

2. Find the value of the following quotients correct to four significant figures:—

$$87069 \div 132, \quad 117338 \cdot 84 \div 379, \quad 185 \div 3 \cdot 65, \quad 8651 \cdot 17 \div 4 \cdot 905.$$

Ans. 659·6, 309·6, 50·68, 1764.

3. Find the value of the following products:—

$$84 \times 25, \quad 126 \times 125, \quad 157 \times 102, \quad 867 \times 1\cdot001, \quad 56 \times 98.$$

Ans. 2100, 15750, 16014, 867·867, 5488.

4. Find the value of the following quotients:—

$$44 \div 2\cdot5, \quad 64 \div 0\cdot375, \quad 125 \div 6\cdot25.$$

Ans. 17·6, $170\frac{2}{3}$, 20.

5. Multiply 21357 by 72366 (in three lines).

Ans. 1545520662.

And 811·15 by 12·14411 (in three lines).

Ans. 9850·6948265.

6. Find the approximate values of the following correct to 1 per cent. (Most of these should be worked mentally.)

$$9\cdot95 \times 10\cdot05, \quad 9\cdot75 \times 102\cdot5, \quad 4\cdot9 \times 5\cdot1, \quad 5\cdot95 \times 0\cdot605, \\ (151)^2, \quad (10\cdot8)^2, \quad (1\cdot01)^3, \quad (9\cdot8)^3, \quad (0\cdot48)^3.$$

Ans. 100, 1000, 25, 3·6, 22500, 116·6, 1·03, 940, 0·11.

7. A boiler under a 10 hours' trial used 4120 lbs. of coal, and the ash and clinker from the coal weighed 114 lbs. What is the ratio of ash and clinker to coal, expressed as a percentage?

Ans. 2·76 per cent.

8. Sea-water contains on an average 5 ounces of salt per gallon. What is the ratio by weight, having given the weight of a gallon of water as 10 lbs.?

Ans. $\frac{1}{32}$.

9. The coefficient of displacement of a vessel is the ratio that the immersed volume bears to the volume of a rectangular having the same three principal dimensions. Taking the case of a vessel whose immersed hull is 400 ft. $\times$ 45 ft. $\times$ 22 ft., and whose displacement is 5500 tons, calculate the coefficient.

Ans. 0·498.

10. A bonus of £95 was divided among four engineers in the ratios of their wages. The wages of the chief, the second, the third and fourth engineers were £20, £15, £12, and £8 respectively. Calculate how much bonus each received.

Ans. £34·55, £25·9, £20·7, and £13·8.

11. A Babbit's metal may contain 10 parts tin, 1 part antimony, and 15 parts copper. Find approximately what weight of Babbit's metal can be made from 120 lbs. tin,

20 lbs. copper, and 15 lbs. antimony. If tin costs 1s. 5d. per lb., copper 10d., and antimony 2s., what would be the cost of the metal to make 1 cwt. of Babbitt's metal?

Ans. $34\frac{3}{4}$ lbs. and £6, 3s. 6d.

12. The areas of the three pistons of a triple-expansion engine were fixed in the ratios of 1 : 2·7 : 5·6, and the area of the H.P. cylinder was 200 sq. in. Calculate the areas of the I.P. and L.P. pistons.

Ans. 540 and 1120.

13. The **specific gravity** or **relative density** of a substance is the ratio of its weight to the weight of an equal volume of water. Taking the weight of a pint of pure water (= 27·274 c. in.) as a pound and a quarter, calculate the specific gravities of the following metals:—

Cast iron	(1 c. in. weighs 0·263 lb.)
Wrought iron	(" " 0·28 ")
Steel (mild)	(" " 0·288 ")
Brass (average)	(" " 0·292 ")
Copper	(" " 0·32 ")
Lead	(" " 0·41 ")

Ans. 5·7, 6·1, 6·3, 6·4, 7, 8·9.

14. The **thermal efficiency** of an oil engine was found to be $\frac{1}{100}$, while the **mechanical efficiency** was 85 per cent. Combine these two ratios for the efficiency from fuel to brake.

Ans. 12·75 per cent.

15. A steam engine and electric generating plant gave under test the following efficiencies:—

- (i) Boiler and steam-pipe efficiency, 60 per cent.
- (ii) Thermal efficiency of engine, 20 "
- (iii) Mechanical efficiency of engine, 88 "
- (iv) Efficiency of dynamo, 92 "

Combine these ratios for the efficiency between the boiler furnace and the dynamo terminals.

Ans. 9·7 per cent.

16. The stroke of an engine is 20 in., and the clearance, expressed as an equivalent length of stroke, is 1·24 in. The cut-off takes place after the engine piston has moved 8 in. from the commencement of the stroke. Calculate (i) the ratio of the clearance volume to the piston displacement; (ii) the apparent expansion ratio; (iii) the real expansion ratio.

Ans. 0·062, 2·5, 2·345.

17. Find approximately what number is to 140 as $4\frac{1}{2}$: $5\frac{1}{2}$.

Ans. 114·54.

18. Find the value of x in each of the proportions :—

(i) $x : 40 :: 67.2 : 50$,

(ii) $6 : x :: 30 : 535$,

(iii) $\frac{22}{374} = \frac{x}{12}$,

(iv) $\frac{3}{4} = \frac{6}{x}$,

(v) $3\frac{1}{4} : \frac{1}{4} = \frac{x}{20}$.

Ans. $\left\{ \begin{array}{l} \text{(i)} 53.76. \\ \text{(ii)} 107. \\ \text{(iii)} \frac{1}{17}. \\ \text{(iv)} 8. \\ \text{(v)} 260. \end{array} \right.$

19. Find the arithmetic and geometric means of 204 and 120.
(*Note.*—The arithmetic mean of two quantities = $\frac{1}{2}$ their sum.)

Ans. 162, 156.4.

20. Find the mean or average of the following numbers :—

12.34, 16.51, 13.23, 5.67, 0.97
9.58, 5.64, 2.10, 105.12, 1.11.

Ans. 17.127.

21. A piece of material is weighed in the two pans of an ordinary balance which is out of adjustment. In one pan its weight is given as 150 lbs. and in the other 142 $\frac{1}{2}$ lbs. Calculate its correct weight. (*Note.*—The true weight is the geometric mean of these two apparent weights.)

Ans. 146.2.

22. A steamer on its trial trip covers the measured mile with the tide in 3 minutes 20 seconds, and in 4 minutes 15 seconds against the tide. What is its correct rate of steaming in knots per hour? (1 Admiralty knot = 6080 ft.).

Ans. 13.7.

23. Find the value, to not more than four places of decimals, of :—

$$\frac{1}{\sqrt{3} - \sqrt{2}}, \quad \frac{\sqrt{49}}{\sqrt{5} - \sqrt{1}}, \quad \frac{\sqrt{17}}{\sqrt{10} - \sqrt{6}}, \quad \frac{\sqrt{4}}{3}, \quad \frac{\sqrt{81}}{3 - \sqrt{2}}, \quad \frac{\sqrt{49}}{4\sqrt{2} \pm 2\sqrt{3}}.$$

Ans. $\frac{1}{2}$, $\frac{7}{8}$, $1\frac{1}{3}$, 0.8944, 0.85, 2.06.

Ans. 3.1456, 1.809, 6.14, 2.04 and 0.779, 0.189 and 0.79.

24. Express the following numbers, using power of 10 :—

3,460,000, 12600, 0.00034.

Ans. 3.46×10^6 , 126×10^3 , 34×10^{-5} .

25. The area of a circle 20 in. diameter is 314.16 sq. in. Calculate the area of a circle whose diameter is 16 $\frac{1}{4}$ in.

Ans. 207.39.

26. Compare the areas of two circles whose diameters are $3\frac{1}{2}$ in. and 7 in. respectively. *Ans.* 1 : 4.

27. The diameters of two circles are a and b respectively, where $b = 2a$. How many circles of diameter a give a total area equal to the circle of diameter b ? *Ans.* 4.

28. Compare the areas of two circles whose diameters are 2.34 and 3.16 respectively. *Ans.* 1 : 1.8.

29. Compare the volume of a sphere $2\frac{1}{4}$ in. diameter with the volume of a sphere 3.71 in. diameter. *Ans.* 1 : 4.5.

30. Three spheres, radii r , $2r$, and $\frac{5r}{2}$. Compare their volumes. If $r = \frac{1}{2}$ in., the volume of the first = 0.5236. Calculate the volumes of the other two. *Ans.* 1 : 8 : 15.62, 4.188 c. in., 8.2 c. in.

31. Compare the torsional strength of a 3-in. shaft having a 20-in. crank with a 4-in. shaft having a 10-in. crank. *Ans.* 1 : 4.74.

32. Compare the strength of a hollow shaft whose two diameters are 10 in. and 5 in. with a solid shaft 8 in. diameter. *Ans.* 1.83 : 1.

33. Compare the strength of a rectangular beam 5 in. wide, 10 in. deep, and 15 ft. span with that of a beam of the same kind of material whose dimensions are 6 in. wide $\times$ 12 in. deep $\times$ 25 ft. span. *Ans.* 1 : 1.036.

34. If a 3-in. shaft will transmit 25 H.P. when rotating at 120 revolutions per minute, what H.P. will a $3\frac{1}{2}$ -in. shaft transmit at 100 r.p.m. (Note.—*H.P. transmitted* $\propto d^3 \times \text{speed}$.) *Ans.* 33.08.

35. If the span of a rectangular beam is increased until the safe carrying capacity of the beam is measured by the weight of the beam, what would be the safe carrying capacity of the beam were its dimensions halved in every respect?

Ans. The beam would support a load equal to the reduced weight of the beam.

CHAPTER II.

TECHNICAL MENSURATION.

If any side of a triangle be produced, the *exterior angle is equal to the sum of the two interior opposite angles.*

$$\angle A = \angle B + \angle C \text{ (fig. 1).}$$

The sum of the three interior angles of any triangle is equal to two right angles.

$$\angle A + \angle B + \angle C = 180^\circ \text{ (fig. 2).}$$

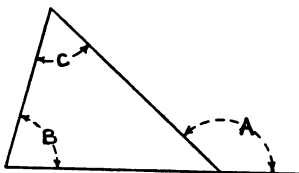


FIG. 1.

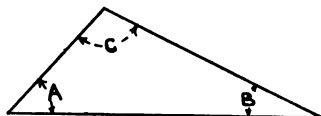


FIG. 2.

The sum of the interior angles of any rectilineal figure is equal to twice as many right angles as the figure has sides, less 4. Thus, for example, in the irregular pentagon (fig. 3),

$$\angle A + \angle B + \angle C + \angle D + \angle E = 2 \times 5 \times 90^\circ - 4 \times 90^\circ ;$$

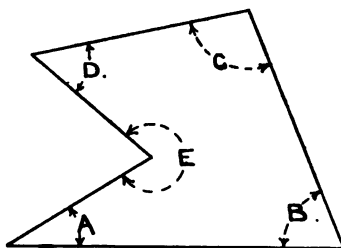


FIG. 3.

or generally, when N = number of sides,

$$\text{Sum of interior angles} = 90^\circ \times (2N - 4).$$

The angles in the same segment of a circle are equal to one another. Thus, if P_1, P_2 are any two points on the arc of the segment, then the angle at P_1 is equal to the angle at P_2 (figs. 4A and 4B).

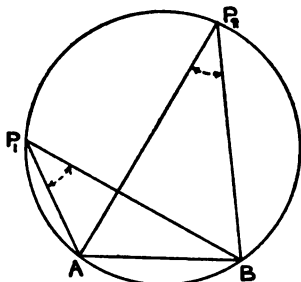


FIG. 4A.

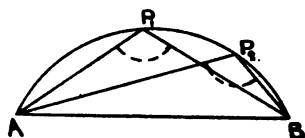


FIG. 4B.

The angle at the centre of a circle is double the angle at the circumference when both are formed by lines drawn from the extremities of the same (or an equal) base or arc. Thus, if P (fig. 5) is any point on the circumference,

$$\angle \text{ at } P = \frac{1}{2} \angle \text{ at } O.$$

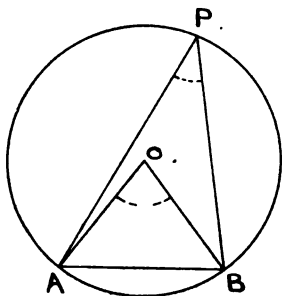


FIG. 5.

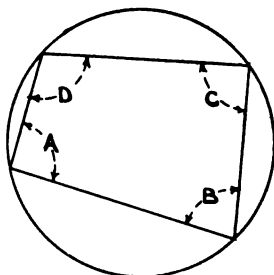


FIG. 6.

The two opposite angles of any four-sided figure inscribed in a circle are together equal to two right angles.

Thus,

$$\angle A + \angle C = 180^\circ,$$

and

$$\angle B + \angle D = 180^\circ \text{ (fig. 6).}$$

The angle in a semicircle is a right angle.

For any position of the point P on the semicircumference,

$$\angle P = 90^\circ \text{ (fig. 7).}$$

Since the three angles of any triangle make up two right angles, it follows that $\angle A + \angle B = 90^\circ$, and that $\angle A$ and $\angle B$ are complementary (fig. 7).

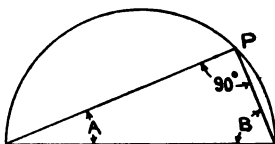


FIG. 7.

Forms with a Right Angle.—*The square on the hypotenuse is equal to the sum of the squares on the sides* (fig. 8).

$$H^2 = B^2 + P^2 \quad \text{or} \quad H = \sqrt{B^2 + P^2}$$

$$B = \sqrt{H^2 - P^2} \quad \text{and} \quad P = \sqrt{H^2 - B^2}.$$

Since $H^2 = B^2 + P^2$,

$$\frac{\pi}{4}H^2 = \frac{\pi}{4}B^2 + \frac{\pi}{4}P^2;$$

i.e., *the circle described on the hypotenuse as diameter is equal in area to the sum of the circles described on the sides as diameters* (fig. 8A).

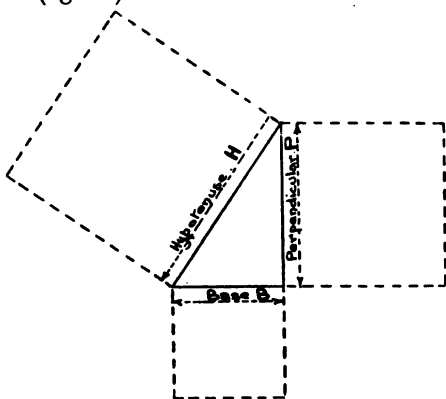


FIG. 8.

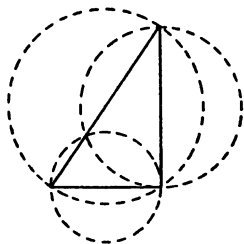


FIG. 8A.

Similar plane forms are those which are of the same form, though not necessarily of the same size. Similar plane forms fulfil the following two conditions:—

- (a) *Their corresponding angles must be equal.*
- (b) *Their sides, or any corresponding lines drawn across the figures, are proportional.*

Take, for instance, two triangles ABC, DEF, such that angle A = angle D, angle B = angle E, and angle C = angle F. Euclid proves that the sides are proportional and the triangles are said to be similar. AB is the same fraction or multiple of AC that DE is of DF.

Similarly,

$$BC : AC :: EF : DF$$

$$AB : BC :: ED : EF, \text{ etc.}$$

Example.—Let the sides of the triangle ABC (fig. 9A) be the dimensions shown, and let the base DF in triangle DEF = 7 in. Find the lengths of the other two sides, ED, EF.

$$AB : AC :: ED : DF$$

$$6 : 10 :: ED : 7 ;$$

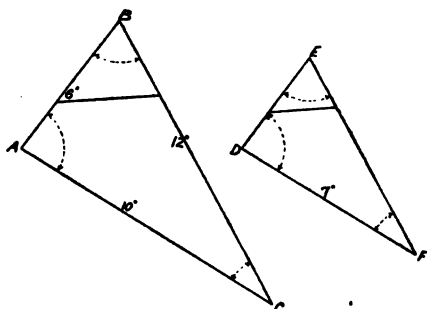


FIG. 9A.

FIG. 9B.

and since in a proportion the product of the extremes is equal to the product of the means, $\therefore 10 \times ED = 7 \times 6 = 42$. $ED = \frac{42}{10} = 4.2$ in.

Again,

$$AB : BC :: ED : EF,$$

$$\therefore 6 : 12 :: 4.2 : EF.$$

$$12 \times 4.2 = 6 \times EF.$$

$$EF = \frac{12 \times 4.2}{6} = 8.4 \text{ in. } \text{Ans.}$$

Figure ABCDE (fig. 10A) is similar to figure $A_1B_1C_1D_1E_1$ (fig. 10B) if the corresponding angles are equal. Any two sides in one figure form the same ratio as the corresponding two sides in the other; or if any line, such as FG, be drawn across ABCDE, and a corresponding line F_1G_1 across $A_1B_1C_1D_1E_1$, such that $\frac{A_1F_1}{A_1E_1} = \frac{AF}{AE}$ and $\frac{C_1G_1}{C_1D_1} = \frac{CG}{CD}$; then these two lines form a proportion when compared with any other two corresponding sides.

Thus, $\frac{FG}{F_1G_1} = \frac{AB}{A_1B_1}$ or $\frac{F_1G_1}{B_1C_1} = \frac{FG}{BC}$, etc., etc.

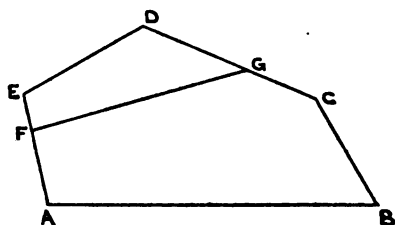


FIG. 10A.

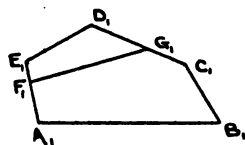


FIG. 10B.

The *plan*, drawn to scale, of a plane area (such as a piece of land) is *similar in form to the actual area represented*; consequently the ratio of any two dimensions on the plan is equal to the ratio of the corresponding two on the actual area.

Circles are similar, and the circumferences are proportional to their diameters. Also, any chords drawn across the circles dividing the diameters in the same ratio are in proportion to the respective diameters.

Sectors of circles are similar when their radii contain equal angles; their arcs are consequently proportional to their radii.

The student should draw out for himself a number of similar figures, and, by actual measurement and comparison, show that the corresponding sides or lines in the two figures, taken two and two, are proportional.

Euclid proves that if two chords of a circle intersect, *the product of the segments of one of them is equal to the product of the segments of the other.*

Hence, if AC, BD (fig. 11A) be two intersecting chords in the circle ABCD, we have the relation: length of AE $\times$ length of EC = length of BE $\times$ length of ED, or briefly,

$$AE \cdot EC = BE \cdot ED.$$

The student may satisfy himself of the truth of this by actually measuring, and taking the product of these segments.

One case of this general relation is worthy of notice, namely, that in which the two straight lines are at right angles to one another, and where one of them passes through the centre of the circle. Here the relation is $AE \cdot EC = BE \cdot ED$; and since AE is in this case equal to EC, it follows that the product of the segments AE and EC is equal to the square of one of them.

Thus, $(AE)^2$ or $(EC)^2 = BE \cdot ED$.

Symbolically, $\left(\frac{c}{2}\right)^2 = a \cdot b$,

$$\text{i.e.} \quad \frac{c^2}{4} = a \cdot b$$

$$c^2 = 4ab.$$

This result has many practical applications, and two or three examples will be given at the end of the chapter.

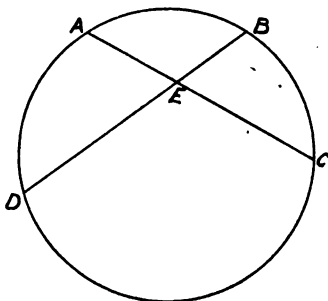


FIG. 11A.

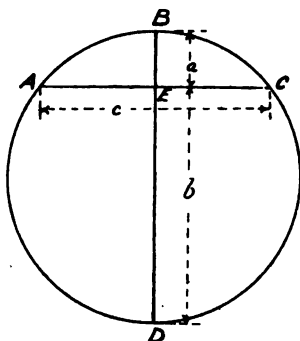


FIG. 11B.

The **Perimeter** or **Circumference** of any plane figure, regarded as a measurable quantity, is the distance round it or the length of its boundary. The term *girth* is often used in practice to denote the distance round a solid object. Thus, for example, we speak of the girth of a steam pipe.

Circumference of Circle.—The ratio of circumference to diameter is the same for all circles, and this ratio is denoted by the Greek letter π . Thus,

$$\frac{\text{Circumference}}{\text{Diameter (D)}} = \pi \quad \text{or} \quad \frac{\text{circumference}}{\text{radius}} = 2\pi.$$

Hence, circumference $= \pi D = 2\pi r$.

The value of π commonly used $= 3.1416$, but for small circles $3\frac{1}{7}$ or $\frac{22}{7}$ is sufficiently accurate. For very large circles, $\pi = 3.14159265 \dots$ etc. may be used for greater accuracy. The error when π is taken as 3.1416 is less than $+\frac{1}{1000}$ in. in a 100-foot circle, and when $\pi = \frac{22}{7}$ about $\frac{1}{7}$ in. in a 10-foot circle.

The relation between the circumference and the diameter may be verified approximately by taking a fine thread, and measuring

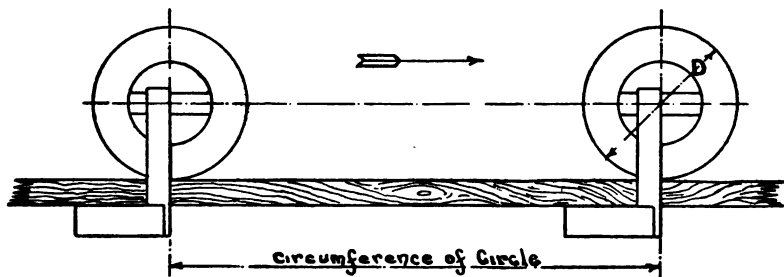


FIG. 12.

the circumference of a turned flange or pulley or other cylindrical object, or by taking a ring of wood (see fig. 12) accurately turned

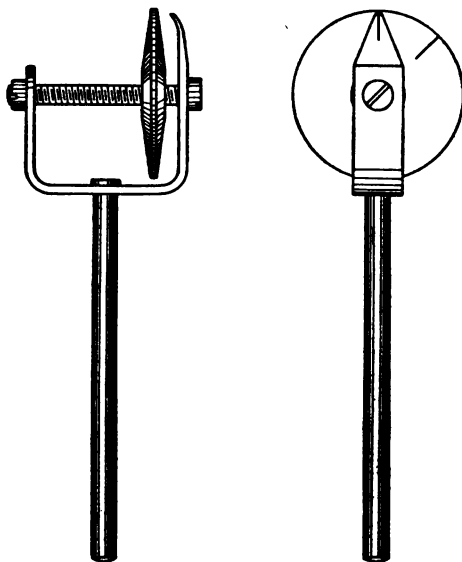


FIG. 13.

on the outside, and rolling it just once round on a level surface. The circumference is then laid out, as it were, and can be measured and compared with the diameter or radius.

The Opisometer.—The perimeters of regular and of irregular curved figures *may be determined by the Opisometer* (fig. 13).

The opisometer consists of a screw of fine pitch, mounted between suitable supports. On this is screwed a round disc turned to a feather edge, and milled in order to prevent slipping. A fine line is engraved on one side, and when the disc is screwed as far as it will go, this line coincides with the zero line on the support. The whole is provided with a handle for convenience in using the instrument.

The method of using the instrument is as follows:—Screw up the disc till the zero marks coincide, and then wheel the disc round the figure whose perimeter it is wished to determine. Then the distance travelled by the zero point of the disc is equal to the

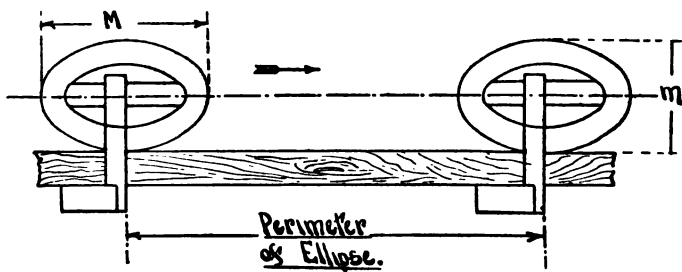


FIG. 14.

perimeter of the figure. Now roll the instrument in a reverse direction along a scale, until the zero marks again coincide; then distance moved along the scale is equal to the perimeter of the figure.

If the perimeter of the figure be great, it is necessary to divide it into several portions, in order that it shall come within the scope of the instrument. The length of each portion is determined as above, and the results are added together.

$$\text{Perimeter of Ellipse}^1 = \pi M \left\{ 0.636 + 0.364 \left(\frac{m}{M} \right)^2 \right\}.$$

Calculation of Areas.

Area.—*Area*, as quantity of surface, is expressed in terms of certain unit squares, the size of the unit chosen depending to some extent upon the size of the surface to be measured.

¹ The authors are indebted to Mr F. Lord for this rule. The rule usually given in text-books is quite incorrect.

Units of Area.—Our smallest unit of area is the square inch, and is the quantity of surface enclosed by the sides of a square of 1-inch side. We have also the square foot, square yard, the acre, the square mile, etc.

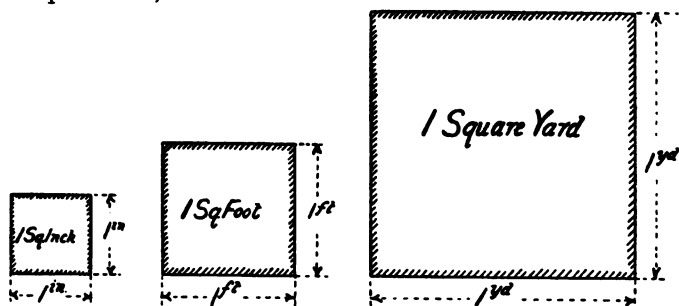


FIG. 15A.

Metric Units.—In the metric system we have the *square metre*, the *square decimetre*, the *square centimetre*, etc.

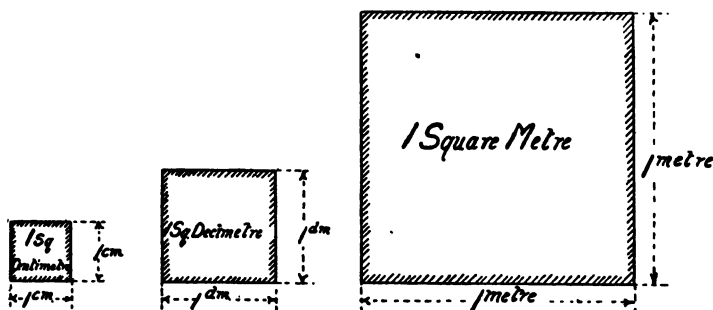


FIG. 15B.

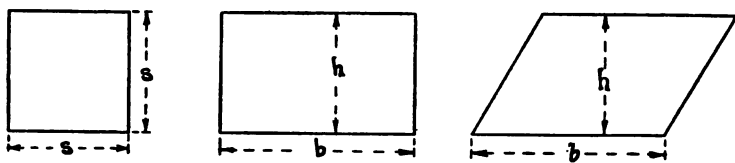


FIG. 16.

Area of *parallelogram* = *base* $\times$ *altitude* = bh .

Area of parallelogram is also = product of two sides into sine of included angle.

Area of triangle :—

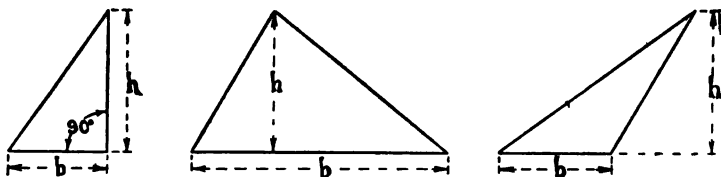


FIG. 17.

(1) Area of each triangle (fig. 17) = $\frac{1}{2}bh$.

(2) Let $S = \frac{1}{2}(a + b + c)$, then

Area of triangle (fig. 18) = $\sqrt{S(S-a)(S-b)(S-c)}$.

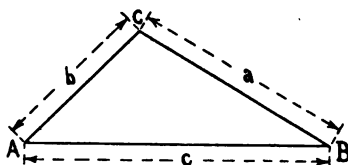


FIG. 18.

(3) Area of triangle (fig. 18) = $\frac{1}{2}$ the product of any two sides into the sine of the included angle.

$$\frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B = \frac{1}{2}bc \sin A.$$

Area of irregular quadrilateral figure (figs. 19A, 19B) :—

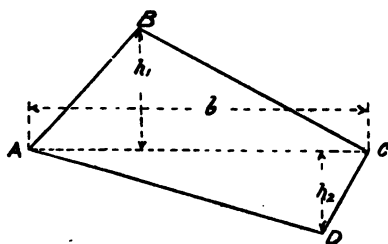


FIG. 19A.

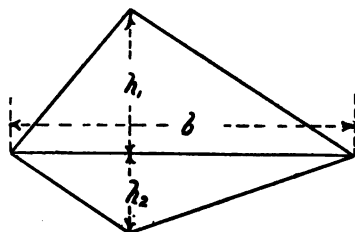


FIG. 19B.

$$\text{Area} = \frac{1}{2}(h_1 + h_2)b.$$

Area of *trapezium* (shaded) (fig. 19c) = average width or length multiplied by distance between parallel sides,

$$= wp = \frac{1}{2}(w_1 + w_2)p.$$

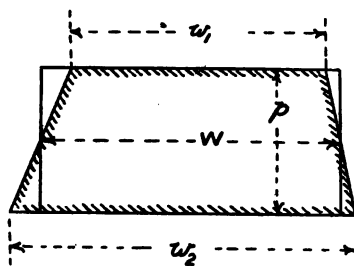


FIG. 19c.

Area of *irregular rectilinear figure* :—

The general method is to divide into triangles as in fig. 20, and then take the sum of their several areas. Special cases arise when it may be convenient to calculate the area from trapeziums and

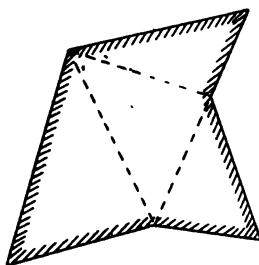


FIG. 20.

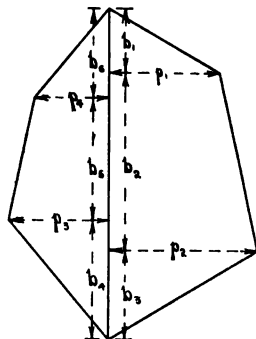


FIG. 21.

triangles—the dimensions being known or determined. Thus in fig. 21 the vertical line A represents a measured chain line, from which the offsets p_1, p_2, p_3 , etc., are measured. Then,

$$\text{Area} = \frac{1}{2}p_1b_1 + \frac{1}{2}(p_1 + p_2)b_2 + \frac{1}{2}p_2b_3 + \frac{1}{2}p_3b_4 + \frac{1}{2}(p_3 + p_4)b_5 + \frac{1}{2}p_4b_6.$$

Area bounded by an Irregular Curve.

Simpson's Rule on the Parabolic Method.—Add together the first ordinate, the last ordinate, twice, the sum of the other odd

ordinates, and four times the sum of the even ordinates. Multiply this sum by the extreme length of the diagram, and divide the

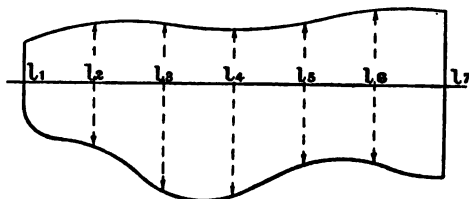


FIG. 22A.

result by three times the number into which the diagram is sectioned.

For example, area of figs. 22A, B, and C

$$= \frac{L}{3 \times 6} [l_1 l_7 + 2(l_3 + l_6) + 4(l_2 + l_4 + l_5)].$$

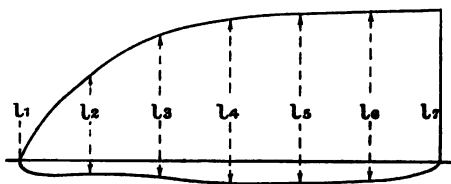


FIG. 22B.

Note.—In the application of this rule it is advisable to divide the diagram into an *even* number of parts, 4 or 6 or 8 or 12, etc.; the greater the number the more accurate will be the determination of the area.

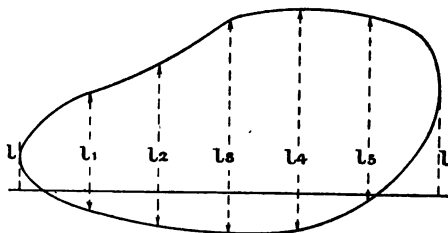


FIG. 22C.

In fig. 22A the first and last ordinates have definite values; in fig. 22B the first ordinate is nil; and in fig. 22C both the first and last ordinates are zero.

Ten Ordinates Method.—Divide the diagram into ten strips of equal width, and measure the middle height of each.

Then,

$$\text{Area} = \frac{h_1 + h_2 + h_3 + \dots + h_{10}}{10} \times L.$$

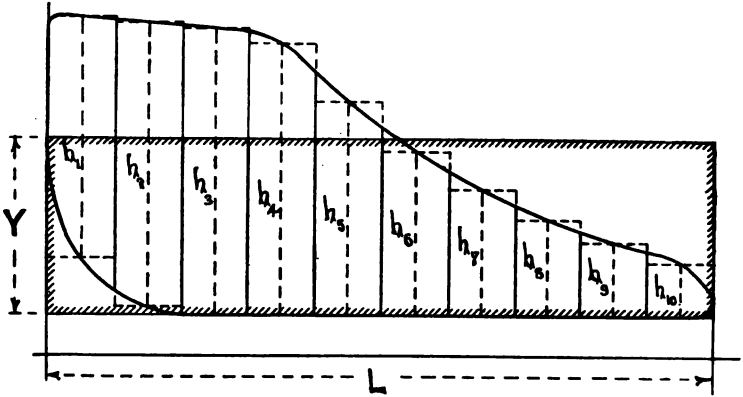


FIG. 23A.

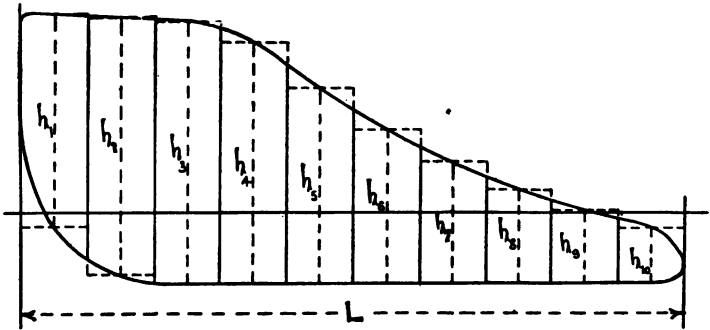


FIG. 23B.

The ordinates should be summed up by marking them off end-to-end on a long strip of paper. It should be noticed that $\frac{1}{10}$ the sum of the ordinates gives the width Y of a rectangle whose area is equal to the area of the curved diagram.

The diagram may be divided into any convenient number of parts—the greater the number the nearer the approach to accuracy.

Approximation to a Rectilinear Figure.—Sometimes the diagram can be blocked out to a straight line diagram as ABCD (see fig. 23c), whose area can readily be calculated.

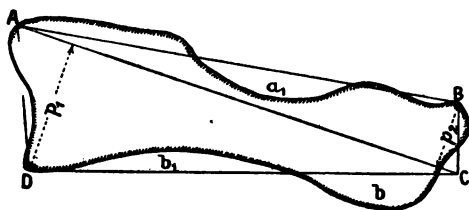


FIG. 23c.

Surface of Revolution.—If a line, straight or curved, revolve about any straight line in its plane, as axis, the area of the surface swept out is found by multiplying the length of the revolving line by the length of path described by its centre of gravity.

In fig. 24, AB is any line having its centre of gravity at G.

Surface swept out = length of curve $\times 2\pi r$.

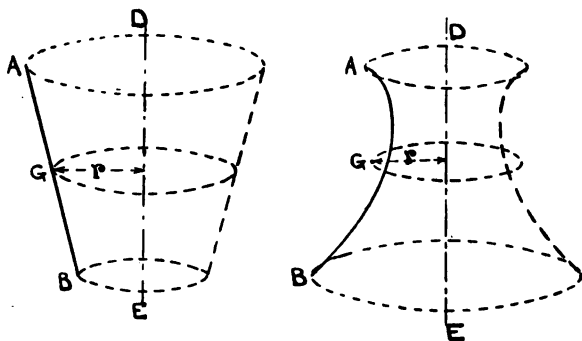


FIG. 24.

Area by the Planimeter.—This instrument is represented in fig. 25A, and consists essentially of two metal bars A and B, and a recording mechanism. One bar carries the recording mechanism, fixed on to a sliding piece D. The bar B has at one end a needle point C, about which the instrument can turn. Just above this point a steadying weight W is attached. The other end of this bar is pivoted in the sliding piece D, which carries the recording mechanism. This sliding piece and its mechanism can be placed at any position along the carrier bar A, providing the screw S is loose. The end of the bar A is terminated by a not too sharp

steel point P, which is caused to move round the boundary of the figure whose area is required. Along the bar A are several marks; at H, for instance, we have marked against the vertical line "01 □", which means that if the line F on the sliding piece be placed coincident with the vertical line H, the instrument is set for recording an area in square inches down to $\frac{1}{100}$ of a square

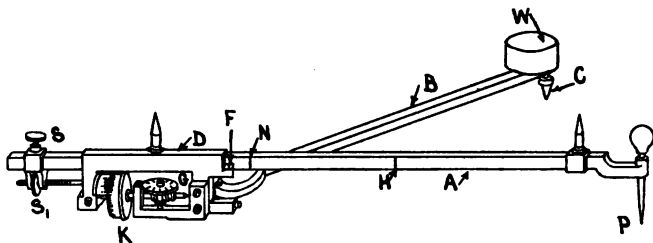


FIG. 25A.—Amaler's Planimeter.

inch. If, however, F be placed coincident with N, then the area is recorded in the metric measure.

In setting, when the sliding piece is brought to about its correct position along the bar, S is screwed down, and the fine adjustment screw S₁ is then used to bring the piece into its exact position with the two lines coincident.

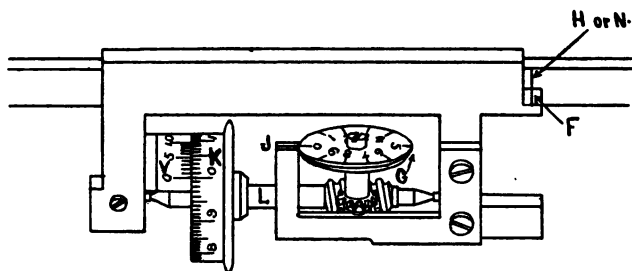


FIG. 25B.—The Recording Mechanism.

An enlarged view of the recording mechanism is shown in fig. 25B, and consists of a hollow drum K, attached rigidly to a small horizontal spindle L, which is centred at each end in conical bearings. Gearing with this spindle by means of a small worm and worm wheel is a horizontal counting disc G, so as to rotate once for every ten revolutions of the roller drum. By reference to the illustration, it will be seen that this disc is divided circumferentially into ten equal divisions, and each division as it

passes the mark engraved on the limb at J records ten square units of area. Thus, for example, if the third division has passed the mark at J, over thirty square units are recorded. If the counting disc were to turn once round, then $10 \times 10 = 100$ square units have been traced out.

The roller drum K is also divided into ten equal divisions round the circumference, which are numbered 1 to 10; and as each division passes the zero mark on the vernier scale V, a square unit of area is recorded—the number of units for a given position of the roller drum being indicated by the last figure which has passed the zero. Further, each of the ten divisions is divided into ten

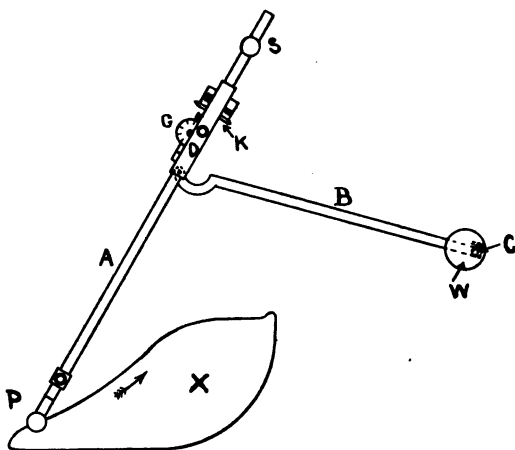


FIG. 25c.

equal parts, so that we get areas to tenths of a square unit by noticing how many of these smaller divisions have passed the vernier zero mark; and lastly, the area can be taken to $\frac{1}{100}$ of a square unit by means of the vernier. Scan along the vernier till a line is found coincident with a line on the roller drum, and then notice the number of small divisions from the zero mark. This gives so many hundredths of square units of area.

Instructions for using the Instrument.—Fig. 25c shows the instrument in position ready for finding the area of any plane figure, such as an indicator diagram.

The instrument rests on the roller wheel, the fixed point C, and the tracing point P. When the point P is tracing round the diagram, the roller wheel rotates sometimes in one direction, sometimes in the other direction, and sometimes it is stationary.

The bar B should be placed at such an angle with the bar A as to allow perfect freedom for the tracing point P in its motion round the curve or boundary. The needle point C is pressed into the paper and the point P carefully traced round the curve, starting from a well-defined point on the diagram in a clockwise direction.

Before tracing round the diagram, the reading of the mechanism should be noted, unless readings are brought to zero before commencing to trace. After the point P has completed the perimeter of the figure, the final reading should be noted, then the difference of the two readings gives the area.

Suppose, on looking at the counting disc, we find that 2 was the last figure that has passed the mark on the limb J, this tells us that the reading is above 20 square units of area. We next look at the roller drum and find, say, that 4.9 divisions have passed the zero mark on the vernier. This represents 4.9 units of area. Next we scan along the vernier till a line is found coincident with a line on the roller drum, and notice how many divisions beyond the zero on the vernier this line represents. This records $\frac{1}{100}$ ths. Suppose the 8th division on the vernier to be coincident with the line on the roller drum; this will correspond to .08 of a square unit of area. The full reading of the instrument is therefore 24.98. If all the readings were zero to begin with, and the point at P had been traced once round the diagram, so that the final reading was 24.98, then the area would be represented by this number. If, however, the reading of the instrument is not zero before tracing round the diagram, then the initial reading would have to be subtracted from the final reading to give the area of the required figure.

Area by Weighing.—This is a very accurate method of finding the area of an irregular figure, but a *good chemical balance* is required.

The method is as follows:—Cut out of a thin piece of card a figure equal and similar to the one whose area is required, and carefully weigh. Then cut out of the same piece of card a square of known area, and weigh. Divide the weight of the square by the number of square units it contains, and we obtain the weight of the unit square. Then the ratio of the weights of the unit square and the figure is in the ratio of their areas.

If the figure whose area is required is the plan to scale on a sheet of drawing paper of some larger area, the same method may be adopted, only it should be remembered that *the ratio of the actual area of a plan to the actual area of the figure represented is given by the square of the representative fraction of the scale to which the plan is drawn.*

Thus, for example, suppose a boiler plate (regular or irregular in form) is drawn to a scale of 3 in. = 1 ft. The representative fraction of the scale = $\frac{1}{4}$; i.e. the drawing is $\frac{1}{4}$ full size.

Then,
$$\frac{\text{Area of drawing}}{\text{Area of plate}} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$$

In other words, the area of the drawing is $\frac{1}{16}$ th the actual area of the plate.

Area of Regular Polygon.—Let n denote the number of sides, r the length of the perpendicular from the centre of the polygon to one of the sides, or the radius of the inscribed circle, and R the radius of the circumscribed circle.

Then,
$$\frac{l}{2} = r \tan \frac{180^\circ}{n} \quad \text{and} \quad r = \frac{l}{2} \cot \frac{180^\circ}{n}.$$

Also,
$$\frac{l}{2} = R \sin \frac{180^\circ}{n} \quad \text{and} \quad R = \frac{l}{2} \operatorname{cosec} \frac{180^\circ}{n}.$$

$$\begin{aligned} \text{Area of polygon} &= n \times \frac{1}{2}lr \\ &= \frac{1}{4}l^2n \cot \frac{180^\circ}{n} \\ &= r^2n \tan \frac{180^\circ}{n} \\ &= \frac{1}{2}R^2n \sin \frac{360^\circ}{n}. \end{aligned}$$

Area of Circle.

$$\text{Area of circle} = \pi r^2 = \frac{\pi}{4} D^2.$$

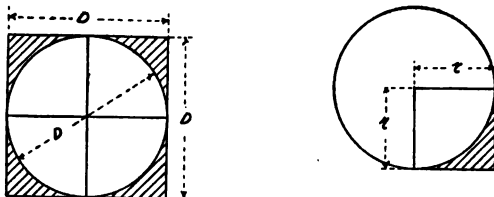


FIG. 26.

Take $\pi = \frac{22}{7}$, 3.1416, etc., 3.14159265 . . . , according to size of circle and accuracy required.

$$\text{Area of circle} = \frac{22}{7} r^2 = \frac{11}{14} D^2,$$

or

$$3.1416 r^2 = 0.7854 D^2.$$

Area of Sector of Circle.

$$(1) \text{ Area of sector} = \frac{1}{2} \text{ arc} \times \text{radius} = \frac{1}{2} l r.$$

$$(2) \quad \text{,,} \quad \text{,,} = \frac{N^\circ}{360} \times \text{area of circle} = \frac{N^\circ}{360} \pi r^2.$$

$$(3) \quad \text{,,} \quad \text{,,} = \frac{1}{2} r^2 \theta \text{ where } \theta \text{ is the circular measure of the angle} = \frac{l}{r}.$$

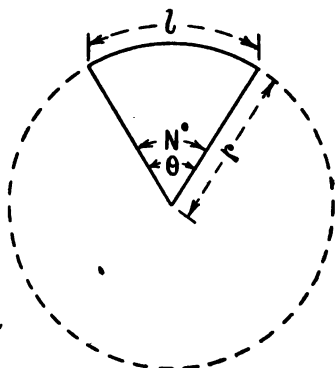


FIG. 27.

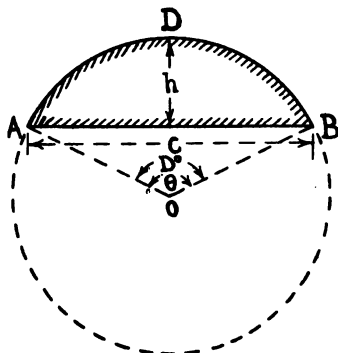


FIG. 28.

Area of Segment of Circle.

Area of segment = area of sector AOB - area of triangle AOB

$$= \frac{1}{2} \text{ arc ADB} \times r - \frac{1}{2} (r - h) c.$$

$$\text{Arc ADB} = \frac{D^\circ}{360} \times 2\pi r \text{ (exactly),}$$

$$\text{or }^1 \quad \text{arc ADB} = \frac{8AD - c}{3} \text{ (approximately).} \quad AD = \sqrt{h^2 + \frac{c^2}{4}}.$$

If θ denotes the circular measure of the angle at O, then

$$\text{Area of segment} = \frac{1}{2} r^2 (\theta - \sin \theta).$$

¹ When arc = semicircle, error is less than 1 per cent.

Area of Annulus.

$$\begin{aligned}\text{Area} &= \text{area of outer circle} - \text{area of inner one} \\ &= \pi(R^2 - r^2) = \pi(R + r)(R - r) = \pi(R + r)w \\ &= \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(D + d)(D - d).\end{aligned}$$

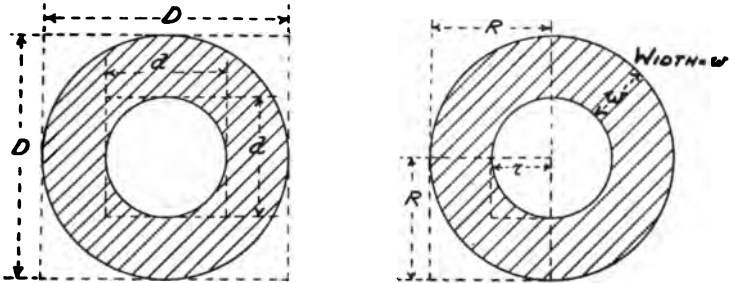


FIG. 29.

$$\begin{aligned}\text{Area of figure ABCD (fig. 30)} &= \text{difference of two sectors} \\ &= \frac{D^\circ}{360} \pi(R^2 - r^2) \\ &= \frac{1}{2}(R + r)(R - r).\end{aligned}$$

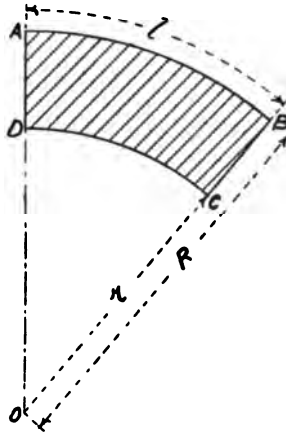


FIG. 30.

Area of Ellipse.

$$\text{Area} = \pi \times \frac{M}{2} \times \frac{m}{2} = \frac{\pi}{4} M \cdot m.$$

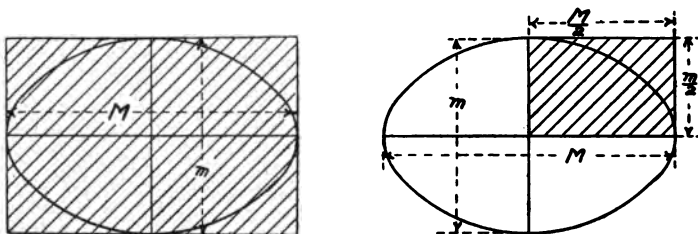
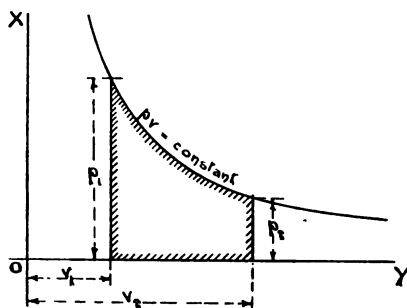


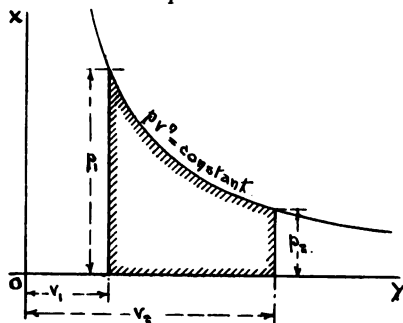
FIG. 31.



$$\text{Area of shaded figure} = p_1 v_1 \log_e \frac{v_2}{v_1} - p_2 v_2 \log_e p.$$

FIG. 32.

The curve is a rectangular hyperbola, since its equation is $pv = \text{constant} = 2.3 p_1 v_1 \log_{10} \frac{v_2}{v_1}$.



$$\text{Area of Shaded figure} = \frac{p_1 v_1 - p_2 v_2}{\eta - 1}$$

FIG. 33.

Area of parabolic segment (fig. 34) (shaded) = $\frac{2}{3}$ of rectangle = $\frac{2}{3}H.C.$

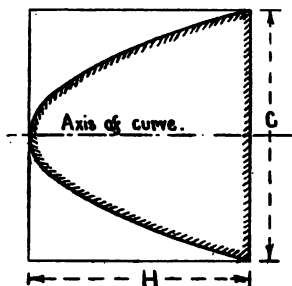


FIG. 34.

Surface of Right Pyramid (or Cone) = $\frac{1}{2}$ perimeter of base $\times$ slant height.

Note.—Slant height = perpendicular from apex to one edge of the base (drawn along one face of pyramid).

Note.—The above rule does not include the area of the base.

Fig. 35 shows a hexagonal pyramid with the development of its surface, including the base surface.

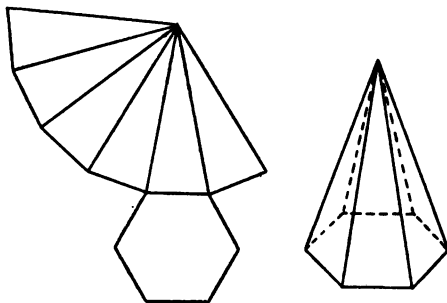


FIG. 35.

In the case of the cone, which is a particular case of the right pyramid with an infinite number of sides,

$$\text{curved surface} = 2\pi rS$$

and

$$S = \sqrt{h^2 + r^2}.$$

Fig. 36 represents the development of the surface of a cone, and it will be seen that the curved surface is equal in area to the sector of a circle.

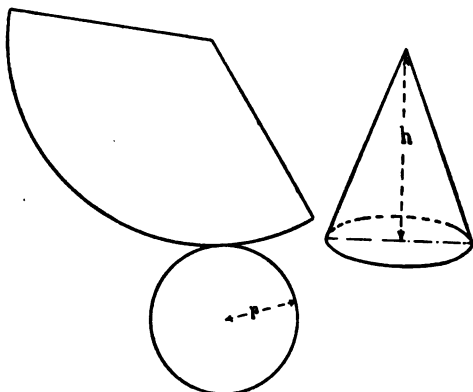


FIG. 36.

Surface of Frustum of Right Pyramid (or Cone) when the plan of the section is parallel to base

$$= \frac{1}{2}(\text{perimeter of base} + \text{perimeter of section}) \times \text{slant height}.$$

Slant height = $\sqrt{(R-r)^2 + h^2}$, where R and r denote the radii of the inscribed circles of the base and section respectively.

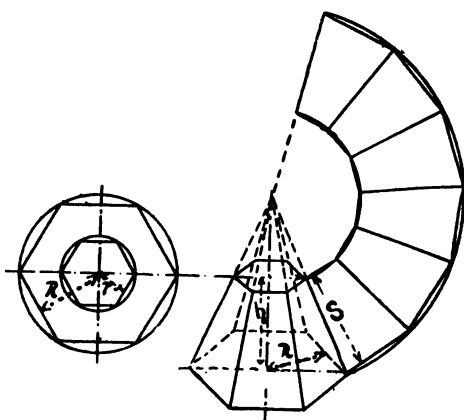


FIG. 37.

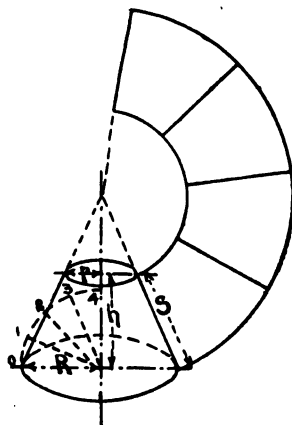


FIG. 38.

In the case of the cone, the development is a sectorial annulus, and area of curved surface = $\pi(R+r)S$

$$= \pi(R+r)\sqrt{(R-r)^2 + h^2}.$$

Surface of Prism (and Cylinder).

The development of the surface is a rectangle, and the method of determining the area obvious.

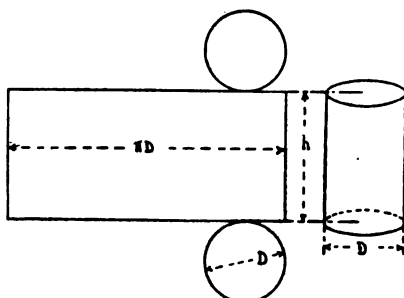
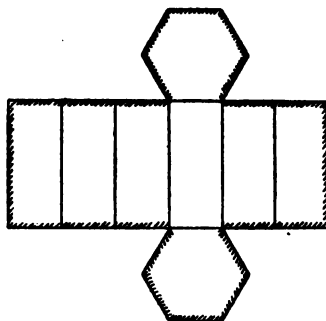


FIG. 39.

Surface of Truncated Cylinders.

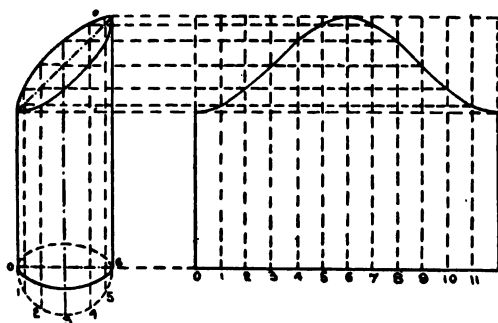


FIG. 40.

The curved surface is equal in area to that of a cylinder on the same base, but whose height is the mean of the two heights (h_1 and h_2).

$$\text{Area} = 2\pi r \times \frac{h_1 + h_2}{2} = \pi r(h_1 + h_2).$$

Area of Surface of Sphere = area of curved surface of cylinder which would just contain the sphere

$$= 2\pi R \times 2R$$

$$= 4\pi R^2$$

= four times area of a primary section of the sphere.

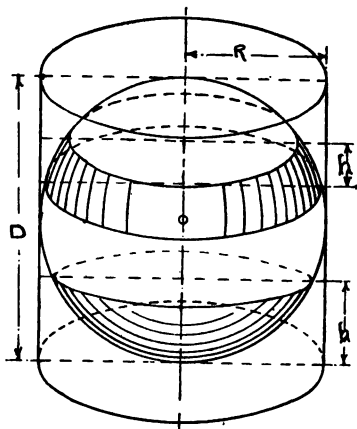


FIG. 41.

Area of Surface of Segment or Zone of Sphere.

Area of surface = area of the projection of the surface on a right circular cylinder = $2\pi R h$, where R denotes radius of sphere, and h the height of the segment or thickness of the zone.

Calculation of Volumes.

Volume regarded as a measurable quantity is expressed in terms of certain unit cubes, the unit chosen depending upon the magnitude of the volume to be measured.

"Capacity," "cubical contents," or "cubical capacity," are terms used synonymously with volume.

Our smallest unit of volume—in the *English* system of capacity

—is the “cubic inch,” being a small cube measuring an inch in each of its three dimensions.

We have also the cubic foot, the cubic yard, etc., in terms of which we express volume.

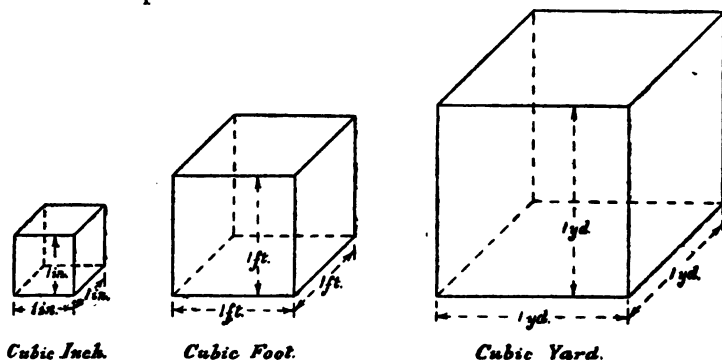


FIG. 42.

In the metric system we have such units as the cubic metre, the cubic decimetre, the cubic centimetre, etc.

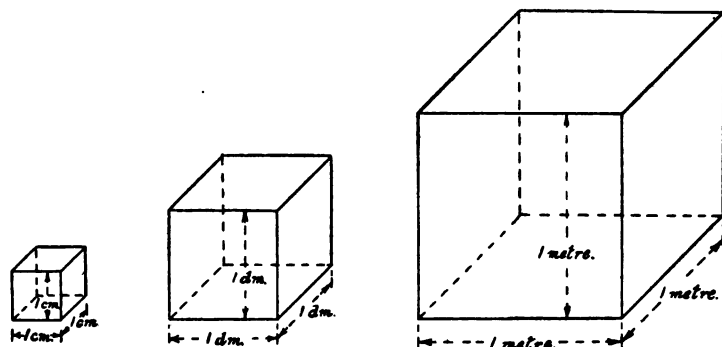


FIG. 43.

Volume of Cuboid, or Rectangular Volume.

$\text{Vol.} = l. b. d$ (fig. 44).

$\text{Vol.} = \text{area of base} \times \text{height or altitude.}$

The truth of this rule for the oblique prism may be made apparent by a simple illustration. Take a pack of cards, and place them one over another so as to form a right prism. The volume we know to be the area of one card (*i.e.* area of base) $\times$ height.

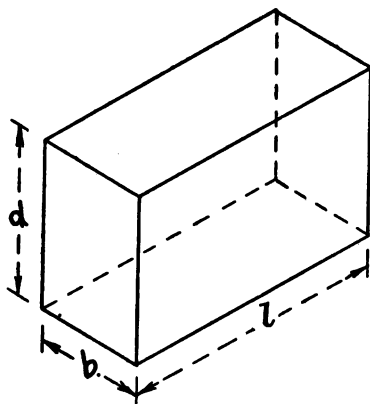


FIG. 44.

Now take a straight-edged ruler, and, placing it vertically in contact with the cards, gradually incline it, causing the cards to slide one over the other, forming an oblique prism of the same

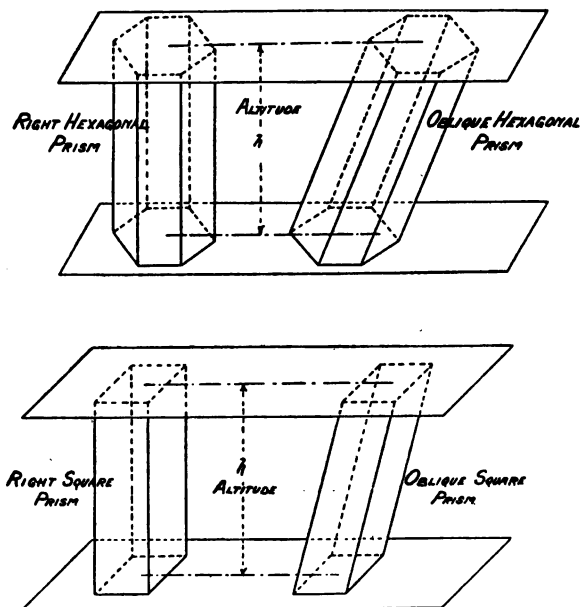


FIG. 45.

base and altitude as before. That the volume of the pack remains the same as before, goes without saying.

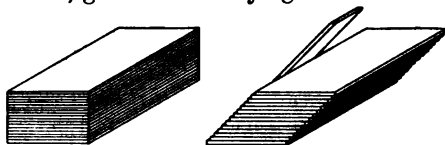


FIG. 46.

Volume of Cylinder = area of one end $\times$ altitude or height

$$= \pi r^2 h = \frac{\pi D^2 h}{4}$$

This rule applies in the case of an oblique cylinder where the axis is inclined to the base, and the student may satisfy himself of the truth of this by substituting pennies for the pack of cards referred to in the previous example.

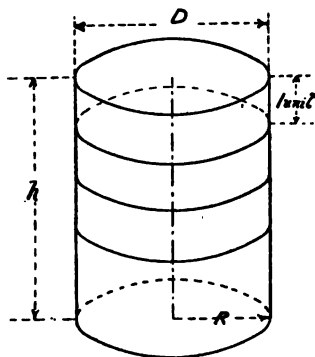


FIG. 47.

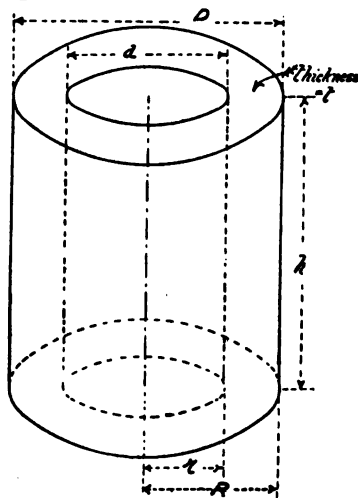


FIG. 48.

Volume of hollow cylinder or cylindrical shell (fig. 48)

= area of cross-section $\times$ height

= mean circumference $\times$ thickness $\times$ height

$$= \pi(R^2 - r^2)h = \pi(R + r)(R - r)h$$

$$= \frac{\pi}{4}(D^2 - d^2)h = \frac{\pi}{4}(D + d)(D - d)h.$$

Volume of Truncated Cylinder or of any Truncated Prism.

Vol. of truncated prism (and cylinder) = area of base $\times$ length of axis intercepted between cutting plane and base.

In fig. 49, ABCD represents the cut surface of the prism by the plane xy . The truncated prism is shown in full dark lines. That the volume of this lower portion is equal to the volume of a prism

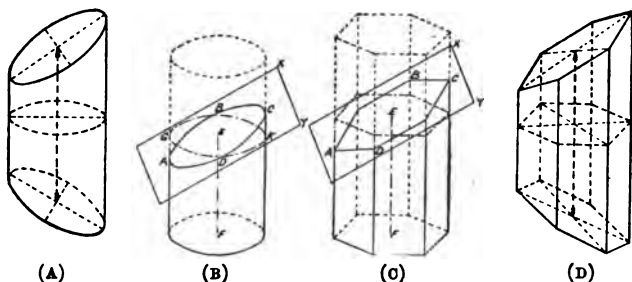


FIG. 49.

on the same base, and of the height EF, is almost apparent, for the wedge-shaped piece BDKC (fig. 49, B, C) may be considered to fill in the wedge-shaped column BDGA, both being equal in all respects. If the prism is truncated at both ends, the vol. is found by multiplying the area of a cross-section at right angles to the axis by the length of the axis intercepted between the cutting planes.

Volume of Segment of Cylinder.

Vol. = area of one end $\times$ length.

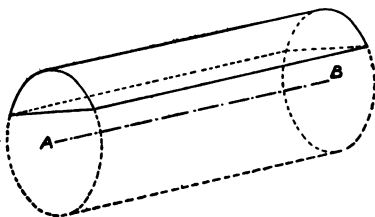


FIG. 50.

Note the section plane is supposed parallel to the axis AB.

Volume of Pyramid (and Cone).

Vol. = $\frac{1}{3}$ of vol. of prism on same base and with equal altitude
 = $\frac{1}{3}$ area of base $\times$ altitude
 = $\frac{1}{3}Ah$.

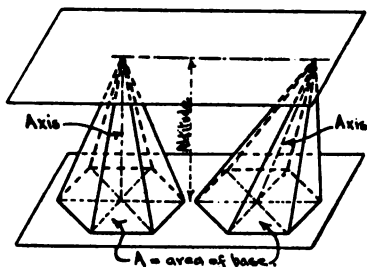


FIG. 51.

Volume of truncated *pyramid* and cone (section plane at right angles to axis).—Fig. 52 represents the frustum of a pyramid. Let A denote the area of the base, a the area of the section, then

$$\text{Vol. of frustum of prism} = \frac{1}{3}h(A + \sqrt{Aa} + a).$$

$$\text{Vol. of frustum of cone} = \frac{1}{3}\pi h(R^2 + Rr + r^2).$$

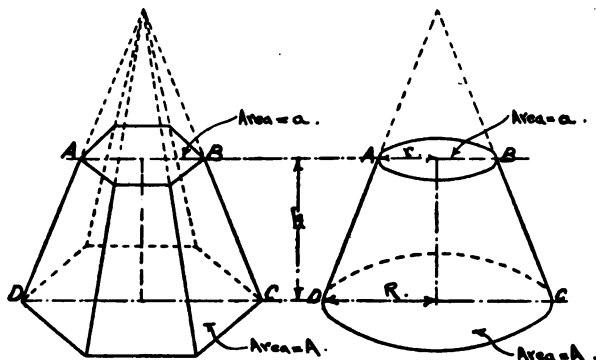


FIG. 52.

Volume of Ring.

To find the volume of a ring of uniform section, multiply the area (A) of the cross-section by the mean circumference.

Vol. of circular ring = $A \times \frac{1}{2}(D + d)\pi$.

Vol. of elliptical ring = $A \times \pi M \left[0.636 + 0.364 \left(\frac{m}{M} \right)^2 \right]$.

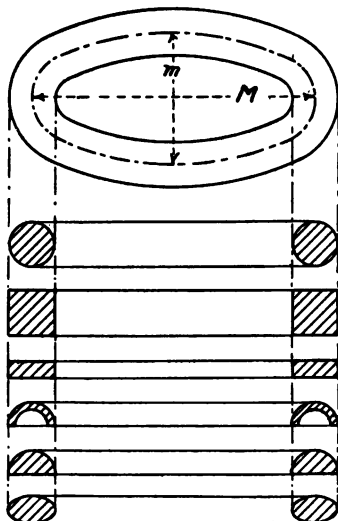


FIG. 53.

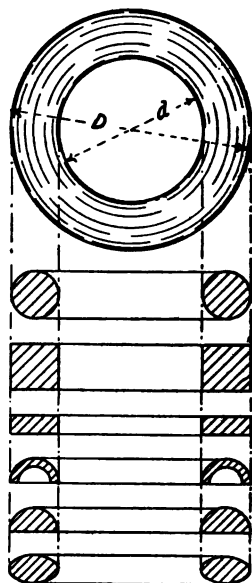


FIG. 54.

Cone, Sphere, and Cylinder.—Very simple relations exist between the volumes of the cone, sphere, and cylinder when they are of the same dimensions. If a cylinder and a cone be

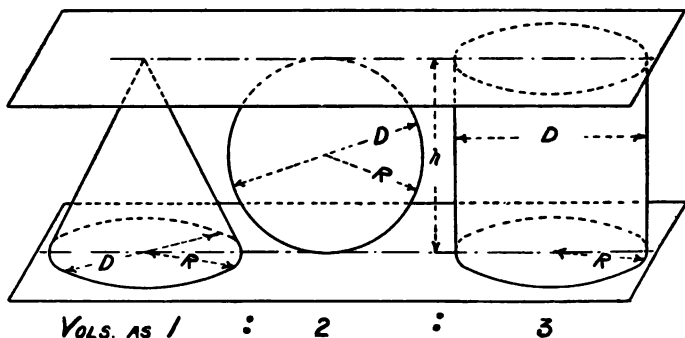


FIG. 55.

taken whose diameters and heights are equal to the diameter of any sphere, then the comparative volumes of the cone, sphere, and cylinder may be represented by the numbers 1, 2, 3.

Thus volume of sphere is twice the volume of cone, and $\frac{2}{3}$ volume of the cylinder.

Volume of a Cone.

Volume of cone = $\frac{1}{3}$ volume of the cylinder on the same base, and having the same altitude

$$= \frac{1}{3} \text{ of } \frac{\pi D^2 h}{4} = \frac{\pi D^2 h}{12} = 0.2618 D^2 h.$$

Volume of Sphere.

Volume of sphere = $\frac{2}{3}$ volume of cylinder which would just contain it

$$= \frac{2}{3} \text{ of } \frac{\pi D^2 \times D}{4} = .5236 D^3$$

$$= \frac{2}{3} \text{ of } \pi R^2 \times 2R = \frac{4}{3} \pi R^3$$

$$= 4.19 R^3.$$

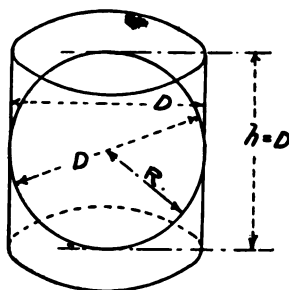


FIG. 56.

Volume of a Hollow Sphere or Spherical Shell (fig. 57).

= vol. of outside sphere - vol. of inside sphere

$$= \frac{4}{3} \pi (R^3 - r^3) = 4.189 (R^3 - r^3)$$

$$= \frac{4}{3} \pi (D^3 - d^3) = 0.5236 (D^3 - d^3).$$

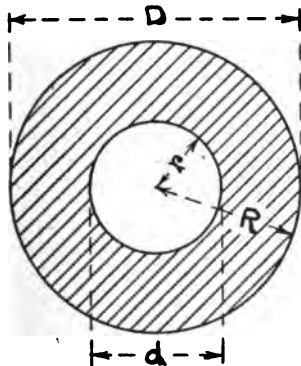


FIG. 57.

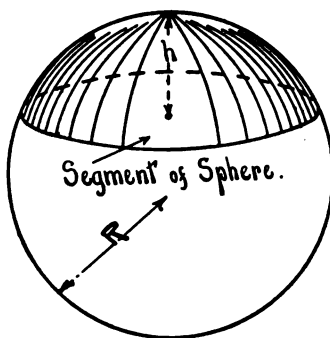


FIG. 58A.

Volume of Segment of Sphere (fig. 58A and 58B)

$$\begin{aligned}
 &= \frac{\pi}{3} (3Rh^2 - h^3) \\
 &= \frac{1}{6}\pi(3r^2h + h^3) \\
 &= \frac{1}{6}\pi(\frac{3}{2}d^2h + h^3).
 \end{aligned}$$

Volume of Zone of Sphere (fig. 59)

$$\begin{aligned}
 &= \frac{\pi}{3} [3R(h_2^2 - h_1^2) - h_2^3 + h_1^3] \\
 &= \frac{1}{6}\pi [\frac{3}{2}h(d_1^2 + d_2^2) + h^3].
 \end{aligned}$$

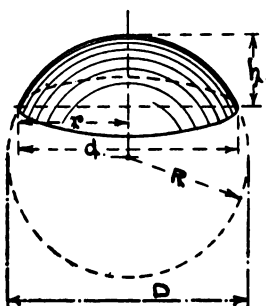


FIG. 58B.

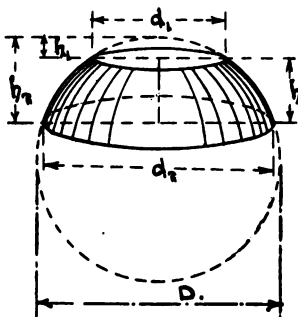


FIG. 59.

Solid of Revolution.—If a plane area revolves about any line in its plane as axis, the volume swept out is equal to the area of the plane multiplied by the length of path described by its centre of gravity.

abcdef (fig. 60A) represents an area of which CG is the centre of gravity and BC the axis of rotation.

Volume swept out by curve = $A \times 2\pi R$.

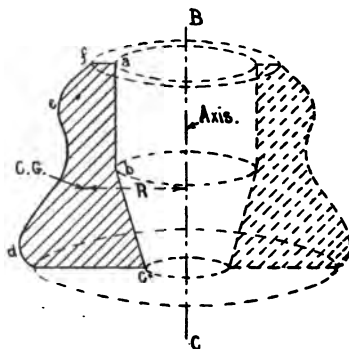


FIG. 60A.

Note.—To find the centre of gravity of any plane figure, cut out the figure from a piece of thin card, or metal plate, and drive small holes through two convenient places in it. Suspend the figure by means of a pin passing through one of the holes. With a plumb line draw a vertical through point of support, then suspend the figure from the pin placed through the other hole, and draw a vertical through this point. The point of intersection of these two lines is the centre of gravity of the figure.

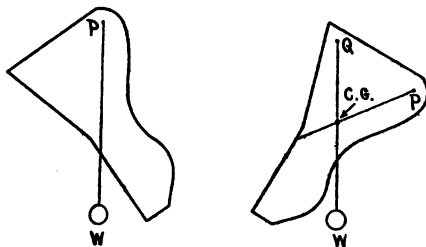


FIG. 60B.

$$\text{Volume of Solid (fig. 61)} = \frac{1}{3}h(A_1 + A_2 + 4A_3).$$

The faces $abcd$ and $ABCD$ are parallel. The sides are trapeziums. $EFGH$ is the mid-section by a plane parallel to the two faces.

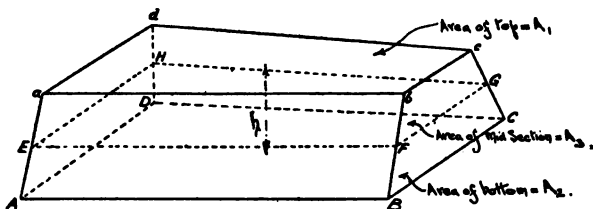


FIG. 61.

In the formula just given, h = perpendicular distance between parallel faces, A_1 and A_2 the areas of the top and bottom faces respectively, and A_3 area of the mid-section.

Note. $EH = \frac{1}{2}(AD + ad)$ $FG = \frac{1}{2}(BC + bc)$

and $EF = \frac{1}{2}(AB + ab)$ $HG = \frac{1}{2}(CD + cd).$

Volume of Form (fig. 62)

= **area of face ABCD** (found by Simpson's rule or mean ordinate method) $\times$ **width**.

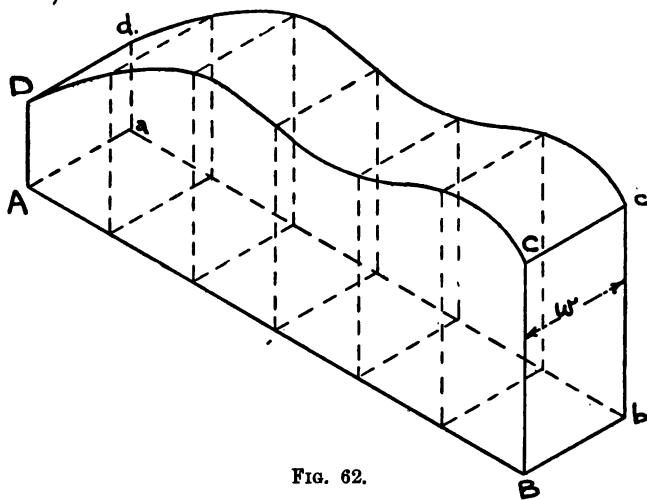


FIG. 62.

Volume of Paraboloid

= $\frac{1}{2}$ vol. of cylinder (see fig. 63)

$$= \frac{\pi D^2 h}{8}$$

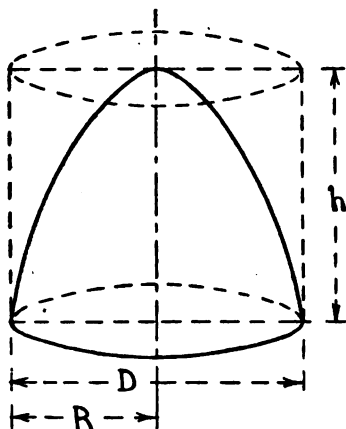


FIG. 63.

Parabola, Ellipse, and Hyperbola.—The general term conic or conic section is given to these curves, and there are two ways of defining them.

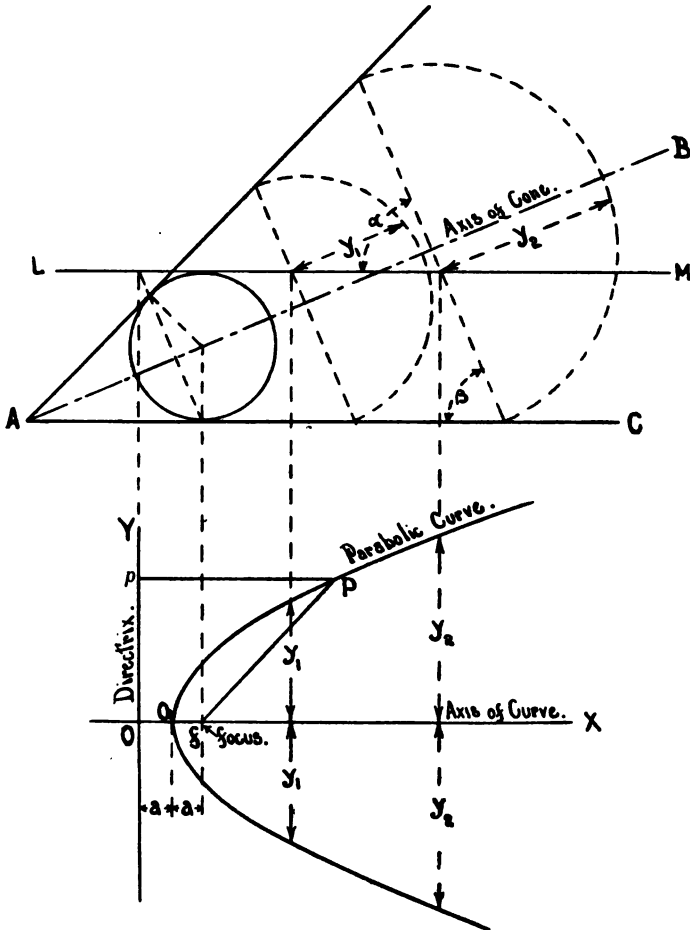


FIG. 64.

(1) A conic section (or conic) is the line of intersection of the curved surface of a right cone by a plane.

(2) A conic (or conic section) is the curve traced out by a point moving in such a way that the ratio of its distance from a fixed point

to its distance from a fixed line is always the same (constant). The fixed point is called the *focus*, the fixed line the *directrix* or *directing line*, and the constant ratio the *eccentricity*. In the parabola

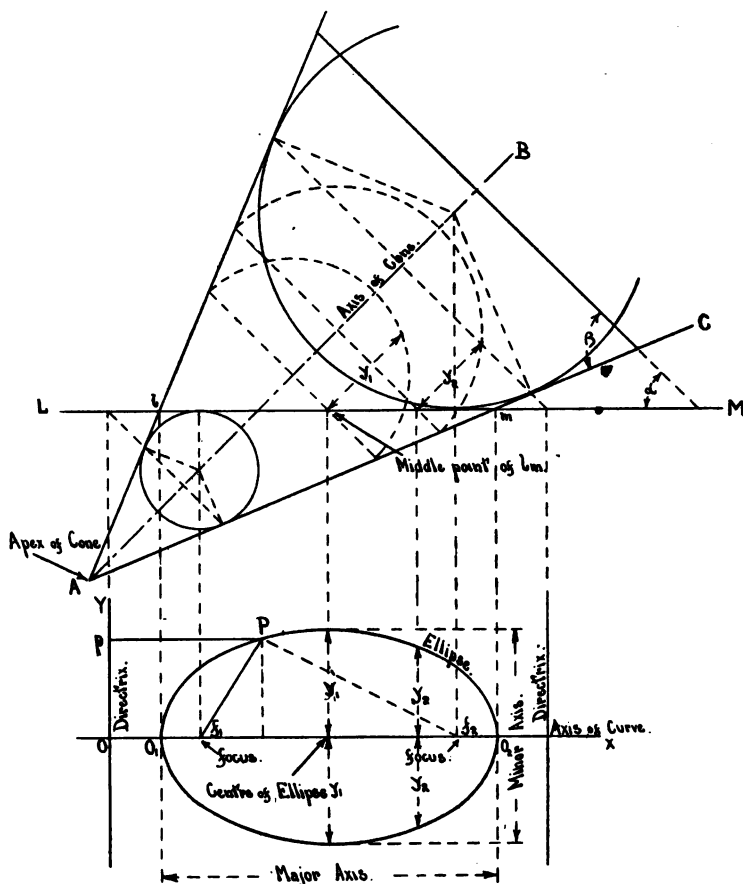


FIG. 65.

the eccentricity is equal to unity, in the ellipse it is less than unity, and in the hyperbola it is greater than unity.

Parabola.—(1) The parabola is the intersection of a right circular cone by a plane parallel to a generating line (or slant side); or (2) the parabola is the curve traced out by a point

moving in such a way that its distance from the focus is equal to its distance from the directrix.

Note that in any conic (a) the focus is the point of contact of the sphere with the section plane (LM) (see fig. 64); (b) the directing line or directrix is the line of intersection of the section plane (LM) with the plane containing the circle of contact of the sphere with the conical surface.

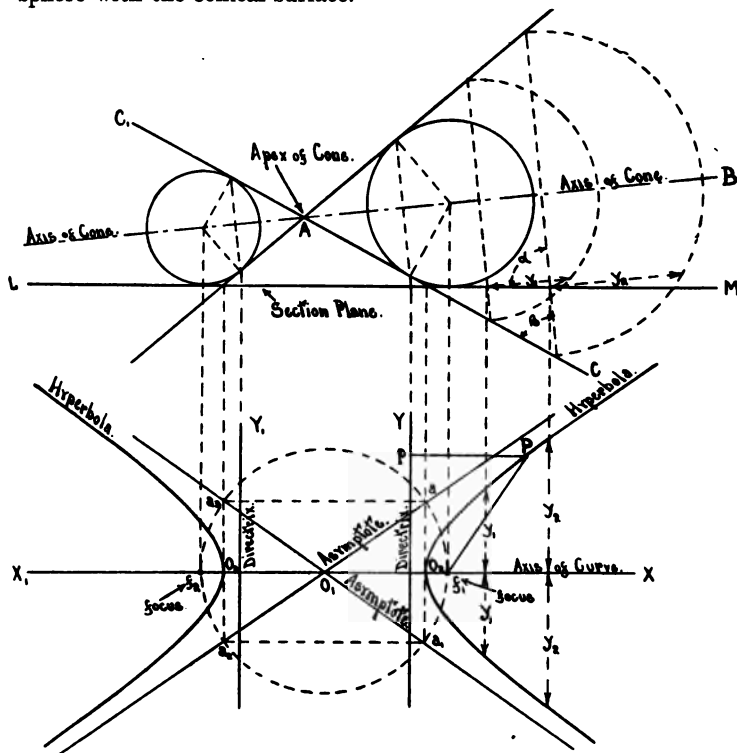


FIG. 66.

Referring to fig. 64, AB is the axis of a right circular cone with vertex at A, and LM is a section plane (shown by its trace) parallel to a generating line AC. The plan of the curve of section by the plane LM is shown by the parabolic curve. If P is *any* point on the curve, then the focal distance fP is equal to the perpendicular distance pP or $\frac{fP}{pP} = 1$.

Ellipse.—(1) The ellipse is the section of a right circular cone by a plane cutting both sides of the conical surface (and such that the angle α is less than the angle β , see fig. 65); or (2) the ellipse is the curve traced out by a point moving in such a way that (for a given ellipse) its distance (f_1P) from the fixed point or focus (f_1) is always the same fraction (less than unity) of its distance (pP) from the fixed line or directrix; (3) the ellipse has this characteristic property, that for any number of positions of the point P on the curve, the sum of the two focal distances is for a given ellipse always the same, and equal to the length of the major axis, *i.e.* $f_1P + f_2P = f_1P_1 + f_2P_1 = O_1O_2$.

Hyperbola.—Fig. 66 represents a double conical surface (apex at A , axis AB) cut by a plane LM such that the angle α is greater than the angle β . The two hyperbolic curves of section are shown in plan. The ratio of the distance f_1P to the distance pP is for any point on a given hyperbolic curve always the same, and greater than unity; *i.e.* eccentricity $= \frac{f_1P}{pP} = \text{constant}$.

If a point O_1 be taken midway between the two apexes O_2, O_3 , and from this, as centre, a circle be described with radius O_1f intersecting the tangents to the curves, as shown at a, a_1, a_2, a_3 , then lines drawn from the centre O_1 through these points of intersection are known as the asymptotes to the hyperbolic curves. When o_1a and o_1a_1 are at right angles to each other, the curve is said to be a rectangular hyperbola.

Circular Measure of an Angle.—A very useful method of measuring an angle, particularly when calculations involving angular velocity are required, is by expressing the value of an angle in what is called its **circular measure**.

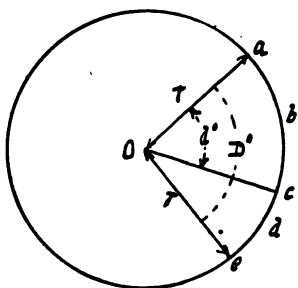


FIG. 67.

The unit of measurement called the **radian** is an angle at the centre of a circle subtending an arc whose length is equal to the radius of the circle.

Thus, referring to fig. 67, let aOb be an angle at the centre O such that arc abc equals radius r , then

this angle is one radian, and other angles are expressed in terms of this unit.

Thus, if d° ($= 57.29578 \dots$) represents the number of degrees in this angle, and D° the number of degrees in any other angle aOe ,

then the angle aOe may be expressed as $\frac{D}{d}$ radians.

A much more convenient method of calculating the circular measure, and one generally adopted, is to divide the subtended arc by the radius.

$$\text{Thus, circular measure of } aOe = \frac{\text{arc } abcde}{r}.$$

$$\text{Generally the circular measure, } \theta = \frac{\text{arc}}{\text{radius}}.$$

$$\text{and} \quad \text{arc} = \theta \times \text{radius}.$$

Example 1.—Calculate the circular measure of each of the following angles :— 360° , 180° , 90° , 1° , and 23° .

$$\text{Circular measure of } 360^\circ = \frac{\text{circumference}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi.$$

$$\text{" " } 180^\circ = \frac{\frac{1}{2} \text{ circumference}}{\text{radius}} = \frac{\pi r}{r} = \pi.$$

$$\text{" " } 90^\circ = \frac{\frac{1}{4} \text{ circumference}}{\text{radius}} = \frac{\pi}{2}.$$

$$\text{" " } 1^\circ = \frac{\frac{1}{360} \text{ circumference}}{\text{radius}} = \frac{\pi}{180}.$$

$$\text{" " } 23^\circ = \frac{\pi}{180} \times 23 = 0.401.$$

Example 2.—If D denote the value of an angle expressed in degrees, then $D \times \frac{\pi}{180} = \theta$ gives the value of the angle in circular measure, i.e. in radians.

Example 3.—Hence, if θ denote the circular measure of an angle, then the same angle expressed in degrees is given by

$$D = \frac{180}{\pi} \theta.$$

Metric Measure and Conversion of Numbers.

The basis of measurement in the metric system is the metre, the other standard units being derived from this unit. The metre was originally intended to be a ten-millionth part of a quadrant of the earth as measured on a geographical meridian from the equator to the pole. It is now the length of a certain bar of platinum at 0° C. , kept under the authority of the French Government, copies of which exist in England. The use of the metric system is also authorised by Act of Parliament in this country.

The unit of mass is the gramme. It is supposed to represent the weight of 1 cubic centimetre of distilled water at 4° C. The kilogramme (which was the original standard of mass) is the mass of a piece of platinum originally intended to balance

the mass of 1 litre (1 cubic decametre, or 1000 cubic centimetres of distilled water at 4° C.). This piece of platinum is kept in Paris with other metric standards, and copies for reference are distributed.

The great value of the metric system of weights and measures is due to the fact that it adopts the decimal divisions for its sub-units, and that larger units are expressed by multiples in 10, 100, 1000, etc., times the fundamental unit, and so such clumsy and time-wasting methods as the British system of weights and measures are avoided.

Latin prefixes are used in connection with the fundamental units to express the *sub-multiples* of length, surface, volume, mass, and weight.

These prefixes are :

<i>Deci</i> ,	signifying	$\frac{1}{10}$.
<i>Centi</i> ,	"	$\frac{1}{100}$.
<i>Milli</i> ,	"	$\frac{1}{1000}$.

Thus, for example, a centimetre is $\frac{1}{100}$ of a metre, and a centigramme the $\frac{1}{100}$ part of a gramme.

Greek prefixes are used for the *multiples* of the fundamental units.

Thus, *Deca*, or *Deka*, indicates 10 times the unit.

<i>Hecto</i>	"	100	"	"
<i>Kilo</i>	"	1000	"	"

For example, the kilogramme is 1000 grammes, and the kilometre 1000 metres.

TABLE I.
Conversion Numbers.

1 inch	=	25·4 millimetres = 2·54 centimetres.
1 foot	=	30·48 centimetres.
1 centimetre	=	0·3937 inches.
1 metre	=	39·37 inches
	=	3·28 feet.
1 cubic inch	=	16·386 cubic centimetres.
1 cubic foot	=	28·315 litres.
1 pint	=	567·93 cubic centimetres.
1 gallon	=	4·548 litres.
1 litre	=	1·76 pints
	=	0·22 gallons
	=	61·027 cubic inches.
1 lb.	=	453·59 grammes.
1 kilogramme	=	2½ lbs.
1 ton	=	1016·05 kilogrammes.

EXERCISES.

1. The span of a common roof truss is 25 ft., and the rise 5 ft. Calculate the length from eaves to ridge. *Ans.* 13.46 ft.

2. If the trusses in the previous exercise are pitched 14 ft. apart, estimate the total wind-pressure supported by each truss in the case when the wind-pressure is normal and of intensity 50 lbs. per sq. ft. *Ans.* 9422 lbs.

3. The spring and span of a circular arch are 2 ft. 6 in. and 7 ft. $4\frac{1}{2}$ in. respectively. Calculate the radius of curvature of the arch. *Ans.* 47.6 in.

4. The internal diameter of a cylindrical boiler is 6 ft. 6 in., and the length 20 ft. During the time the feed-pump was stopped the water level fell 6 in. as shown by the gauge-glass, the lower level being 12 in. above the centre of the boiler. Calculate the variation in the volume of the water contained by the boiler between these two levels. *Ans.* About 60 cub. ft.

5. Calculate (a) the strongest beam, (b) the stiffest beam, rectangular in each case, that could be cut out of a cylindrical log of timber 15 in. diameter.

Hint.—For the strongest beam, divide the diameter into three equal parts, and erect perpendiculars from the points of division in opposite directions to intersect the circumference; join these intersections to the extremities of the diameters. For the stiffest beam, divide the diameter into four equal parts, and erect perpendiculars from the two extreme divisions. Then make the necessary calculations.

Ans. Strongest beam measures 8.66 in. $\times$ 12.25 in.
Stiffest ,, ,, 7.62 in. $\times$ 12.9 in.

6. A road rises with a gradient of 1 in 75 from its commencement to a point $1\frac{1}{2}$ miles (on a horizontal datum). It then falls with a gradient of 1 in 100 to its termination at a further distance of 140 chains (on a horizontal datum). What is the difference of level between the beginning and the end?

Note.—The length of a chain is 66 ft. *Ans.* 13.2 ft.

7. The breadth of a screw propeller at a distance of 28 in. from the centre of the shaft is $29\frac{1}{2}$ in., while the distance from the rudder post to the near edge of the blade is 2 ft. 2 in., and the distance from the rudder post to the further edge is 3 ft. 4 in. Calculate the pitch of the propeller. *Ans.* 7 ft. $10\frac{3}{4}$ in. full.

8. If an observer is 50 ft. above the sea-level, what is the distance away of the visible horizon?

Hint.— $(h + 4000)^2 = x^2 + (4000)^2$, where h = height of observer (in miles), x the distance of the horizon in miles, and 4000 miles the radius of the earth. The student should first show that this gives $x = \sqrt{1\frac{1}{2}h_1}$, where h_1 is the height in feet.

Ans. 8.66 miles.

9. A funnel 4 ft. 6 in. diameter is built up of three plates in girth, and the lap of the joints is $1\frac{1}{2}$ in. Calculate the width of each plate.

Ans. 4 ft. $10\frac{1}{16}$ in.

10. How many revolutions per minute does the driving wheel of a locomotive engine make, if the wheel is 6 ft. diameter, and the engine is travelling 36 miles an hour?

Ans. 168.15.

11. A flywheel 15 ft. diameter rotates at 80 revolutions per minute. Calculate the velocity of its rim.

Ans. 3770

12. The pitch of a screw propeller is 18 ft. 6 in., and the speed of the vessel when the engines make 150 r.p.m. is 22 knots per hour. Calculate the percentage slip.

Ans. $18\frac{1}{2}$ per cent.

13. The diameter of the paddle-wheels of a steamer to centre of floats is 27 ft., and the wheel makes 31 r.p.m. while the vessel is steaming at the rate of 19.56 knots per hour. Calculate the percentage slip.

Ans. 24.6.

14. A helical spring made up of four complete turns has a mean diameter of 5 in., and a length of 8 in. Calculate the length of the coils.

Ans. 5 ft. $3\frac{1}{4}$ in. full.

15. The steam output from a boiler is 3488 lbs. per hour, and the area of the water surface is rectangular, and measures 20 ft. $\times$ 6 ft. Calculate the mean velocity of steam leaving the water surface, having given the density of the steam as 2.732 cub. ft. per lb.

Ans. 1.32 ft. per minute.

16. If the internal diameter of the main steam pipe from the boiler referred to in the previous exercise is $5\frac{1}{2}$ in., calculate the velocity of the steam as it leaves the boiler.

Ans. 16.04 ft. per second.

17. Find the heating surface given by the following hot-water pipes:—

155 ft. run of 2-in. pipe.

36 ft. 8 in. " 3-in. "

3 ft. 6 in. " $4\frac{1}{2}$ -in. "

Ans. 114 sq. ft.

18. What is the cooling surface given to a condenser by 850 tubes $\frac{3}{4}$ in. diameter and 6 ft. long?

Ans. 1000 sq. ft.

19. How many pipes must an economiser have to furnish 2000 sq. ft. of heating surface, if the pipes are to be 4 in. diameter and 14 ft. long? *Ans.* 137 tubes.

20. What is the tube surface of a water-tube boiler having 112 tubes 4 in. diameter by 19 ft. long? *Ans.* 2228 sq. ft.

21. The shell of a large Lancashire boiler, 9 ft. 0 in. outside diameter and 30 ft. long, is made up of four rings of steel plating $\frac{1}{8}$ in. thick alternately in and out, the first belt being an outer one. The longitudinal joints are butt joints, and the circumferential joints are lap joints, the overlap being 6 in. Calculate the area in sq. ft. and the volume in cub. ft. of the shell plating. *Ans.* 842 sq. ft. and 69·7 cub. ft.

22. The end-plate of a Lancashire boiler is 9 ft. diameter, and the two furnace tubes are each 3 ft. 9 in. diameter. Calculate the area of the end-plate exposed to the steam pressure, and the total pressure on this area for a working steam pressure of 200 lbs. per sq. in. *Ans.* 41·53 sq. ft. and 534 tons.

23. What is the diameter of a piston whose area is 164 sq. in.? *Ans.* 14·4 in.

24. Calculate the diameter of a circular plate whose weight would be the same as that of a rectangular plate measuring 2 ft. 6 in. $\times$ 3 ft. 2 in., both plates to be of the same thickness and material. *Ans.* 3 ft. $2\frac{1}{2}$ in.

25. The sides of a triangular plate of wrought iron, $\frac{1}{2}$ in. thick, measure 3 ft. 4 in. $\times$ 2 ft. $3\frac{1}{2}$ in. $\times$ 4 ft. 6 in. Calculate the weight of the plate.

Note.—The weight of a sq. ft. $\frac{1}{8}$ in. thick = 5 lbs. *Ans.* 74 lbs.

26. The area of an isosceles triangular plate is 306 sq. in., and the base measures 2 ft. 3 in. Calculate the length of each side. *Ans.* 2 ft. $2\frac{3}{8}$ in.

27. The two parallel sides of a trapezium measure 3 ft. $2\frac{1}{2}$ in. and 5 ft. 2 in., and the distance between these sides is 2 ft. $4\frac{1}{4}$ in. Calculate the dimensions of a square plate having an equal area. *Ans.* 3 ft. $1\frac{1}{2}$ in.

28. The steam space of a cylindrical boiler measures on the water surface 6 ft. 4 in., and the height of the top of the boiler above the water surface is 2 ft. 4 in. If the boiler is 20 ft. long, what is the volume of the steam space? *Ans.* 167·4 cub. ft.

29. A wrought-iron plate 1 in. thick, in the form of a sector of a circle, measures along each of its straight edges 4 ft. 7 in., and the included angle is $54\frac{1}{2}^\circ$. Calculate the weight of the plate. *Ans.* 400 lbs.

30. A $\frac{1}{2}$ -in. cylindrical liner for an air-pump has a mean diameter of $18\frac{1}{2}$ in. How many cubic inches of metal does it contain? Length = 25 in. *Ans.* 364 cub. in.

31. A cylinder 15 in. diameter is required to hold 40 gallons of water. What must be its height, having given that 1 gallon occupies $6\frac{1}{4}$ cub. ft.? *Ans.* 5 ft. $2\frac{1}{2}$ in.

32. Calculate the capacity in gallons of a cylindrical vessel 7 ft. diameter and 5 ft. high. *Ans.* 1203 gallons.

33. An elliptical manhole cover supports a steam pressure whose intensity is 150 lbs. per sq. in. The two largest diameters are 16 in. and $10\frac{1}{2}$ in. Calculate the total pressure on the cover. *Ans.* 8·89 tons.

34. A hemispherical dome is 8 ft. $6\frac{1}{2}$ in. diameter. What is the area of its curved surface? *Ans.* 114 sq. ft.

35. Calculate the third dimension of a rectangular cistern to hold 100 gallons, the other two dimensions being 2 ft. 6 in. $\times$ 3 ft. 4 in. *Ans.* 1 ft. $11\frac{1}{8}$ in.

36. A gallon contains 277·274 cub. in. Find the number of gallons in a cubic foot. *Ans.* $6\frac{1}{4}$.

37. A rectangular coal bunker measures 20 ft. 4 in. $\times$ 4 ft. 3 in. $\times$ 10 ft. 6 in. Calculate the weight of coal it would hold, having given that 1 ton of coal in stowage requires 45 cub. ft. *Ans.* 20·1 tons.

38. A room 15 ft. long, 18 ft. high, and 12 ft. wide is to be ventilated by a fan so that the air discharge every 20 minutes may be equal to the volume of the room. What quantity of air must the fan pass per minute? *Ans.* 162 cub. ft.

39. Jointing is required for a manhole which is elliptical along its mean perimeter, and measures $16\frac{1}{4}$ in. and $10\frac{3}{4}$ in. along its axes. What must be the length of the jointing? *Ans.* 3 ft. $4\frac{1}{2}$ in.

40. A railway embankment filling has the following dimensions:—Width at top, 20 ft., at base, 45 ft.; height, 11 ft.; length at top, 1020 yds., at base, 960 yds. Find its volume in cubic yards. *Ans.* 39172 cub. yds.

41. What weight of water does a Cornish boiler, 6 ft. 6 in. $\times$ 25 ft. long, with one flue 3 ft. 6 in. diameter, contain, if the ratio of water space to steam space = 3 : 1? *Ans.* 12·4 tons.

42. Calculate the weight of a cast-iron ball $2\frac{1}{2}$ in. diameter, taking the weight of a cub. in. of cast iron as 0·27 lb. *Ans.* 2·21 lbs.

43. A spherical safety-valve weight of cast iron is required to have a mass equal to that of a 15-lb. weight. What must be its diameter?
Ans. 4.73 in.

44. If the coal-bunkers of a steamer hold 480 tons of Welsh coal, and if this is sufficient for a voyage of 2000 knots, calculate the approximate distance which could be steamed if the bunkers were filled with a steam coal of different and inferior quality, having given that this coal requires more space, weight for weight, in the ratio of 45 : 40, and that the steaming properties of the two qualities are as 1.18 : 1.
Ans. 1506 knots. .

CHAPTER III.

CALCULATION BY COMMON LOGARITHMS.

IN almost all practical calculations where logarithms are employed the system in use is that of common logarithms.

In this system 10 is taken as the *base*, and the *logarithm* of any number is the *index* of the power to which 10 is raised to produce that number.

Consider the following lines:—

$$\begin{aligned} 10^0 &= 1; & 10^1 &= 10; & 10^2 &= 100; & 10^3 &= 1,000; \\ 10^4 &= 10,000; & 10^5 &= 100,000; & 10^6 &= 1,000,000; \\ 10^n &= \text{some number (call } N) \text{ depending on the numerical} \\ &\quad \text{value which } n \text{ may be given.} \end{aligned}$$

Here it will be noticed that the *base* 10 has been raised to whole number integral powers, the square, the cube, fourth, etc., giving numerical values of the powers of 10. Now in the *common system of logarithms* we agree to call the *index* (i.e. the number indicating the power), to which the base 10 is to be raised, the *logarithm of the number produced*.

Thus,	0 is the log of	1 to base 10.
	1 " "	10 "
	2 " "	100 "
	3 " "	1,000 "
	.	.
	.	.
	.	.
	n is the log of N to base 10.	

So that when the number is an exact or integral power of 10, its logarithm* is equal to the number of ciphers following the 1, or, in other words, consists of an integral part only, and is one less than the number of figures (including the *one* and the ciphers).

The next thing to be understood is that when the number of which the log is required lies between 0 and 1, its log is less than 0, i.e. it is a negative quantity.

* Usually written "log" for brevity.

Again, if the number is greater than 1 and less than 10, its log lies between 0 and 1—0 being the log of 1 and 1 the log of 10. These and other results which follow may be stated as follows:—

When a number lies between	0 and	1	its log is a negative quantity
"	"	1	" lies between 0 and 1
"	"	10	" " 1 " 2
"	"	100	" " 2 " 3
"	"	1000	" " 3 " 4
"	"	10000	" " 4 " 5

and so on.

It follows therefore that *in such cases the log of a number consists of two parts*—an integral part and some decimal fraction. The integral part is called the index or *characteristic*, and the decimal part is called the *mantissa*.

Thus, for example, if we take (from the tables) the log of 76·31, which we find to be 1·8826, the integral part 1 is the characteristic and the decimal part ·8826 the mantissa.

Writing this in the form of an equation, we obtain $10^{1\cdot8826} = 76\cdot31$, but such a statement as this the student will probably fail to realise fully unless he has studied the theory of logarithms. Nevertheless he may accept the principle, and will have no difficulty in understanding how to work out calculations.

The characteristic of the logarithm of a number can be always determined by inspection, and is not stated in a table of logarithms. For any quantity whose value is not less than unity, it consists of one less than the number of figures to the left of the decimal point, or in the integral part of the number whose logarithm is required.

The mantissa of the logarithm of any number is the same for the same set of figures wherever the decimal point may be placed.

Thus,	log of 5697·	3·7556
	" 569·7	2·7556
	" 56·97	1·7556
	" 5·697	0·7556

The characteristic of the logarithm of a decimal fraction (less than unity) is to be taken as negative, and one greater than the number of ciphers which follow the decimal point—the mantissa being always kept positive.

To illustrate this point take, for example, the decimal fraction 0·4578. It can be shown that its real log is entirely negative and equal to $-0\cdot3393$, for 0 is the log of 1 and 0·4578 is less than 1, so its log must be less than 0 and therefore negative.

Now for logarithmic calculations negative values of mantissæ would be most inconvenient, so where they occur we agree to make the mantissa positive by adding some integer and then

subtracting the same quantity from the logarithm to keep up the equality, thus,

$$\begin{aligned}\log 0.4578 &= -0.3393; \text{ add } 1; \\ \text{so } \therefore \text{ " " } &= (1 - 0.3393) - 1; \\ \text{so } \therefore \text{ " " } &= 0.6607 - 1.\end{aligned}$$

This is written* $\log 0.4578 = \bar{1}.6607$, the mantissa or decimal part being positive, while the characteristic or integral part is negative.

Again, the real $\log$ of $0.04578 = -(1.3393)$. Here 2 is first added and subtracted as under :

$$\begin{aligned}\log 0.04578 &= \{ +2 - (1.3393) \} - 2; \\ \text{" " } &= 2.6607.\end{aligned}$$

Once more, $\log 0.004578 = -(2.3393)$; add 3 and subtract 3.

$$\begin{aligned}\text{Thus, " " } &= \{ 3 - 2.3393 \} - 3; \\ \therefore \text{ " " } &= 3.6607.\end{aligned}$$

If another cipher is placed to the right of the decimal point, 4 is first added and the $\log$ subtracted from it, and then $\bar{4}$ stands for the negative characteristic.

An inspection of the examples just given will show that, for the same set of significant figures, irrespective of the position of the decimal point, the mantissa is always the same and positive, while the characteristic is negative and one more than the number of ciphers following the decimal point: hence the rule given in italics above.

Sometimes in calculations the resulting $\log$ is entirely negative, both the characteristic and mantissa, and when this is the case we must transform it into a $\log$ with a positive mantissa and a negative characteristic. The method of doing this and of dealing generally with negative values is similar to that just explained. We shall, however, deal more fully with this presently.

Having explained the elementary principles upon which the structure of common logarithms depend, the next step is to show how by their aid arithmetical calculations can be made. Before reading any further, however, the student will find it advisable to refer to p. 77 dealing with the method of using the logarithm tables.

Calculations by Logarithms.—The following four rules are stated and illustrated by means of examples :—

(a) **Multiplication.**—*To find the product of two or more numbers, take the sum of the logarithms of the numbers and refer to a table of*

* The negative sign is written *over* the characteristic to indicate that it only is negative.

antilogarithms: in connection with this sum will be found the value of the product.

(b) **Division.**—*To find the quotient of a division, subtract the logarithm of divisor from the logarithm of the dividend, and find in a table of antilogarithms the number whose antilogarithm is the difference.*

(c) **Involution.**—*To find any power of a number, multiply the logarithm of the number by the index of the power: the result gives the logarithm of the answer which may be obtained from the table of antilogarithms.*

(d) **Evolution.**—*To obtain any root of a number, divide the logarithm of the number by the index of the root: the result is the logarithm of the required root of the number, and by reference to the table of antilogarithms the root may be obtained.*

Example 1.—Multiply 42 by 36.

By rule (a), $\log (42 \times 36) = \log 42 + \log 36$.

$$\log 42 = 1.6232$$

$$,, \quad 36 = 1.5563$$

$$3.1795 = \log (42 \times 36).$$

Now referring to the table of antilogs, it will be seen that 0.1795 is the mantissa of the log of 1512, so that these figures give the required product. The characteristic 3 shows that there are four digits in the product to the left of the decimal point.

Example 2.—Multiply 234 by 35.

By rule (a), $\log (234 \times 35) = \log 234 + \log 35$.

$$\log 234 = 2.3692$$

$$,, \quad 35 = 1.5441$$

$$3.9133 = \log \text{ of product.}$$

Antilog from table = 8191, which is a sufficiently near approximation to the actual product 8190 obtained by ordinary multiplication. Here also the characteristic 3 shows there are four digits in the product to the left of the decimal point.

Example 3.—Find the value of 39.67×8.52 .

Log of product = $\log 39.67 + \log 8.52$.

$$\log 39.67 = 1.5985$$

$$8.52 = 0.9304$$

$$2.5289 = \log \text{ of product.}$$

Antilog from table = 3380, and since index is 2, there are three digits in the integral part of the product, which is therefore 338·0. By actual multiplication the product is **337·9884**, so we see that by logarithmic calculation an approximation is arrived at quite sufficient for all practical purposes.

(If in calculations for very special work greater accuracy is required than could be obtained by the use of four-figure logarithms, a table of seven-figure logarithms would have to be used.)

Example 4.—Find the value of $\frac{97\cdot58}{3\cdot14}$.

By rule (b), $\log \frac{97\cdot58}{3\cdot14} = \log 97\cdot58 - \log 3\cdot14$.

$$\log 97\cdot58 = 1\cdot9894$$

$$,, \quad 3\cdot14 = 0\cdot4969$$

$$1\cdot4925 = \log \text{ of quotient.}$$

Antilog from tables = 3109, and since characteristic is 1, there are two digits in the integral part of quotient,

$$\therefore \frac{97\cdot58}{3\cdot14} = \mathbf{31\cdot09}.$$

Example 5.—Find the third power of 12·25.

By rule (c), $\log (12\cdot25)^3 = \text{three times the log of } 12\cdot25$.

$$\log 12\cdot25 = 1\cdot0881$$

3

$$3\cdot2643 = \log \text{ of third power.}$$

Antilog from tables = 1838, and therefore $(12\cdot25)^3 = \mathbf{1838}$ (approx.).

Example 6.—Find the cube root of 798.

By rule (d), $\log \sqrt[3]{798} = \frac{1}{3} \text{ of the log of } 798$.

$$\log 798 = 2\cdot9020$$

$$\frac{1}{3} \text{ of } 2\cdot9020 = 0\cdot9673 = \log \text{ of } \sqrt[3]{798}.$$

$$\text{Antilog} = 9274,$$

$$\therefore \sqrt[3]{798} = \mathbf{9\cdot274} \text{ (approx.).}$$

Example 7.—Find the value of $\frac{33000 \times \sqrt[5]{(32.6)^2} \times 125.6}{\sqrt[3]{343} \times 84.9}$.

The log of required result = log of numerator – log of denominator (by rule (b)).

$$\text{log of numerator} = \text{log } 33000 + \text{log } \sqrt[5]{(32.6)^2} + \text{log } 125.6 \text{ (rule (a))}$$

$$,, \text{ denominator} = \text{log } \sqrt[3]{343} + \text{log } 84.9 \text{ (rule (a))}$$

$$\text{log } \sqrt[5]{(32.6)^2} = \frac{2}{5} \text{log } 32.6 \text{ (rules (c) and (d))}$$

$$= \frac{2}{5} \text{ of } 1.5132 = 0.6053$$

$$,, \sqrt[3]{343} = \frac{1}{3} \text{log of } 343 = 0.8451 \text{ (rule (d))}.$$

Summing the logs of numerator,

$$\text{log } 33000 = 4.5185$$

$$,, \sqrt[5]{(32.6)^2} = 0.6053$$

$$,, 125.6 = 2.0990$$

$$7.2228 \text{ log of numerator,}$$

and

$$\text{log } \sqrt[3]{343} = 0.8451$$

$$,, 84.9 = 1.9289$$

$$2.7740 \text{ log of denominator.}$$

$$\text{log of quotient} = 7.2228 - 2.7740 = 4.4488$$

$$\text{antilog } .4488 = 2810,$$

$$\therefore \text{quotient} = 28100 \text{ (approx.). } \textit{Ans.}$$

Calculations by Logarithms involving Negative Characteristics.

We have already referred (see p. 71) to logarithms which have negative characteristics: but as calculations are constantly arising involving their use, it is advisable to notice further a few points which may present a little difficulty to the beginner.

Suppose in a calculation it is required to divide the log of a number having a negative characteristic by a given positive quantity. First taking the case where the characteristic is an exact multiple of the divisor, the process of division is quite simple.

$$\text{Thus, for example, } \frac{1}{3} \text{ of } 3.2208 = 1.0736,$$

$$\text{or } \frac{1}{2} \text{ of } 4.3284 = 2.1642.$$

But if the characteristic is not an exact multiple of the divisor, the least integral number which will make it so is to be subtracted, and then finally added to keep up the equality.

$$\begin{aligned}\text{Thus,} \quad \frac{1}{3} \text{ of } 2.3874 &= \frac{1}{3} \{ \overline{2} - 1 + 0.3874 + 1 \} \\ &= \frac{1}{3} \{ \overline{3} + 1.3874 \} \\ &= \overline{1} + 0.4625,\end{aligned}$$

$$\text{i.e.} \quad \frac{1}{3} \text{ of } 2.3874 = \overline{1}.4625.$$

$$\begin{aligned}\text{Again,} \quad \frac{1}{4} \text{ of } 7.3892 &= \frac{1}{4} \{ \overline{7} - 1 + 0.3892 + 1 \} \\ &= \frac{1}{4} \{ \overline{8} + 1.3892 \} \\ &= \overline{2}.3473.\end{aligned}$$

Next consider the *addition* of logs having both positive and negative characteristics.

Find the sum

$$\begin{array}{r} \overline{4}.5378 \\ 2.3820 \\ \overline{1}.2330 \\ \overline{1}.6085 \\ \hline \end{array}$$

$$\overline{3}.7613 = \text{the sum.}$$

All the mantissæ are positive and their sum is 1.7613. But the characteristics are negative in lines 1, 3, and 4, and positive in line 2. The sum of the negative characteristics = $\overline{6}$ and the sum of the positive + 2, to which latter must be added the integral part 1 from the sum of the mantissæ, making a positive total 3. Now the algebraic sum of $\overline{6}$ and plus 3 = $\overline{3}$. Hence the algebraic sum of the mixed logs is $\overline{3}.7613$.

Subtraction.—From $\overline{4}.7388$ take 2.8756.

$$\begin{array}{r} \text{Thus,} \quad \overline{4}.7388 \\ 2.8756 \\ \hline \overline{7}.8632 \end{array}$$

Subtracting the mantissæ in the ordinary way, we have finally to take 3 from $\overline{4}$, and $\overline{4} - 3 = -7$ (written $\overline{7}$).

Again, from $\overline{2}.4409$ take $\overline{3}.5899$.

$$\begin{array}{r} \text{Thus,} \quad \overline{2}.4409 \\ \overline{3}.5899 \\ \hline 0.8510 \end{array}$$

Subtracting the mantissa we have finally,

$$\overline{2} - (-3) - 1 = \overline{2} + 3 - 1 = 0.$$

Example 8.—Find the value of $\frac{0.0674 \times \sqrt[3]{0.799} \times 52.64}{0.586}$.

log of quotient = log of numerator – log of denominator

$$\begin{aligned}\log \text{ numerator} &= \log 0.0674 + \frac{1}{3} \log 0.799 + \log 52.64 \\ &= \bar{2}.8287 + \frac{1}{3}(\bar{1}.9025) + 1.7213 \\ &= \bar{2}.8287 + \bar{1}.9675 + 1.7213 \\ &= 0.5175.\end{aligned}$$

$$\log \text{ denominator} = \bar{1}.7679.$$

$$\begin{aligned}\log \text{ quotient} &= 0.5175 - (\bar{1}.7679) \\ &= 0.5175 - (-1) - (+0.7679) \\ &= 0.5175 + 1 - 0.7679 \\ &= 0.7496.\end{aligned}$$

$$\text{antilog} = 5.618. \quad \text{Ans.}$$

Use of Logarithm Tables.—Reference has already been made to logarithm tables, of which two are given in the following pages. Both tables are four-figure tables. **Table II.** gives the mantissa part of any number up to four significant figures which may or may not be followed or preceded by any number of ciphers, and it must be borne in mind that the position of the decimal point among the numbers whose log is required does not affect the value of the mantissa.

The vertical column of two figures on the extreme left of the table gives the first two figures of the number whose log is required, counting from left to right; the third figure is indicated in the first series of horizontal numbers along the *top*, while the fourth is given in the second series on the right. In the body of the table the logarithms of these numbers are set out.

The log of any two significant figures is shown in the second vertical column under the 0.

Thus, mantissa of 39 = 5911,
so that log of 39 = 1.5911,

the characteristic 1 being determined by inspection and the rule already explained

We have pointed out more than once that the position of the decimal point does not affect the value of the mantissa, and also that the logs of any multiple or sub-multiple of a number by powers of 10 can be immediately written from the log of the number.

Thus, from log 39 = 1.5911 we get log 3.9 = 0.5911
and log .39 = $\bar{1}.5911$ " " 0.039 = $\bar{2}.5911$
" " 390 = 2.5911 " " 3900 = 3.5911

and so on.

TABLE II.—LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4 8 12	17 21 25	29 33 37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4 8 11	15 19 23	26 30 34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3 7 10	14 17 21	24 28 31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3 6 10	13 16 19	23 26 29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3 6 9	12 15 18	21 24 27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3 6 8	11 14 17	20 22 25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3 5 8	11 13 16	18 21 24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2 5 7	10 12 15	17 20 22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2 5 7	9 12 14	16 19 21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2 4 7	9 11 13	16 18 20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2 4 6	8 11 13	15 17 19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6	8 10 12	14 16 18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6	8 10 12	14 15 17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6	7 9 11	13 15 17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5	7 9 11	12 14 16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2 3 5	7 9 10	12 14 15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2 3 5	7 8 10	11 13 15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2 3 5	6 8 9	11 13 14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2 3 5	6 8 9	11 12 14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1 3 4	6 7 9	10 12 13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1 3 4	6 7 9	10 11 13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1 3 4	6 7 8	10 11 12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1 3 4	5 7 8	9 11 12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1 3 4	5 6 8	9 10 12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1 3 4	5 6 8	9 10 11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1 2 4	5 6 7	9 10 11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1 2 4	5 6 7	8 10 11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1 2 3	5 6 7	8 9 10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1 2 3	5 6 7	8 9 10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1 2 3	4 5 7	8 9 10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1 2 3	4 5 6	8 9 10

TABLE II.—LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1 2 3	4 5 6	7 8 9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1 2 3	4 5 6	7 8 9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1 2 3	4 5 6	7 8 9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1 2 3	4 5 6	7 8 9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1 2 3	4 5 6	7 8 9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1 2 3	4 5 6	7 7 8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1 2 3	4 5 5	6 7 8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1 2 3	4 4 5	6 7 8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1 2 3	4 4 5	6 7 8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5

TABLE II.—LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

Again, find the log 374. The mantissa 5729 is given in the vertical column under 4 and in the horizontal line from 37, the first two figures in the column on the extreme left.

Thus, $\log 374 = 2.5729$.

Logs of multiples and sub-multiples by powers of 10 are written down from this value as just explained above.

Now suppose we require the log of a number having four significant figures, and take for example the numbers 2298. The mantissa for the first three figures $229 = 3598$ and is found in the vertical column under 9, and the horizontal line from 22, the first two figures. To this mantissa must be added the 15 shown in the vertical column under 8 on the extreme right (in the second series) and in the horizontal line from 22.

$$\begin{array}{r} 3598 \\ 15 \\ \hline \end{array}$$

Thus, mantissa 2298 = .3613

Table III. gives the numbers (called antilogarithms) corresponding to given mantissæ; that is to say, the numbers corresponding to their logarithms.

In this table the mantissæ are arranged down the first vertical column and along the top of the table—the numbers of which these are the logs being set out in the body of the table.

In Example 7 the log of the answer works out to 4.4488, and turning to the table of antilogs corresponding to the mantissa 4488, we find 2810; and since the characteristic is 4, the quotient or answer is 28100.

In Example 8, log of quotient or answer = 0.7496, and the antilog from table = 5.618.

These illustrations will be quite sufficient to show the student the method of finding the log of a given number or the number corresponding to a given logarithm.

Naperian or Hyperbolic Logarithms are very largely used for many engineering calculations, some of which are dealt with in several chapters of this work. This being so, we add Table IV., giving Naperian logarithms of numbers from 1 to 44.

The base of the Naperian system is uncommensurable, and always denoted by the letter e (or the Greek ϵ); its value is represented by the sum addition of the series.

$$2 + \frac{1}{2} + \frac{1}{2 \times 3} + \frac{1}{2 \times 3 \times 4} + \text{etc.}$$

Its value to five places of decimals is 2.71828.

TABLE III.—ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
-00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0 0 1	1 1 1	2 2 2
-01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0 0 1	1 1 1	2 2 2
-02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0 0 1	1 1 1	2 2 2
-03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0 0 1	1 1 1	2 2 2
-04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0 1 1	1 1 2	2 2 2
-05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0 1 1	1 1 2	2 2 2
-06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0 1 1	1 1 2	2 2 2
-07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0 1 1	1 1 2	2 2 2
-08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0 1 1	1 1 2	2 2 3
-09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0 1 1	1 1 2	2 2 3
-10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0 1 1	1 1 2	2 2 3
-11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0 1 1	1 2 2	2 2 3
-12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0 1 1	1 2 2	2 2 3
-13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0 1 1	1 2 2	2 2 3
-14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0 1 1	1 2 2	2 3 3
-15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0 1 1	1 2 2	2 3 3
-16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0 1 1	1 2 2	2 3 3
-17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0 1 1	1 2 2	2 3 3
-18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0 1 1	1 2 2	2 3 3
-19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0 1 1	1 2 2	3 3 3
-20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0 1 1	1 2 2	3 3 3
-21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0 1 1	2 2 2	3 3 3
-22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0 1 1	2 2 2	3 3 3
-23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0 1 1	2 2 2	3 3 4
-24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0 1 1	2 2 2	3 3 4
-25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0 1 1	2 2 2	3 3 4
-26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0 1 1	2 2 3	3 3 4
-27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0 1 1	2 2 3	3 3 4
-28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0 1 1	2 2 3	3 4 4
-29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0 1 1	2 2 3	3 4 4
-30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0 1 1	2 2 3	3 4 4
-31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0 1 1	2 2 3	3 4 4
-32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0 1 1	2 2 3	3 4 4
-33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0 1 1	2 2 3	3 4 4

TABLE III.—ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
34	2188	2193	2198	2208	2208	2213	2218	2223	2228	2234	1 1 2	2 3 3	4 4 5
35	2239	2244	2249	2254	2259	2266	2270	2276	2280	2286	1 1 2	2 3 3	4 4 5
36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1 1 2	2 3 3	4 4 5
37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1 1 2	2 3 3	4 4 5
38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1 1 2	2 3 3	4 4 5
39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1 1 2	2 3 3	4 5 5
40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1 1 2	2 3 4	4 5 5
41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1 1 2	2 3 4	4 5 5
42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1 1 2	2 3 4	4 5 6
43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1 1 2	3 3 4	4 5 6
44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1 1 2	3 3 4	4 5 6
45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1 1 2	3 3 4	5 5 6
46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1 1 2	3 3 4	5 5 6
47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1 1 2	3 3 4	5 5 6
48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1 1 2	3 4 4	5 6 6
49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1 1 2	3 4 4	5 6 6
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1 1 2	3 4 4	5 6 7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1 2 2	3 4 5	5 6 7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1 2 2	3 4 5	5 6 7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1 2 2	3 4 5	6 6 7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1 2 2	3 4 5	6 6 7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1 2 2	3 4 5	6 7 7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1 2 3	3 4 5	6 7 8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1 2 3	3 4 5	6 7 8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1 2 3	4 4 5	6 7 8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1 2 3	4 5 5	6 7 8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1 2 3	4 5 6	6 7 8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1 2 3	4 5 6	7 8 9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1 2 3	4 5 6	7 8 9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1 2 3	4 5 6	7 8 9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1 2 3	4 5 6	7 8 9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1 2 3	4 5 6	7 8 9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1 2 3	4 5 6	7 9 10

TABLE III.—ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1 2 3	4 5 7	8 9 10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1 2 3	4 6 7	8 9 10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1 2 3	5 6 7	8 9 10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1 2 4	5 6 7	8 9 11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1 2 4	5 6 7	8 10 11
72	5248	5260	5272	5284	5297	5309	5321	5333	5345	5358	1 2 4	5 6 7	9 10 11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1 3 4	5 6 8	9 10 11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1 3 4	5 6 8	9 10 12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1 3 4	5 7 8	9 10 12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1 3 4	5 7 8	9 11 12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1 3 4	5 7 8	10 11 12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1 3 4	6 7 8	10 11 13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1 3 4	6 7 9	10 11 13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1 3 4	6 7 9	10 12 13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2 3 5	6 8 9	11 12 14
82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2 3 5	6 8 9	11 12 14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2 3 5	6 8 9	11 13 14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2 3 5	6 8 10	11 13 15
85	7070	7086	7112	7129	7145	7161	7178	7194	7211	7228	2 3 5	7 8 10	12 13 15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2 3 5	7 8 10	12 13 15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2 3 5	7 9 10	12 14 16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2 4 5	7 9 11	12 14 16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2 4 5	7 9 11	13 14 16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2 4 6	7 9 11	13 15 17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2 4 6	8 9 11	13 15 17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2 4 6	8 10 12	14 15 17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2 4 6	8 10 12	14 16 18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2 4 6	8 10 12	14 16 18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2 4 6	8 10 12	15 17 19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2 4 6	8 11 13	15 17 19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2 4 7	9 11 13	15 17 20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2 4 7	9 11 13	16 18 20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2 5 7	9 11 14	16 18 20

Common logarithms were originally calculated from the Naperian logarithms.

To find the Naperian log of a number when a table of Naperian logs is not available, multiply the common log by 2.3026.

TABLE IV.—HYPERBOLIC LOGARITHMS.

No.	Hyp. Log.	No.	Hyp. Log.	No.	Hyp. Log.	No.	Hyp. Log.	No.	Hyp. Log.
1.0	0.0000	4.0	1.3863	7.0	1.9459	10.00	2.3026	20.0	2.9957
1.1	0.0953	4.1	1.4110	7.1	1.9601	10.25	2.3279	20.5	3.0204
1.2	0.1823	4.2	1.4351	7.2	1.9741	10.50	2.3513	21.0	3.0445
1.3	0.2624	4.3	1.4586	7.3	1.9879	10.75	2.3749	21.5	3.0680
1.4	0.3385	4.4	1.4816	7.4	2.0015	11.00	2.3979	22.0	3.0911
1.5	0.4055	4.5	1.5041	7.5	2.0149	11.25	2.4201	22.5	3.1135
1.6	0.4700	4.6	1.5261	7.6	2.0281	11.50	2.4430	23.0	3.1355
1.7	0.5306	4.7	1.5476	7.7	2.0412	11.75	2.4636	23.5	3.1570
1.8	0.5876	4.8	1.5686	7.8	2.0541	12.00	2.4849	24.0	3.1781
1.9	0.6419	4.9	1.5892	7.9	2.0669	12.25	2.5052	24.5	3.1987
2.0	0.6931	5.0	1.6094	8.0	2.0794	12.50	2.5262	25.0	3.2189
2.1	0.7419	5.1	1.6292	8.1	2.0919	12.75	2.5455	26.0	3.2581
2.2	0.7885	5.2	1.6487	8.2	2.1041	13.00	2.5649	27.0	3.2958
2.3	0.8329	5.3	1.6677	8.3	2.1163	13.25	2.5840	28.0	3.3322
2.4	0.8755	5.4	1.6864	8.4	2.1282	13.50	2.6027	29.0	3.3673
2.5	0.9163	5.5	1.7047	8.5	2.1401	13.75	2.6211	30.0	3.4012
2.6	0.9555	5.6	1.7228	8.6	2.1518	14.00	2.6391	31.0	3.4340
2.7	0.9933	5.7	1.7405	8.7	2.1633	14.25	2.6567	32.0	3.4657
2.8	1.0296	5.8	1.7579	8.8	2.1748	14.50	2.6740	33.0	3.4965
2.9	1.0647	5.9	1.7750	8.9	2.1861	14.75	2.6913	34.0	3.5263
3.0	1.0986	6.0	1.7918	9.0	2.1972	15.00	2.7081	35.0	3.5553
3.1	1.1314	6.1	1.8083	9.1	2.2083	15.5	2.7408	36.0	3.5835
3.2	1.1632	6.2	1.8245	9.2	2.2192	16.0	2.7726	37.0	3.6109
3.3	1.1939	6.3	1.8405	9.3	2.2300	16.5	2.8034	38.0	3.6376
3.4	1.2238	6.4	1.8563	9.4	2.2407	17.0	2.8332	39.0	3.6636
3.5	1.2528	6.5	1.8718	9.5	2.2513	17.5	2.8621	40.0	3.6889
3.6	1.2809	6.6	1.8871	9.6	2.2618	18.0	2.8904	41.0	3.7136
3.7	1.3083	6.7	1.9021	9.7	2.2721	18.5	2.9173	42.0	3.7377
3.8	1.3350	6.8	1.9169	9.8	2.2824	19.0	2.9444	43.0	3.7612
3.9	1.3610	6.9	1.9315	9.9	2.2925	19.5	2.9703	44.0	3.7842

EXERCISES.

1. Using the four-figure log tables (II.) write down the logarithms of the following numbers:—

4769, 324.6, 2300, 6.73, 0.5942, 0.003961, 0.0243.

Ans. 3.6784, 2.5113, 3.3617, 0.8280, 1.7739, 3.5978, 2.3856.

2. Using the table of antilogs (III.), write down the numbers whose logs are :—

$$2.316, \quad 1.296, \quad 0.7400, \quad 0.061.$$

$$\text{Ans. } 207.0, 19.77, 5.495, 1.151.$$

3. Find the values of the following operations :—

$$(98.31)^3, \quad \sqrt[4]{3974}, \quad \sqrt[5]{(12.32)^2}.$$

$$39.62 \times 897.4, \quad 0.6159 \times 0.0321, \\ 0.7854 \div 2.694, \quad 0.0039 \div 9.764.$$

$$\text{Ans. } 950100, 7.939, 2.73.$$

$$\text{Ans. } 35570, 0.01977, 0.2915, 0.0003995.$$

$$4. \text{ Find the value of } \frac{\sqrt[3]{(29.6)^2} \times 33.6}{\sqrt{3.206} \div 2.6} \quad \text{Ans. } 466.8.$$

$$5. \text{ Find the value of } \frac{0.56 \times \sqrt[3]{0.65} \times \sqrt{2564}}{0.396 \times 20.4} \quad \text{Ans. } 3.04.$$

6. Calculate, using the table of common logs, the Napierian logs of 9.3, 23.5, 684.3.

$$\text{Ans. } 2.23, 3.157, 6.528.$$

For additional exercises the student will find ample scope in the numerous problems given in the following chapters.

CHAPTER IV.

MECHANICAL CALCULATION DEVICES.

The ordinary 10-inch or Gravet Slide Rule.—If the student will refer to the worked examples in the previous chapter on logarithms, he will notice that each calculation ultimately reduces down to the mere addition or subtraction of the logarithms of the terms involved, and we now proceed to show how these additions and subtractions can be performed mechanically by moving one graduated scale over another.

By way of simple illustration, first notice how *numbers represented by units of length (e.g. inches) can be added or subtracted mechanically by means of two ordinary foot-rules*. Let it be required to find the sum of $4 + 2$. Referring to fig. 68, call the upper circle or scale X and the lower one Y. Move the rule X to the right until the commencement of the scale coincides with one of the terms in the addition, say 2, on the lower scale Y, and read the sum of the two numbers 6 on the scale Y immediately below the second term 4 on X. Here the length of 4 inches has been added or placed end to end with the length of 2 inches, the sum being registered as 6 on the lower scale. Similarly,

$2 + 5$ is given as 7 on the Y scale.

$2 + 6$ „ 8 „

The process of subtraction is obviously the reverse of this, and hardly needs explanation. Thus, referring to fig. 68, p. 88,

$$ac - bc_1 = ab \quad \text{or} \quad 6 - 4 = 2$$

$$ae - be = ab \quad \text{or} \quad 8 - 6 = 2,$$

so that to subtract a length (or number) on the X scale from a length (or number) on the Y scale, place the number to be subtracted on the X scale coincident with the number from which it has to be taken on the Y scale, and read the difference on the scale Y just below the left index or end of the scale X.

At this stage we should strongly recommend the student to obtain two equal scales divided throughout into inches and tenths

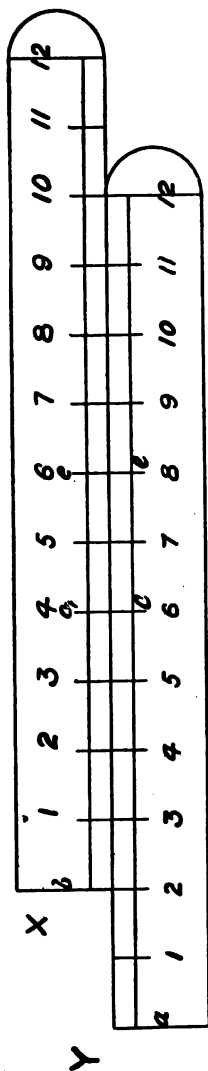


Fig. 88.

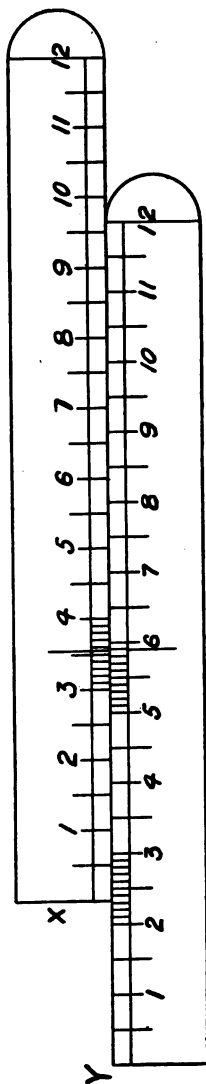


Fig. 89.

of an inch or centimetres and millimetres, and to practise the mechanical addition and subtraction of numbers. Having two

such scales divided into inches and tenths, let it be required to find the sum of 2·32 and 3·57. Referring to fig. 69, which shows sufficient of the divisions for the purpose of illustration, we notice the end of the X scale is brought between the 2·30 and the 2·40 divisions on the Y scale, its actual position being determined by imagining the fourth tenth divided into ten equal parts giving hundredths or second places of decimals, the end of the X scale being brought to the second of these imaginary divisions. The distance from the end of the Y scale to the end of the X scale will then represent 2·32. Having fixed the position of the first term in the sum on the lower scale, take a fine hair and, scanning along the X scale, stretch it across the position marking the second term 3·57 (see fig. 69). The required sum, 5·89, is read on the Y scale just below the stretched hair, the second place of decimals being judged as before.

In the examples given we have taken the inch unit of length to represent unity in number, so that the numbers 1, 2, 3, 4, etc., were represented by 1 inch, 2 inches, 3 inches, 4 inches, etc.; but we would here remark that the inch unit of length may be taken as representing any multiple or submultiple of 10, as 10, 100, 1000, 10,000, etc., or ·1, ·01, ·001, ·0001, etc., and the sum of two or more sets of numbers be found in the same way.

Hence, if 1 inch in length = 10 units in number, the divisions on the scale 1 inch, 2 inches, 3 inches, 4 inches, etc., would represent 10, 20, 30, 40, etc., intermediate tenths of an inch, and the imaginary divisions would give units and tenths.

Again, by referring to fig. 69, we should then read $23\cdot2 + 35\cdot7$ as 58·9. Or if 1 inch in length = 100 units in number, then $232 + 357$ is read 589. Further, if 1 inch in length represents the submultiple 0·1 in number, then $0\cdot232 + 0\cdot357$ is read on the scale as 0·589, and so on.

The next stage in the explanation of the use of the slide rule is to make clear the principle on which a logarithm scale is constructed. Any length ab , such as 10 inches, upon which to construct a logarithmic scale may be taken, and this length we shall imagine to be divided into a thousand equal divisions, first by dividing it into 10 equal parts, then by subdividing these 10ths into 10, and finally each of the subdivisions into 10 again (see fig. 70A). Now, dealing with the numbers 1, 2, 3, 4, ... to 10, distances are marked and figured above the scale numbers corresponding to the logarithms, thus:—

The log of 2 being ·301, the figure 2 is placed at the 301st division on the scale;

- The log of 3 being $\cdot477$, the figure 3 is placed at the 477th division on the scale ;
 The log of 4 being $\cdot602$, the figure 4 is placed at the 602nd division on the scale ;
 The log of 5 being $\cdot699$, the figure 5 is placed at the 699th division on the scale ;
 The log of 6 being $\cdot778$, the figure 6 is placed at the 778th division on the scale ; and so on.

Note that, since the log of $1=0$, unity is marked on the extreme left of the scale.

In fig. 70A *numbers with their corresponding logarithms* are figured on the line representing the logarithms.

The logarithms of numbers between any two of those considered throughout the entire range of the scale would be determined and figured in a similar manner. Thus, for example, take the following numbers between 1 and 2, viz.: 1.1, 1.2, 1.3, 1.4, etc., having logarithms 041, 079, 114, 146, etc.; the small numbers 1, 2, 3, 4, etc., would be placed on the scale between 1 and 2, opposite the 41st, 79th, 114th, and 146th divisions respectively. These divisions are shown in fig. 70A, between 1 and 2, with a further subdivision for logarithms of numbers with a second place of decimals. Now if the student will imagine this process of division and figuring to be carried throughout the entire length of the scale, he will readily see that ultimately a complete logarithmic scale is arrived at.

With the slide rule itself two precisely similar scales are used in conjunction for the addition and subtraction of logarithms. Fig. 70B shows two scales, C and D, set to give say the sum of the logs of 2 and 3; and bearing in mind that *the sum of these logs is equal to the log of the product 2×3* , we have at one and the same time the product then marked against its logarithm 778, though the product and *not* the logarithm is indicated.

Or again, the difference $\cdot301$ of these logarithms gives the log of $\frac{6}{3}$, the quotient 2 being figured against its logarithm.

Now if the student has followed the explanation given, and the principles upon which the logarithmic scale is constructed, he will have no difficulty in seeing that it is unnecessary to mark the logarithms of the numbers on the scale; it is quite sufficient for the purpose of calculation to place the numbers against the division representing their logarithms.

The main divisions along the scale have been taken as representing units in number, but from the principles already explained it

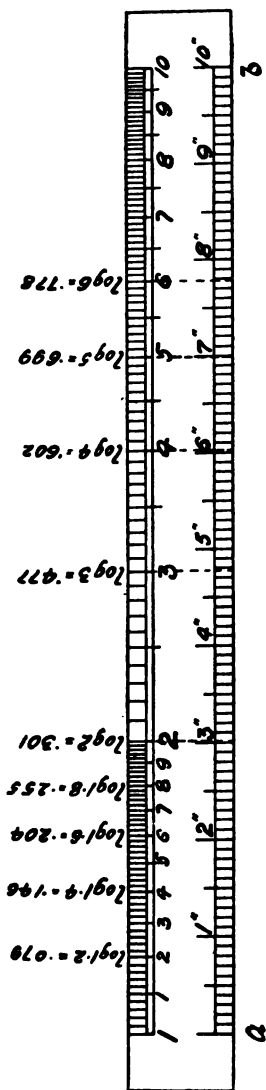


FIG. 70A.

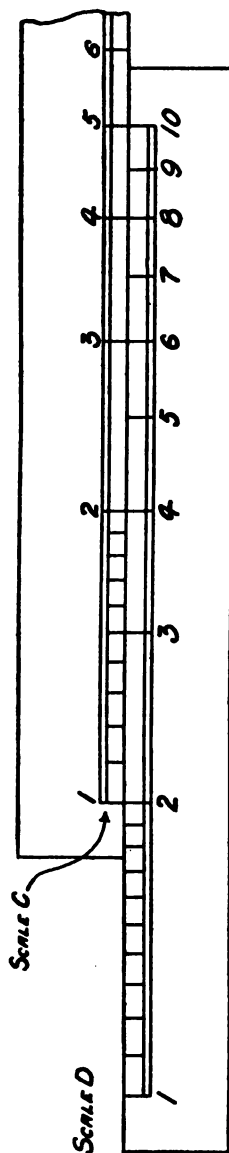


FIG. 70B.

follows that they may represent multiples or submultiples of 10; thus the division 1 to 10 may stand for 10, 20, 30, . . . to 100,

or 100, 200, 300 to 1000
 „ 0.1, 0.2, 0.3 „ 1.0
 „ 0.01, 0.02, 0.03 „ 0.1

the subdivisions being the same multiples or submultiples of 1 or 10 as the main division.

Having illustrated the principles involved in the construction and use of a pair of logarithmic scales, the student will now have very little difficulty in appreciating the great practical utility of the Gravet slide rule in its most portable and useful form, and with which we shall now deal. It is, of course, assumed that the student has provided himself with a slide rule.

The essential features of the Gravet slide rule are represented in fig. '71, Plate I. It consists of the stock, which is a little over 10 inches long, carrying two scales engraved on celluloid faces, the upper scale being designated the A scale and the bottom one the D scale. Along the length and down the middle of the stock works a slider fitting easily in grooves. On the celluloid facing of the slider are also two scales: the upper or B scale, which is identical in construction with the A scale on the stock, is marked so that its graduations move over the A scale, while the lower or C scale is identical with and works over the D scale. The cursor consists of a small metal frame carrying a piece of transparent horn or glass upon which is engraved a thin line which stands across the scales and is capable of being moved along the stock from end to end. The use of the cursor will appear presently. The extreme lines marking the ends of the scales on the slider are spoken of as right or left-hand index of A or B according as reference is made to the right end of the scale or the left. It will be noticed that the divisions on the C and D scales are just twice the length of the corresponding divisions on the A and B scales.

The operations of multiplication, division, proportion, etc., may be worked out either by means of A and B scales or by the C and D scales. Some people prefer one pair of scales, while others prefer the other, but both pairs are required to be used in certain calculations.

Multiplication and Division.—We shall first state, in the form of two rules, the methods of finding the product and quotient of numbers; though, if the student has fully understood the principles on which these calculations are based, he will not require to remember rules for the purpose.

Rule 1.—To find the product of two quantities, move the slide until the index* of the C scale coincides with one of the factors on the D scale. Then bring the cursor line over the second factor on the C scale, and read the product below the line on the D scale.

Rule 2.¹—To find the quotient of one quantity divided by another, move the slide† until the divisor on the C scale coincides with the dividend on the D scale, when the quotient will be found on the D scale coincident with the index on the C scale.

In working calculations by means of this instrument, the slide sometimes projects to the right of the stock and sometimes to the left, its position on one side or the other being dependent on the figures or terms in the calculation.

Example 1.—Find the product of 1.275×3.43 .

By Rule 1 the index of C is placed coincident with 1.275 on the D scale, and the cursor line brought over the second factor 3.43 on the C scale, the product **4.375** being read below the cursor line on the D scale (fig. 71A, Plate I.).

Example 2.—Find the value of 12.75×34.3 .

Exactly the same setting of the instrument as in Example 1 is made, only in this case the divisions have a tenfold value, and the product is **437.5**.

Example 3.—Multiply 1275 by 34.3.

The same setting of the instrument as in Examples 1 and 2. We have still the same significant figures in the product, i.e. 4375.

The number of digits in the integral part of the product in this and all cases where the slide projects to the right (C and D scales being used) is equal to the sum of the digits in the integral parts of the factors, less one. In this example there are four digits in the first factor and two in the second. Hence in the product there must be $4 + 2 - 1 = 5$. Thus the required product = **43750**, a very near approximation to the result obtained by actual multiplication, the error being exactly 1 part in 2500.

Example 4.—Find the value of 127.5×0.343 .

Reading of scale as before = 4375.

Number of digits in product = $3 + 0 - 1 = 2$.

$\therefore$ Product = **43.75**.

* Right or left index of C, as the case may be.

† The exact position of the slider is determined by bringing the cursor line over the dividend on the D scale, and bringing the divisor on the slider up to this.

Example 5.—Multiply 12.75 by 0.0343.

Reading of scale = 4375.

Number of digits in first factor = 2.

“ “ “ second “ = -1 (count the ciphers following the decimal point).

Sum of digits in product = $2 - 1 - 1 = 0$.

∴ Product = **0.4375**.

Example 6.—Multiply 0.01275 by 0.00343.

Reading of scale = 4375.

Sum of digits = $-1 + (-2) = -3$.

Number of digits in product = $-3 - 1 = -4$.

Hence product = **0.00004375**.

Example 7.—Divide 43750 by 34.3.

First bring the cursor over 4375 on the D scale, and move the slide to the right until the numbers in the divisor 343 are coincident with this line. The required quotient **1275** will be found on the D scale coincident with the left index of the slide (refer to Rule 2, p. 93, and fig. 71A, Plate I.).

Example 8.—Find the value of $\frac{4375}{3.43}$.

With the same setting of the slide as in Example 7, the required quotient is given as **1275**.

Example 9.—Divide 43.75 by 3.43.

Again, with the same setting of the slide, we have the quotient **12.75**.

In the last three exercises on division it is a simple matter to determine by inspection the number of significant figures preceding the decimal point on the left, but in some calculations the number of figures is not quite so obvious. It will be found, however, that *in all cases where the slide projects to the right, the C and D scales being used, the required number of figures in the quotient is equal to the sum of the digits in the dividend minus the sum of the digits in the divisor, plus one*, and this the student can verify in the particular exercises given.

Example 10.—Find the value of $\frac{43.75}{0.0343}$.

The numbers in the quotient stand as 1275, but the position of the decimal point has to be determined.

Number of digits in dividend = 2.

“ “ divisor = -1.

∴ “ “ quotient = $2 - (-1) + 1 = 4$.

∴ Quotient = **1275**.

Example 11.—Divide $\cdot 04375$ by $0\cdot 00343$. The quotient contains the figures 1275 as before.

Number of digits in dividend = - 1.

 " " divisor = - 2.

Hence " " quotient = - 1 - (- 2) + 1 = 2.

 " $\therefore$ Quotient = **12·75**.

Example 12.—Divide $\cdot 0004375$ by $\cdot 0343$.

Reading of quotient on scale = 1275.

Number of digits in quotient = - 3 - (- 1) + 1 = - 1.

$\therefore$ Quotient = **0·01275**.

Having now taken a number of calculations with the slide projecting to the right, we shall now consider a product or quotient with the slide projecting to the left; the same two rules given on p. 93 apply, only *in this case the number of digits in the product of two quantities is equal to the sum of the number of digits of the factors, while the number of digits in the quotient is found by subtracting the number of digits in the divisor from the number of digits in the dividend.*

Example 13.—Multiply 752 by 189·5.

Bring the right index of the slide to one of the factors 752 on the D scale, slide the cursor over second factor 189·5 on the C scale, and read the significant figures 1425 in the product below the cursor line on the D scale (see fig. 71c, Plate I.).

Number of digits in product = 3 + 3 = 6.

$\therefore$ Product = **142500** (approx.).

Example 14.—Divide 1425 by 189·5.

With the same setting of the slide as in the previous exercise we obtain the same significant figures 752.

Number of digits in quotient = 4 - 3 = 1.

$\therefore$ Quotient = **7·52**.

Continued Multiplication.—To find the product of three or four numbers, first choose two of the factors and find their product in the manner already explained. The cursor being over the product on the D scale, move the slide until the right or left index, as the case may be, coincides with the line on the cursor; then move the cursor over the third factor on the slide, and read the product of the three factors on the D scale coincident with the index on C.

For a product of four or more factors this process would be repeated, multiplying each product obtained by an additional

factor until the value of the product of all factors in the multiplication is arrived at.

Example 15.—Find the value of $16.84 \times 21.5 \times 9.475$.

Using C and D scales, we might first place the right index of C over 9.475 on the D scale, then placing the cursor over a second factor on the C scale, say 16.84, we obtain the product of these two quantities below the cursor on the D scale. It is not necessary to read this product, but we might remember that, since the slide projects to the left, the number of digits this product contains is equal to the sum of the digits in the factor, viz. 3. Now bring the left index of C to the cursor and then slide the cursor to the third factor 21.5 on the C scale and read the significant figures 343 in the product on the D scale immediately below the cursor line. Since the slide then projects to the right, the number of digits in this continued product must equal $3 + 2 - 1 = 4$. Hence the product $16.84 \times 21.5 \times 9.475 = 3430$.

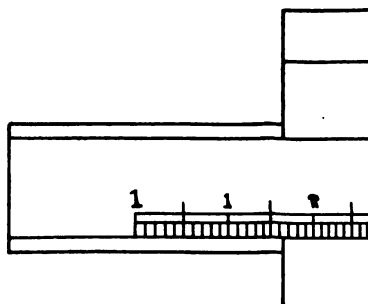
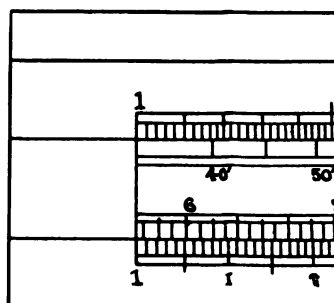
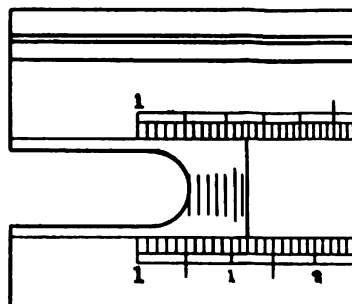
Combined Multiplication and Division.—At this stage it is hardly necessary to give any lengthy description of the method of finding the value of an expression in the form of $\frac{a \times b \times c \times \dots}{d \times e \times \dots}$.

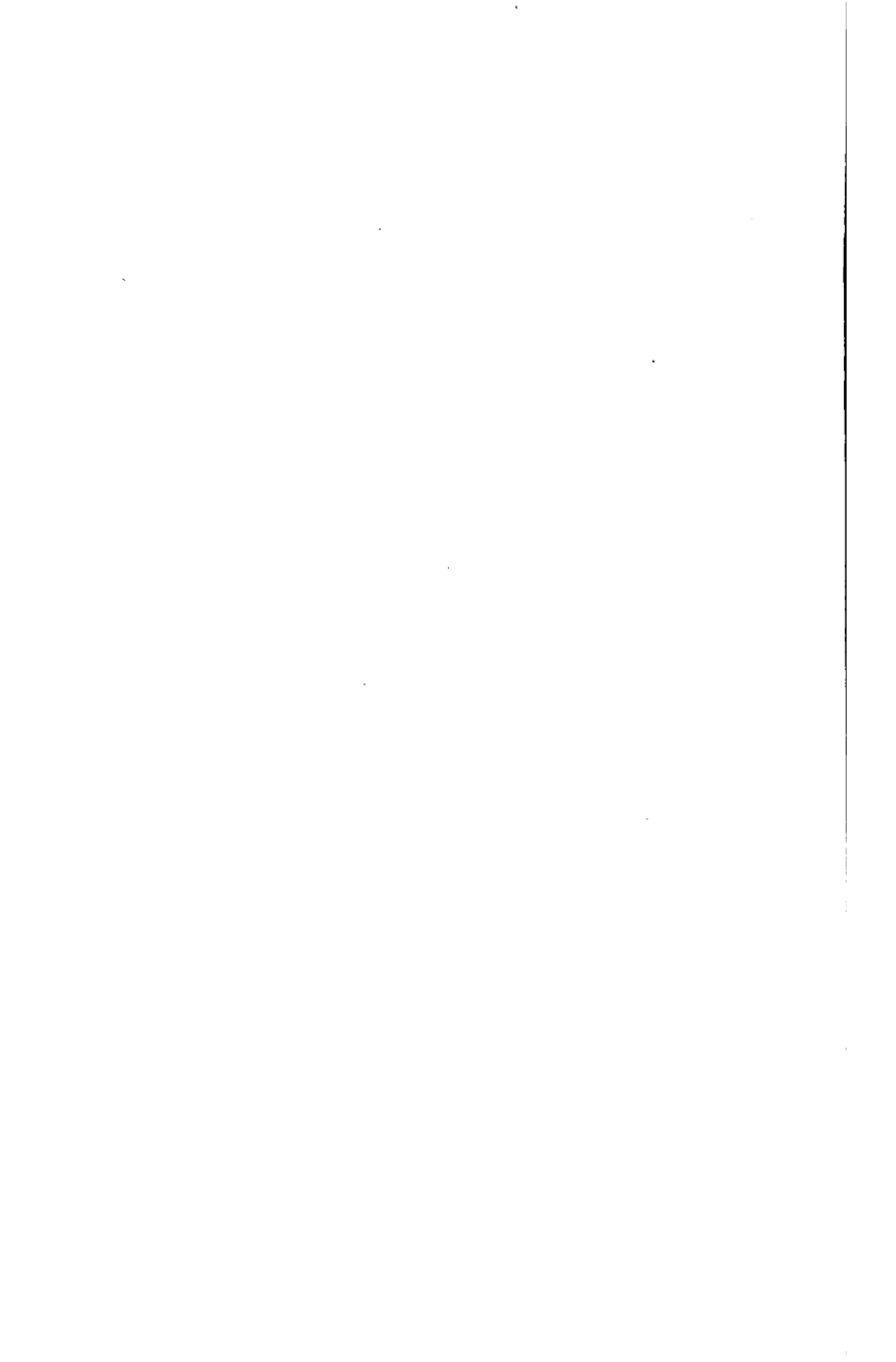
where $a, b, c, d, e, f, \dots$ represent any numbers. It will be sufficient to emphasise the fact we have already stated, that products are merely additions by the slide rule and quotients subtractions, and to illustrate the method of dealing with such a quantity by an example.

Example 16.—Find the value of $\frac{42.7 \times 7.53 \times 1.74}{33.4 \times 0.0925}$.

Here we shall find it convenient to commence with a division. Set the cursor to 42.7 on the D scale and move the slide until 33.4 on the C scale is below the line: the quotient is given on the D scale coincident with the index of C. It is not necessary, however, to read this quotient, but we may remember the number of digits (1) it contains. This quotient may then be multiplied by the second factor 7.53 on the C scale by bringing the cursor over that division, thus defining the product on the D scale. Since the slide still projects to the right, the number of figures in the operation, so far, by rules already given, is 2. Now, leaving the cursor in its present position, move the slide until the next divisor 925 falls below the line. The quotient and product of this by 1.74 is read as 1812 on the D scale coincident with the left index on the C scale. Number of digits in this result = 3.

The number of digits may be determined either by rough cancelling or by repeated application of the two rules already given.





Proportion, Ratios, Percentages.—We have already stated (p. 13) that in any proportion the product of the extremes equals the product of the means, so that the fourth term in proportion equals the product of the second and third terms divided by the first; thus, if we represent our proportions so— $a : b :: c : d$, $d = \frac{b \times c}{a}$. Hence the fourth term can usually be found by one setting

of the slide and cursor by dividing one of the factors b or c by a and multiplying by the second factor in the numerator.

Since, however, we have shown that a proportion consists of a pair of equal ratios, calculations on the slide rule can be readily dealt with if we remember that for a given setting of the slide any two numbers on the C scale are to each other in the same ratio as the numbers on the D scale immediately below them. This we may state as follows:—

Any number on the C scale : the number below it on the D scale
 $::$ any other number on the C scale : the number below it
 on the D scale; or in this way:—

Any number on the C scale : any other number on the C scale
 $::$ the number below the first on the D scale : the number
 below the second on the D scale.

Thus, for example, take the setting of the slide shown in fig. 71A, Plate I., we get $33 : 42 :: 78.6 : 100$. This illustration shows at once how we may use the slide rule to determine percentages by setting the first terms of the proportion on the one scale over the second term on the other and reading the percentage over the extreme index of the scale.

Example 17.—A student at an examination obtains 375 marks out of 580, what is the percentage on the maximum? Here 375 on the slide is set over 580 on the stock, and the percentage 64.7 is read on the slide over the index on the stock.

Example 18.—Find the fourth term x in the proportion

$$26.2 : 59.5 :: 125 : x.$$

Set 59.5 on the B scale below 26.2 on the A scale and read the fourth term in the proportion 283 on the B scale below the third term 125 on the A scale. The proportion then stands:—

$$26.2 : 59.5 :: 125 : 284.$$

It is thus obvious that, given any three terms, we could determine the fourth by one setting of the slide.

Example 19.—Find a number of ratios each equal to $\frac{17}{16}$.

Set 17 on the slide over 16 on the stock, then scan along the two scales and pick out any coincident pairs of numbers, one taken from each scale. Thus we get $\frac{42.5}{40}, \frac{50}{47},$ (approx.) $\frac{600}{565}, \frac{68}{64}, \frac{85}{80}, \frac{100}{94.2}.$

Determination of Powers and Roots.—In the chapter on logarithms it was explained that the logarithm of the power of any number was equal to the logarithm of the number itself multiplied by the index of the power, and also that the logarithm of the root of a number is equal to the logarithm of the number divided by the index of the root. This is the mathematical principle underlying the determination of powers and roots by means of the slide rule. In the description given of the Gravet slide rule it was pointed out that the intervals of length on the C and D scales were just twice the length of the corresponding intervals on the A and B scales, so that a logarithm or length on the D scale, defined only by the number of which it is the logarithm, has opposite to it on the A scale twice this value; and since the log of the square of a number is equal to twice the log of the number, it follows immediately that the numbers on the A scale are the squares of the numbers on the D scale, and *vice versa* the numbers on the D scale are the square roots of numbers on the A scale. Hence, to find the square of a number, place the cursor line over the number on the D scale and read its square below the line on the A scale; or, to find the square root of any number, place the cursor over the number on the A scale and read the square root below the line on the D scale.

Example 20.—Find the square of 28.8.

The cursor line being placed over 28.8 on D, we read opposite to it the number 83 on the A scale; and in order to determine the number of digits in the square of a number, we should remember the rule that, if the answer is on the left scale of A, the number of digits is equal to twice the number minus one, and if on the right scale twice the number. The result 83 is on the left scale, consequently the number of digits would be $(2 \times 2) - 1 = 3$, so that the required square is 830 (approx.).

Example 21.—Find the square root of 437.

The cursor line being placed over 437 on A, we read below it the number 209 on D, and counting the number of periods, as in ordinary arithmetic, in the number 437, we obtain two, thus the required square root is 20.9. It is unnecessary to give a rule for

the number of digits in a square root, as this can be arrived at by inspection.

The determination of the **fourth power** of any number is also a simple matter. Place the index of C over the number on D, whose fourth power is required, and over the number on C read the fourth power on A. The reason for this is not far to seek, for we notice that for any setting of the slide, its index marks on the A scale the square of the number coincident with it on the D scale. Then when the cursor is moved to the number on the C scale whose fourth power is required, underneath the line on the B scale appears also the square of the number, so that on the A scale up to the index of B is the square of the number, and on the B scale from the index to the cursor line also the square of the number, so that it follows that the product of these two squares must fall on the A scale below the cursor line, thus giving, as we should see clearly by taking a particular example, the square of the square of the quantity, which is, of course, the fourth power.

Example 22.—Find the fourth power of 13·35.

Place the index of C to 13·35 on D, move the cursor to 13·35 on the C scale, and above it on the A scale read the fourth power **31700**. The **fourth root** of a quantity could be found either by the double operation of extracting the square root of the square root, or by the reverse of the method already given for finding the fourth power of a number. If the latter method is adopted, the cursor would be placed over the number on scale A whose fourth root is required, and the scale B would then be moved until the reading of the A scale to the index was equal to the reading on the B scale to the cursor line.

Example 23.—Find the fourth root of 77·4.

Place the cursor over 77·4 on the A scale and move the slide until the reading on the A scale coincident with the index of B is equal to the reading on the B scale below the cursor line, when the fourth root, **2·96**, will be found on the D scale below the left index of slide.

On the back of the slide are engraved three separate scales for finding (1) the **logarithms of numbers**, (2) the values of the **sines of angles**, and (3) the values for the **tangents of angles**. On the stock of the slide rule at each end there is a gap bevelled on the reverse side carrying two marks. First considering the gap on the extreme right with the rule turned over, we notice that the lower mark stands over the logarithmic scale, while the upper one is over the sine scale. To find the logarithm of a number, place the left index of the C scale (front face)

coincident with the number whose logarithm is required. Then reverse or turn over the scale, and read the logarithm on the lower mark in the right-hand gap. It is hardly necessary to point out that the scale of logarithms gives the mantissa of the logarithm, the characteristic being determined as already explained.

Examples.—Find the logarithms of the numbers 1 to 20, and refer to the table of common logarithms, pp. 78 to 80, to test the degree of accuracy.

To find the sine of an angle, with the stock of the rule reversed, pull out the slide until the upper mark on the gap coincides with the angle whose sine is required. Then reverse the slide rule to show the front face, and read the value of the sine on the B scale opposite to the right-hand index of the stock.

Examples.—Find values for the sines of the following angles, 12° , 16° , 18° , 27° , 34° , 50° , and 6° , and check the results by reference to a table of sines.

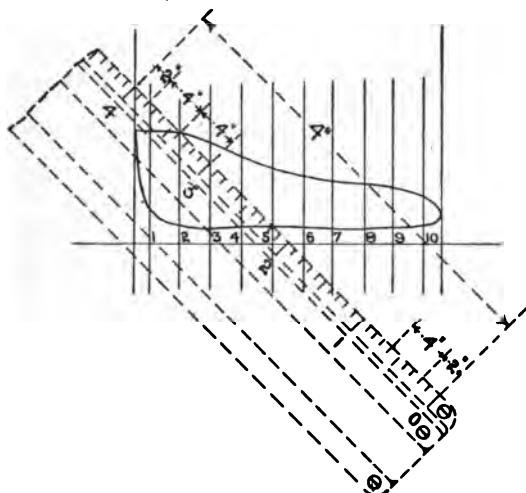
To find the tangent of an angle, reverse the rule, pull out the slide, until the value of the angle on the bottom scale coincides with the lower mark in the left-hand gap of the stock. Then reverse the rule to show the front face, and read the value of the tangent on the C scale above the left index on the D scale.

Examples.—Find the values of the tangents for angles of 5° , 10° , 15° , 20° , 30° , 40° , and 50° , then refer to table of tangents, and test the degree of accuracy.

Improved Pointed Cursor for Slide Rule.—A recent improvement has been introduced in the modern slide rule by the addition of a pointed cursor sliding over a fixed scale on the bevelled edge of the rule. This scale is an open one (*i.e.* not logarithmic), and the rules are supplied with scales graduated either in inches and fiftieths or centimetres and millimetres. One very important use to which this additional scale and pointed cursor can be put is in the calculation of the area of an irregular figure by mechanically summing up the ordinates. We will take, for example, the determination of the mean effective pressure from an indicator diagram (one of the most useful purposes to which the pointed cursor lends itself when a planimeter or other measuring instrument is not available). By dividing the indicator diagram into ten strips and drawing the centre line through each, as shown in fig. 72, p. 101, the length of each of the ten ordinates, representing pressures, can be successively added upon the rule without writing down a single figure. When the total length of the ten ordinates is thus obtained, then, by simply moving the decimal point one place to the left, the mean length of the ordinates

representing the mean height of the diagram in inches is given. This mean length has only to be multiplied by the scale of the spring to give the mean effective pressure in lbs. per square inch. With a little practice the m.e.p. can be found in this way almost as quickly as with a planimeter, and to within a sufficient degree of accuracy for all practical purposes. The method of using the rule whilst adding up the ordinates is shown clearly in fig. 73 and needs no further explanation.

Another important use for the pointed cursor is in the plotting

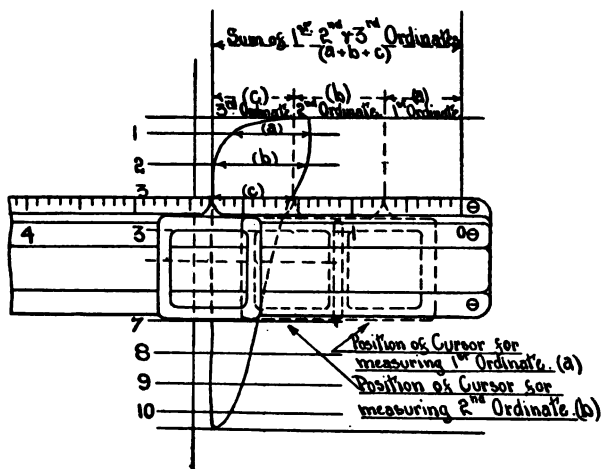


Method of Dividing Diagram.

FIG. 72.

of lengths proportional to the logarithms of numbers. If the cursor be set to a series of numbers on one of the logarithmic scales, the lengths marked off by the pointer on the open scale will be proportional to the logarithms of such numbers. This is of very great value to engineers, as affording a ready means of drawing any curve of the series $p.v.^n = \text{a constant}$ by a graphical method. Take, for example, the adiabatic expansion curve of 1 lb. of dry saturated steam following the law $p.v.^{1.135} = \text{a constant}$, from say, 170 lbs. pressure to 30 lbs. pressure. For this purpose it will be necessary to draw two diagrams, one the ordinary $p.v.$ diagram, shown in fig. 74, p. 103, and the second a logarithmic diagram, as

in fig. 75, p. 104, the ordinates of the latter being proportional to the logarithms of the pressures, and the horizontal distances or abscissæ proportional to the logarithms of the volumes. Commencing with the *p.v.* diagram, fig. 74, on squared paper, draw the two axes OX and OY and graduate the latter to represent absolute pressures from 0 to 170 lbs. Draw also the two axes OX and OY of fig. 75. Now mark off any series of points along OY, such as 1, 2, 3, 4, etc., on the *p.v.* diagram and plot their corresponding pressures. Mark off a corresponding series of points 1, 2, 3, 4, etc., on the logarithmic diagram, fig. 75, proportional to the



Mechanical Addition of Ordinates.

FIG. 78.

logarithms of the pressures by means of scale A on the slide rule. For instance, the pressures taken in the figures are 170, 157, 143, 128 lbs., therefore the distances 01, 02, 03, 04, etc., in fig. 75 are proportional to the logarithms of these numbers. Now construct the volume scale for fig. 74 along OX and set off the volume of the steam equal to the weight that is being expanded. If this be 1 lb., make AB, fig. 74, equal to 2.649 cubic feet, the specific volume of 1 lb. of dry saturated steam at 170 lbs. absolute pressure. Also set off along *ab*, fig. 75, a distance from scale A of rule equal to the logarithm of 2.649, and draw the line *bc*, making an angle θ to the horizontal such that $\tan \theta = 1.135$, the

exponent of the expansion. Next project the lines 1, 2, 3, 4, etc., across on fig. 75, as shown, and measure the lengths of the

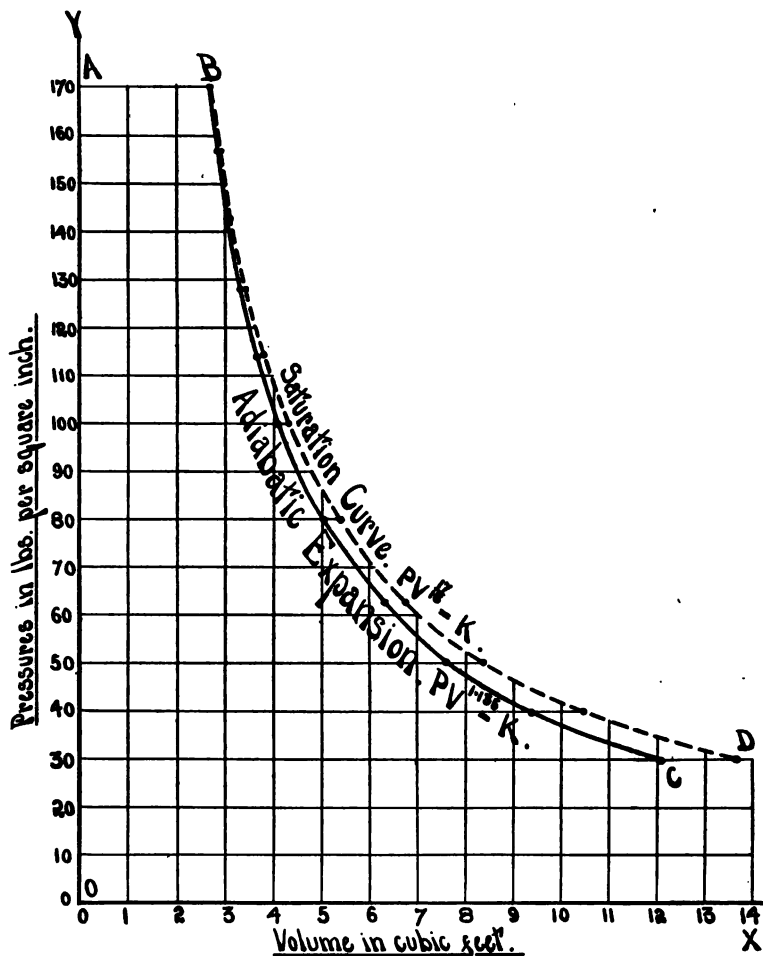


FIG. 74.

ordinates intersected by bc on the same logarithmic scale A of the rule by means of the pointed cursor. These lengths will represent the volumes occupied by the steam at the various pressures taken, and they are transferred successively direct to fig. 74 by means of

the volume scale. They will then form points on the expansion curve BC which should be drawn in (see fig. 74).

The student should plot a similar curve by the same method for the expansion of dry saturated steam, altering the angle of bc (fig. 75) such that $\tan \theta = \frac{1}{17}$. He will thus have a graphic representation of the difference between the two curves, showing the condensation that occurs during adiabatic expansion of dry

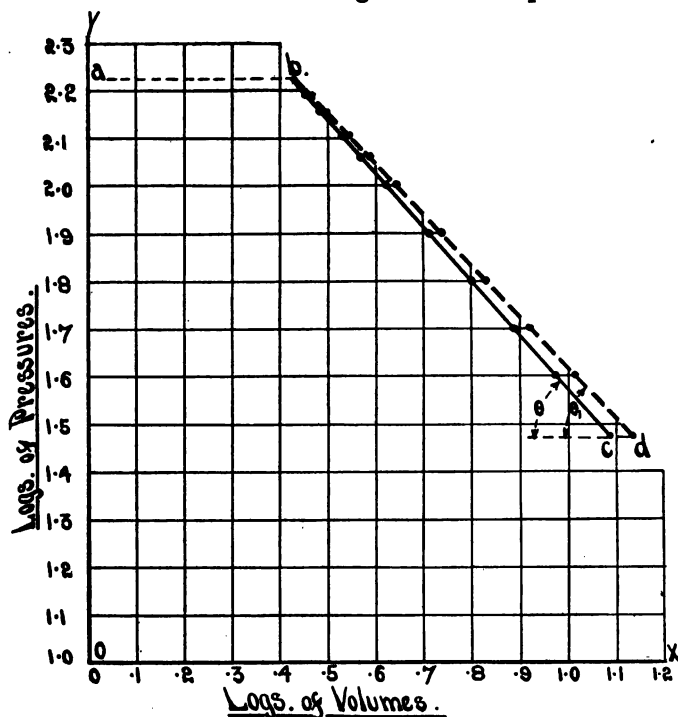


FIG. 75.

saturated steam. This curve is shown dotted in fig. 75 and marked bd .

The Watch or Pocket Calculator.—The circular form of the slide rule of a size and shape suitable for pocket use has been on the market for several years, but it has recently come more into prominence. The original rule had a double metallic face, one at the front and one at the back, rotated together by a milled nut, with separate movable pointers, but the more popular form has

only a single face on the dial. It is shown in fig. 76, and consists of a circular disc or dial engraved with five concentric scales marked A, B, C, D, and E in diagram. It is provided with two hands or pointers F and G, the former being fixed to the casing of the instrument, while the latter can be given rotary motion by means of the milled nut H at the side. The complete dial can

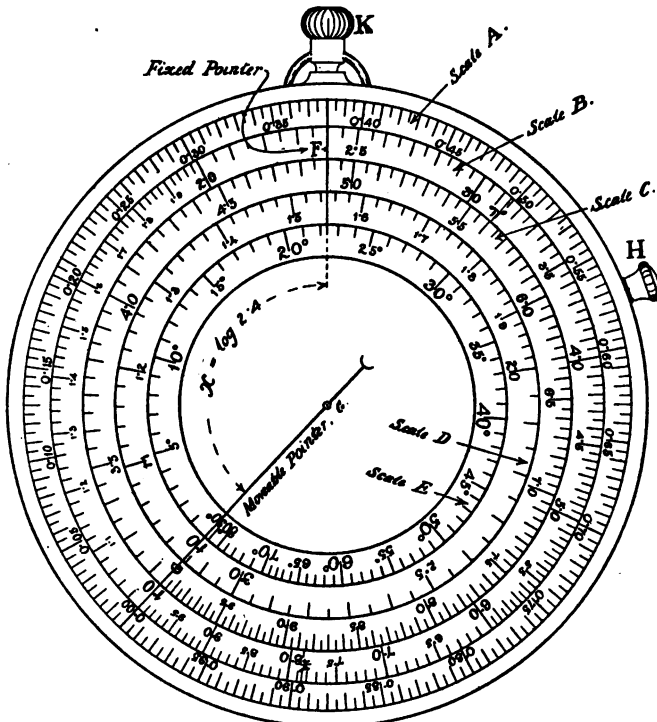


FIG. 76.

also be revolved by means of the milled head K at the top of the instrument.

Turning now to the scales engraved on the dial, the outer scale A is an open or evenly-divided scale graduated into 200 equal parts, and used to find logarithms of numbers or the sines of angles. The second scale B is a logarithmic scale, similar to the first part of the top scale A on an ordinary slide rule, the graduations being proportional to the logarithms of numbers from 1 to 10,

as explained in the previous paragraphs on the slide rule. This scale is mainly used for ordinary multiplication and division. The next two scales C and D are logarithmic scales of double the length of scale B, and are similar to the bottom scale D of an ordinary slide rule; that is, the two scales C and D are really continuous, and form one complete scale equal in length to two revolutions of the dial, and are primarily used for taking powers and roots,

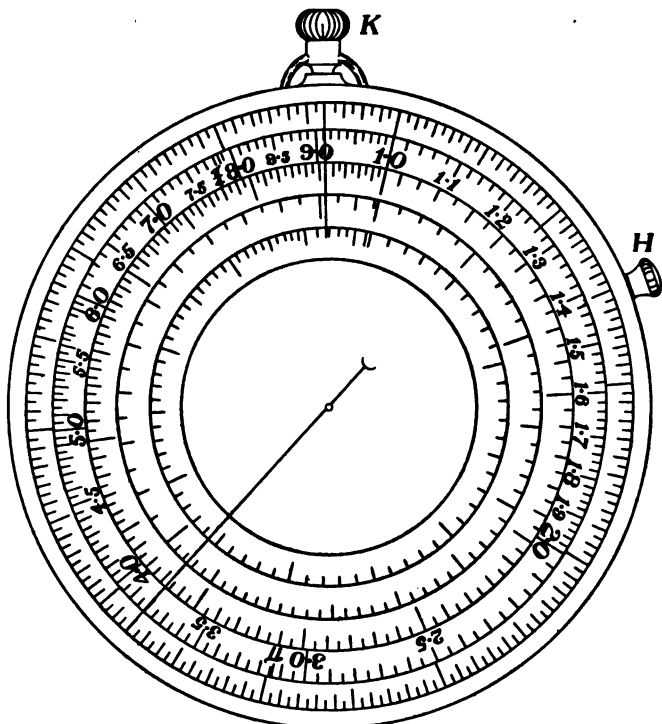


FIG. 77.

although they can be used for multiplication and division, as will be explained later. The inner scale of all, scale E, is a logarithmic scale of angles, graduated from 0° to 90° , for use in obtaining various trigonometrical functions.

As the student is now familiar with the use of the ordinary slide rule, a detailed account of the principles of the construction of this instrument is unnecessary. We now pass on to a number of examples descriptive of the method of using it.

Multiplication of two or more numbers.—To multiply any two numbers, say 2.4×3.8 , revolve the disc until 2.4 on scale B is under the fixed pointer F, as shown in fig. 76, and by means of the milled nut H set the movable pointer or hand G to 1. The circular distance x between the two pointers as measured on scale B will thus be equal to the logarithm of 2.4. Now rotate the

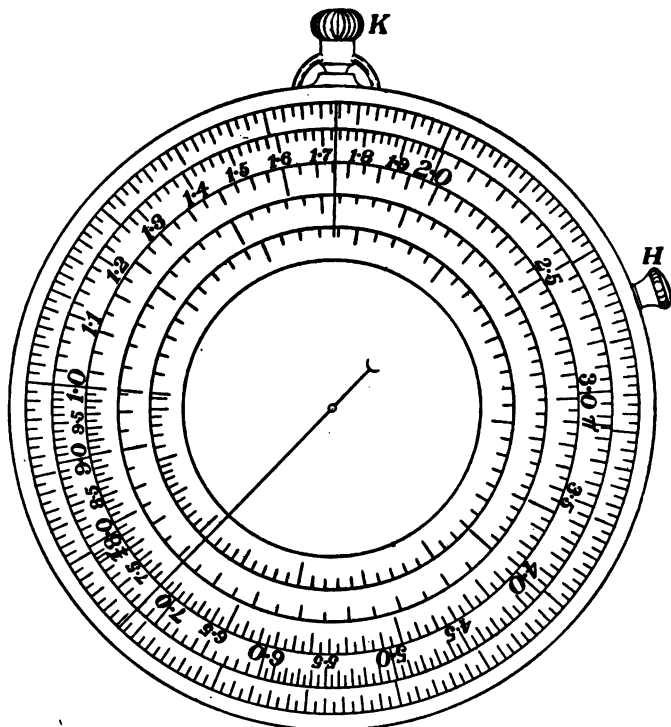


FIG. 78.

dial by means of knob K until the movable pointer G is over 3.8 on scale B, and read the answer 9.12 under the fixed pointer F on scale B, as shown in fig. 77, p. 106. It should be noted that by turning the dial the second time we are really adding a distance y equal to the logarithm of 3.8 to the distance x already set between the two pointers, so that the total distance $x + y$ is recording a distance on scale B equal to $\log 2.4 + \log 3.8$ or the logarithm of the product.

The rule for fixing the position of the decimal point is similar to that adopted in the ordinary slide rule. Where the answer is read off the disc without making one complete revolution (i.e. $x + y$ is less than 360°), the number of digits in the integral part of the product is one less than the sum of the integral digits composing the two numbers or factors. Thus in the example just

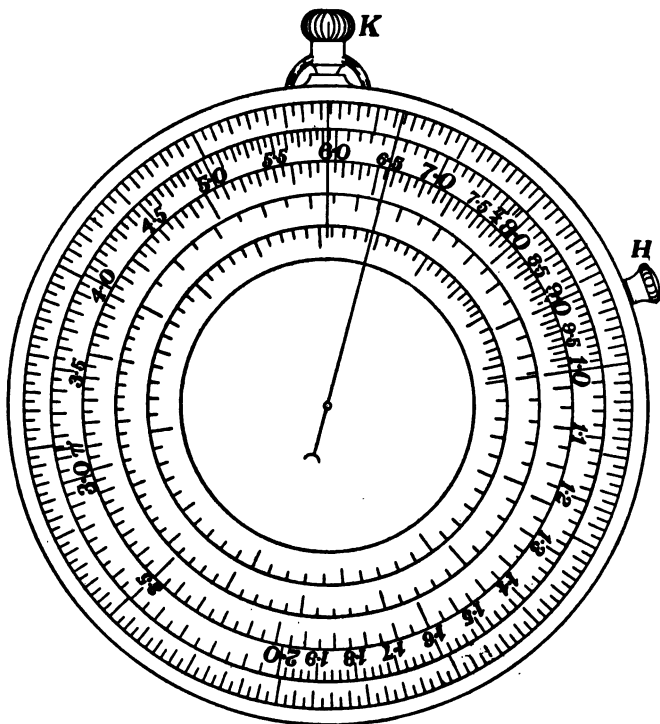


FIG. 79.

given the sum of the integral digits in the two factors is 2, and subtracting 1 we obtain one integer to the left of the decimal (9.12). If the sum of $x + y$ exceeds 360° , the number of integers in the product is equal to the sum of the whole numbers in the two factors, as for instance $2.4 \times 7.2 = 17.3$ (see fig. 78).

To multiply more than two numbers together, say $2.4 \times 3.8 \times 6.5$, the same process is observed, except that after setting the disc to 3.8 the movable pointer is again brought back to 1, and the dial

again rotated until 6.5 on scale B is underneath the movable pointer G (see fig. 79, p. 108), when the answer 59.3 is read below the fixed pointer F. In this way the extra distance z equal to the logarithm of 6.5 is added to the distance $x+y$ already obtained, so that the total distance $x+y+z$ gives the logarithm of the answer.

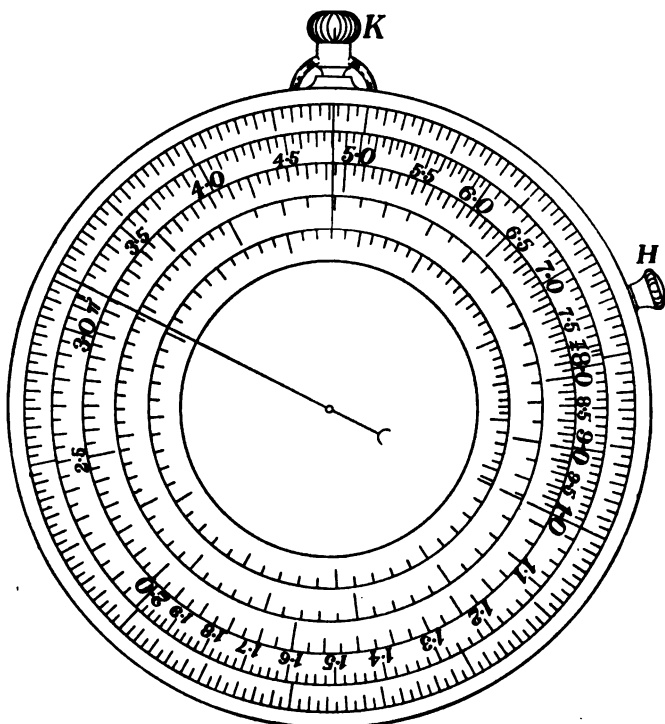


FIG. 80.

Exercises for practice by the student :—

$$\begin{array}{ll} 2.8 \times 3.0 = 8.4. & 4.45 \times 22.5 = 100. \\ 1.72 \times 4.3 = 7.4. & 0.064 \times 270 = 17.3. \\ 1.085 \times 0.415 = 0.45. & \end{array}$$

The Process of Division.—To find the quotient of one number by another number, as for instance $4.8 \div 3.2$, set the dial so that 4.8 on scale B is immediately underneath the fixed pointer F, and bring the movable pointer G over 3.2 on the same scale by means

of the milled nut H. In this way we represent two distances or lengths x and y on the dial (see fig. 80, p. 109), the former being equal to the log 4·8 and the latter to the log 3·2, and it only remains to measure the difference between the two distances ($x - y$) on the logarithmic scale B. This is done by rotating the dial by the milled head K until its zero or 1 is under the movable

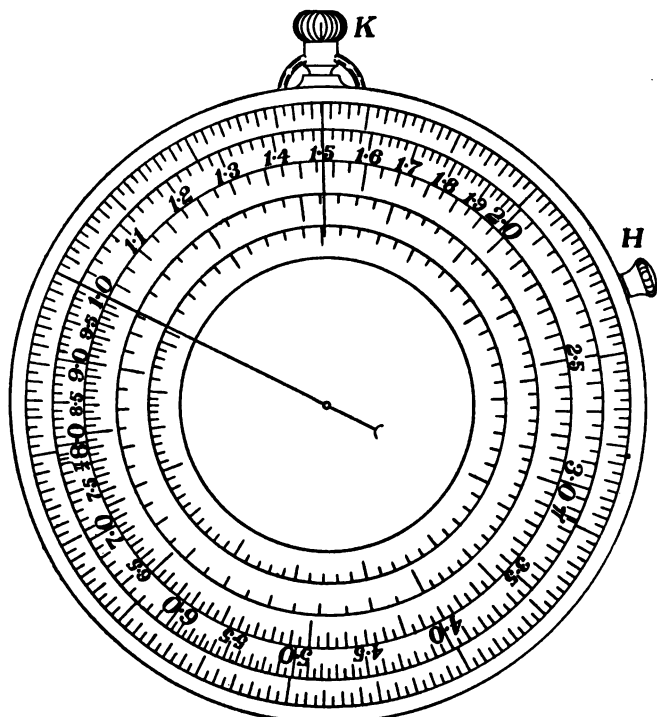


FIG. 81.

pointer G as shown in fig. 81, when the answer 1·5 is read from scale B underneath the fixed pointer F. By remembering the two simple rules that the multiplication of numbers is effected by adding their logarithms, and the division of numbers by subtracting the logs of divisors from the logs of dividends, the student should have no difficulty, after a little practice, in using this very handy form of calculating instrument.

The rule for fixing the position of the decimal point in the

quotient is, where the distance y is less than x , subtract the number of digits in y from those in x and add 1. Where, however, y is greater than x , the quotient will have as many whole numbers before the decimal point as the difference between the numbers of digits in x and y . Where the numbers of digits in x and y are equal, but y is greater than x , the quotient will be a decimal quantity, less than 1 and greater than $\frac{1}{10}$; but where the number of digits in y exceeds that in x , that is, where $x - y$ is a minus quantity, the answer will be a decimal quantity with as many ciphers following the decimal point as $(x - y)$.

For example, $\frac{4.8}{3.2}$; $x = 4.8$, $y = 3.2$, *i.e.* digits in x and y are equal, therefore quotient has $(1 - 1) + 1 = 1$ figure before the decimal point.

Again, $\frac{30}{6}$; $x = 30$, $y = 6$, *i.e.* digits in x are 2 and in y , 1, but distance y on logarithmic scale is greater than x , therefore the answer will have $(2 - 1) = 1$ number before the decimal point.

Again, $\frac{3}{600}$; here $x = 3$ and $y = 600$, and the number of digits 1 and 3 respectively, so that $(x - y) = (1 - 3) = -2$. The quotient will therefore be a decimal quantity with two ciphers following the decimal point, *i.e.* answer = 0.005. Where, however, y is greater than x , but the logarithmic distance y is less than x , *i.e.* the mantissa of the log y is less than the mantissa of the log x , the answer will have $(x - y) + 1$ ciphers following the decimal point; as in $\frac{6}{300}$, $x = 6$, $y = 300$, *i.e.* integers in x and y are 1 and 3 respectively, but the mantissa of the log y is less than the mantissa of the log x , so that answer will have $(x - y) + 1 = (1 - 3) + 1$ or 1 cipher following the decimal point; therefore the quotient = 0.02. Usually it will be easier to fix the position of the decimal point in the answer by inspection of the numbers in the case of simple examples, but with more complicated exercises the rules will form an additional check upon the position of the decimal point.

The student will do well to notice particularly the difference in the operations when multiplying as compared with division. The first operation is the same in each case, *viz.* set the first figure to the fixed pointer; but when multiplying, the movable pointer is set to 1, whereas in division it is set to the divisor. In each case the dial has to be rotated for the third operation, and in each case the answer is read off the fixed or top pointer F. With the pocket calculator, as with the slide rule, it is "practice makes perfect," only more so in the case of the former.

Exercises in division :—

$$8.4 \div 4.8 = 1.75.$$

$$12.5 \div 3.6 = 3.47.$$

$$2.45 \div 15 = 0.163.$$

$$6.85 \div 263 = 0.026.$$

$$0.32 \div 650 = 0.000492.$$

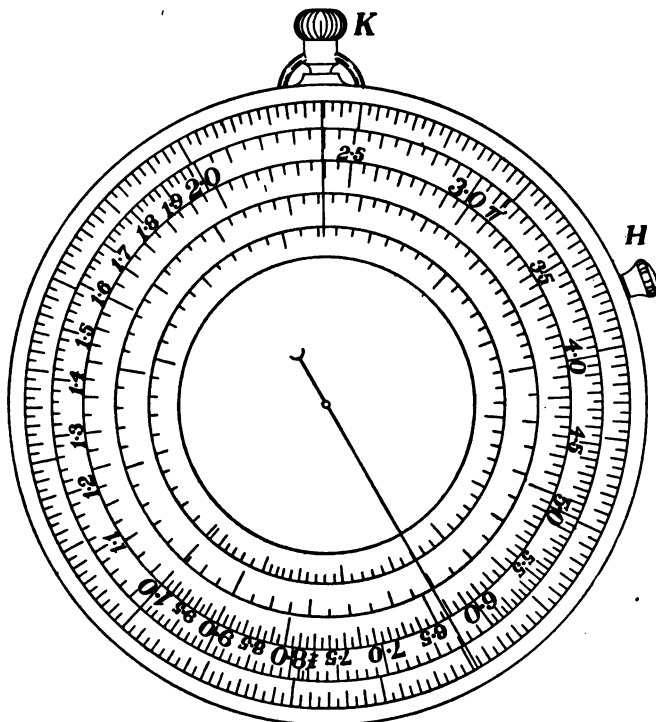


FIG. 82.

Combined Multiplication and Division.—This is only a combination of the two previous rules.

Example 1.—Find the value of $\frac{1.43 \times 6.28}{3.75}$.

Set 1.43 on the dial to the fixed pointer F, and bring the movable pointer to 3.75; then rotate dial until 6.28 is under the movable pointer G (see fig. 82), and read the answer 2.39 under the fixed pointer. The position of the decimal point is fixed by inspection.

Example 2. $\frac{34.5 \times 3.14 \times 2.5}{4.6 \times 8.3}.$

Set 34.5 (or 3.45) on the dial under the fixed pointer, and bring the movable pointer to 4.6; then turn the dial round until 3.14 (π) is under the movable pointer G. Now again set the movable pointer to 8.3 by means of nut H, and afterwards bring the dial round until 2.5 is under the movable pointer G; then the answer (7.09) can be read under the fixed pointer F. It should be remembered that the fixed pointer is only used in the first and last operations—the intermediate operations consisting of rotating the dial and the movable pointer alternately.

Exercises for practice:—

$$\begin{aligned}\frac{4.85 \times 3.68 \times 0.75}{6.82 \times 0.285} &= 6.88 \\ \frac{0.56 \times 2.45 \times 3.14}{144} &= 0.0299 \\ \frac{1.65 \times 6.48}{0.72 \times 0.044} &= 337.\end{aligned}$$

To find the mantissa of the logarithm of any number n to a base of 10 (i.e. ordinary logs), rotate the dial until the given number n on scale B is underneath the fixed pointer F, and read the answer on scale A immediately above.

Thus,	log 2	= 0.301	log 168	= 2.225
	„ 5	= 0.699	„ 0.45	= 1.653
	„ 25.4	= 1.405	„ 0.0057	= 3.756.

It must be remembered that scale A only gives the “mantissa” and not the complete logarithm, and that the characteristic or index must be added by inspection.

Conversely, to find the number corresponding to any given logarithm, set the mantissa of the logarithm on scale A, under the fixed pointer F, read the answer on scale B immediately beneath, then determine the decimal point according to the value of the characteristic of the log, thus:—

antilog of	0.845	= 7.0
„	3.658	= 4550
„	1.959	= 0.91
„	2.616	= 0.0413
„	5.903	= 0.00008.

Involution.—To find the square root of any number n , set the number n on scale B, immediately underneath the fixed

pointer F, and read the answer from either scale C or D according to the number of significant figures in n . If n has an *even* number of figures before the decimal point, the square root will be found on scale C; but if *odd*, it is given on scale D. The position of the decimal point in the root will be found by inspection.

For example :—

$$\begin{aligned}\sqrt{25} &= 5 && \text{(scale C)} \\ \sqrt{2.5} &= 1.58 && \text{(scale D)} \\ \sqrt{81} &= 9 && \text{(scale C)} \\ \sqrt{0.81} &= 0.9 && \text{(scale C)} \\ \sqrt{8.1} &= 2.84 && \text{(scale D)} \\ \sqrt{15} &= 3.87 && \text{(scale C)} \\ \sqrt{150} &= 12.25 && \text{(scale D)}.\end{aligned}$$

Evolution. To find the square of any number.—This can be done either by multiplying the number by itself on scale B, or by setting the number on scale C or D under the fixed pointer, and reading the square off scale B immediately above, thus,

$$\begin{aligned}(4.5)^2 &= 20.2 && 85^2 = 7220 \\ (0.222)^2 &= 0.0493.\end{aligned}$$

To multiply and divide using scales C and D.—In some cases where the approximate value of the answer is known, it may be preferable to perform the usual operations for multiplication and division on the scales C and D instead of on scale B, as the former are more openly divided, and the answer can be read with a greater degree of accuracy. There is, however, a great liability of reading the result on the wrong scale; and for that reason alone, it is not desirable to use these scales except on such occasions when the approximate value of the answer is known. It is advised, therefore, that the student should limit the uses of the scales C and D to examples of involution and evolution.

To obtain the simpler trigonometrical functions of angles.—Scale E (the innermost scale of all) is a logarithmic scale of angles, and the sines of all angles up to 90° can be read directly by setting the dial so that the fixed pointer F is immediately over the required angle on scale E, and reading the value of the sine off the fixed pointer on scale A.

$$\begin{aligned}\text{Thus,} \quad \sin 30^\circ &= 0.500 \\ &,, \quad 67\frac{1}{2}^\circ = 0.924 \\ &,, \quad 142^\circ = (-\sin 38^\circ) = -0.616.\end{aligned}$$

The cosine of any angle can also be found by taking the sine of its complement.

Thus, $\cos 42^\circ = \sin 48^\circ = 0.743$, and the tangent could be found (if desired) by dividing the sine by the cosine, thus,

$$\tan 22^\circ = \frac{\sin 22^\circ}{\cos 22^\circ} = \frac{0.375}{0.927} = 0.404.$$

Fuller's Spiral Slide Rule.—When very great accuracy is required, the ordinary form of slide rule or watch calculator should not be used, but the suitable calculating rule for very accurate calculations as introduced by Professor Fuller, M.Inst.C.E. This rule consists of a light cylinder C (see fig. 83) sliding and turning on a cylindrical stem D, which can be held in the left hand by the handle E. Upon the cylinder C is printed a logarithmic scale in spiral form, the total length of which is 41 feet 8 inches. Its accuracy, therefore, is equal to an ordinary slide rule of double this length. Inside the circular stem D is another light brass cylinder G, which carries two movable pointers A and B fixed to a brass strip, whilst the bottom of the stem is provided with a fixed pointer F, at the end of a light brass strip. The stem of the instrument D has printed upon it a number of useful tables and constants. The spiral scale is divided into 1000 parts, the lengths of which are proportional to the logarithms of the numbers represented from 100 to 1000, the divisions from 100 to 650 being further divided into 10 parts, and those from 650 to 1000 into 5 divisions each. In this way four significant figures can always be read correctly, and very often the fifth figure ensuring an accuracy of $\frac{1}{10000}$ or $\frac{1}{100}$ of 1 per cent. The principles underlying the use of this instrument for ordinary

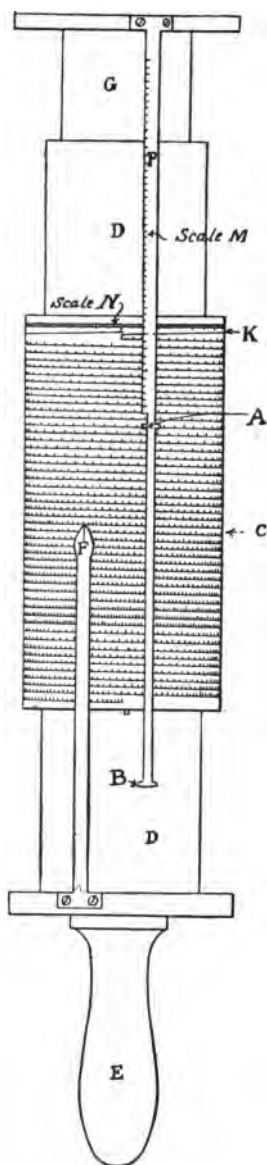


FIG. 83.

multiplication and division are exactly the same as for ordinary slide rules.

To multiply two or more numbers together, say $x \times y$.—Bring the number corresponding to x on the barrel C to the fixed pointer F and set A (the top movable pointer) to 100. Then turn the barrel C round, and slide it axially, to bring the number corresponding to y on it against either the top (A) or bottom (B) movable pointer, and read the answer from the fixed pointer F.

To multiply three or more numbers together, say $22.45 \times 3.1416 \times 1.5625 \times 0.27$. Set 22.45 to F, bring A to 100, set 3.1416 to A, and again bring A to 100, set 1.5625 to B, again bringing A to 100, and finally set 0.27 on the barrel to A and read the answer (29.754 from the fixed pointer F).

To determine the position of the decimal point, proceed as with logarithms by taking the algebraic sum of the indices of the various factors and add 1 for every time the instrument is set to B. Thus in the exercise just given the sum of the indices is

$$(1 + 0 + 0 + 1) + 1 \text{ (for setting to B)} = 1,$$

and the product therefore will consist of two figures before the decimal point.

Exercises for practice :—

$$\begin{aligned}(4.875)^2 \times 107.53 \times 0.285 &= 728.32 \\ 35.74 \times 28.274 \times 2.25 \times 156.85 &= 356620 \\ 6.0833 \times 32.182 \times 60 &= 11,747.\end{aligned}$$

Division of any two or more numbers.—To obtain the quotient of any two numbers, say $\frac{x}{y}$, set F to x , and bring either A or B to y , then bring 100 on the barrel to A, and read the quotient from the fixed pointer F.

$$\text{For example, } \frac{165.4}{25.4}.$$

Set 165.4 on the barrel to the fixed pointer F, and by means of the inner cylinder G bring B (the lower movable pointer) to 25.4. Then turn the barrel round and raise it until 100 on the barrel comes opposite A, the top movable pointer, and read the answer (6.5118) from the fixed pointer. The same sequence of operations applies when there is more than one factor in the division ;

for example, to find the value of $\frac{1728}{110.2 \times 0.5625}$, set 1728 on the barrel to the fixed pointer F and bring A to 110.2, then move 100 on the barrel to A, again bring B to 0.5625 (562.5 on the

logarithmic scale), and finally turn 100 on the barrel to A, reading the quotient (27·877) from the fixed pointer.

The position of the decimal point in the quotient is fixed by subtracting the algebraic sum of the indices of all the factors in the divisor from the index (or sum of the indices, as the case may be) of all the factors in the dividend and then subtracting 1 from the difference for every time B is set to a factor in the divisor—adding 1 for every time a factor in the numerator is brought to

B. Thus in the first example $\left(\frac{165\cdot4}{25\cdot4}\right)$ the difference of the indices in numerator and denominator is $(2 - 1) = 1$, and subtracting 1 (because B was set to 25·4) this leaves 0 for the index of the quotient, which must therefore have a value greater than unity but less than 10 (quotient = 6·5118). Applying the rule to the second example given, we have difference of indices = $[3 - (2 + 1)] = (3 - 1) = 2$, and subtracting 1 (because B was brought to 5·625, a factor of the denominator) we have the index of the log of the quotient = $(2 - 1) = 1$; the quotient therefore consists of *two* figures before the decimal point (27·877).

Exercises for practice :—

$$\frac{6288}{44\cdot585 \times 67\cdot42} = 2\cdot0919$$

$$\frac{147\cdot62 \times 3\cdot5625 \times 33000}{3\cdot1416 \times 18545} = 297\cdot88.$$

This example being one of combined multiplication and division, can be solved very simply on the spiral slide rule in the following manner:—Set 14762 (omitting all consideration of the decimal point) to F the fixed pointer, and bring B to 31416; then bring 35625 on the barrel to B and set B to 18545, finally bringing 330 to B and reading the answer (29788) from the fixed pointer. Where the number of factors in the numerator is one more than in the denominator the instrument need not be set to 100, the various factors being taken alternately by bringing A or B to the various factors in the denominator and setting the succeeding factors of the numerator to either A or B as the case may be. The index of the quotient in this example will be $(6 - 4) = 2$ for the difference in the sum of the indices of the factors in numerator and denominator respectively, plus 2 for having to set two of the factors in the numerator to B, and minus 2 for bringing two of the factors in the denominator to B, making the final value $(2 + 2 - 2) = 2$.

Involution and Evolution.—The operations required in raising a number to a given power or finding the value of a root of a

number by means of the Fuller spiral slide rule are accomplished by means of two additional scales M and N shown in fig. 83, p. 115. The vertical scale M is an evenly-divided scale of fifty divisions, graduated on the brass strip P, carrying the movable pointers between the top movable pointer A and the inner cylinder G. The length of each of the divisions on this scale M is equal to the pitch of the spiral on the barrel D; and the divisions are marked .02, .04, .06, etc., to unity at the top. The other scale N is a horizontal, openly-divided scale of 200 divisions, printed on the barrel C circumferentially round the top, and marked .001, .002, .003, etc., to .02, forming a supplementary or continuation scale to M. The mantissa of the logarithm of any number is found by bringing the number on the barrel C to the top movable pointer A, and reading the first two figures of the mantissa off scale M and the remaining two or three figures off the scale N. Scale M is read by observing the height of the inclined line K on the barrel C immediately underneath the scale N, as read from scale M; whereas the scale N is read from the left-hand side of the brass strip P, immediately over the movable pointer A. The student should practise the readings of the logarithms correctly with a table of logs before him, before passing on to the operations of involution and evolution with this form of rule. Thus, to obtain log 2, set 200 on the barrel to A and read .30 from scale M opposite the inclined line K, and the remainder of the mantissa .001 on scale N from the left-hand side of the brass strip P. By adding the two readings together (.30 + .001) the complete mantissa of the logarithm is found (.301), and the characteristic or index being 0, the complete logarithm is 0.301.

Examples.—Log $44 = 1.6435$ (M reads .64 and N .0035), the index 1 being added by inspection. Log $450 = 2.6532$ (M reads .64 and N 0.132). Log $0.78 = \bar{1}.8921$ (M reads .88 and N = .0121). Sometimes the fifth figure in the mantissa can be read (if required), as, for example, log $24.37 = 1.38685$ (.38 on M and .00685 on N); but this degree of accuracy is seldom required in practical work.

To raise any number x to a given power y , *i.e.* to find the value of x^y , the mantissa of the logarithm of x is found as above, and the complete logarithm (with the index added) multiplied by y in the usual way. The product gives the value of the complete logarithm of the power, and by locating the position of the decimal point according to the usual rules, the value of the mantissa is given. By setting M to the first two figures of this mantissa, and turning the barrel horizontally so that N gives the remaining two or three figures, the required power is read from the top movable pointer A, the decimal point being located according to the value of the index of the logarithm of the product $\log x \times y$.

For example $(3.54)^{5.687}$. Set 3.54 on barrel C to A, and read .54 from M and .009 from N. The complete logarithm of 3.54 is therefore 0.549, which, multiplied by 5.687 on the rule in the usual way, gives 3.1222, the log of the answer. Taking the mantissa portion only, set .12 on M to the inclined line K and turn barrel so that .0022 on N is against the left-hand side of the brass strip P (taking care that M still reads .12), and the answer 13248 is given by the movable pointer A. The position of the decimal point is now located from the known index of the logarithm (3). The answer is therefore **1324.8**.

Again, to find the value of $(4.5678)^{0.6789}$. Find the log of 4.5678 equal to 0.6597 and multiply by 0.6789, giving 0.44788. This is found to be the log of 28045, and as the index is 0, the required answer is **2.8045**.

Again, to find the value of $\sqrt[6.78]{345.67}$. Find the log of 345.67, which is 2.53866, and *divide* it by 6.78, giving as a quotient 0.37443. This is the logarithm of **2.3683**.

Note that in setting M to the first two figures of the mantissa attention should be paid to the position of line K; that is, the barrel C should be so adjusted vertically that the line K passes underneath scale M at a height approximately corresponding to the full value of the mantissa, and not set to the first two figures only. Thus in the last example line K should pass underneath scale M at three-quarters the distance between the two graduations .36 and .38; otherwise there is a liability of reading the pointer A on the wrong line of the spiral.

Professor Perry's Rule with Log-log Scale.—An improved form of slide rule has recently been introduced by Professor Perry, D.Sc., F.R.S., which possesses an additional scale at the bottom replacing the D scale, graduations being proportional to the logarithms of the logarithms of the numbers. By this addition many calculations can be performed more directly than with the ordinary form of rule.

For example, one of the most useful purposes to which the log-log scale lends itself is that of raising a number to any given power, fractional or otherwise; and, conversely, finding the root of a given number.

The rule as usually made consists of four scales, the first three of which are the A, B, and C scales of the ordinary slide rule. The bottom scale on the rule (to which we shall refer as scale E) is the log-log scale, and, as pointed out, this is graduated so that the distances are proportional to the logarithms of the logarithms of the numbers represented. Thus we have the two scales C and E, one sliding against the other. The former is the ordinary logarithmic scale and the latter the so-called log-log

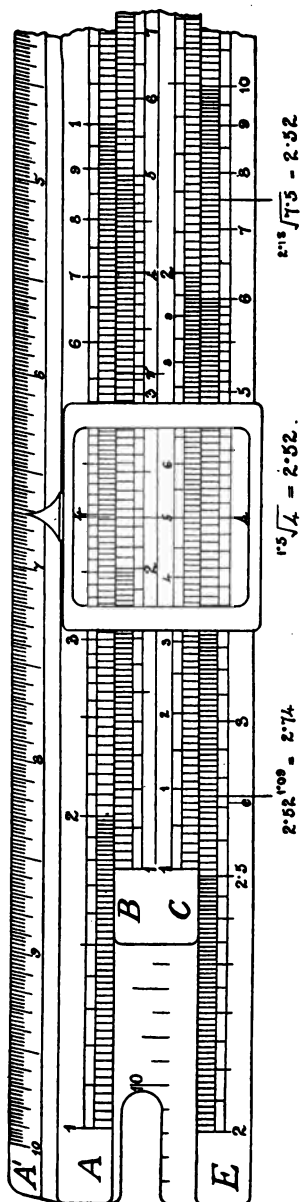


FIG. 84.

scale. We shall now describe some of the more important problems which may be readily solved by its aid.

To raise a number n to any given power p . — This is represented algebraically by the formula $n^p = x$, where x represents the answer, or, taking logs of both sides, $p \times \log n = \log x$; and, again, taking logs of both sides, $\log p + \log \log n = \log \log x$. It is thus seen that by adding the logarithm of p to the logarithm of the logarithm of n , we get the logarithm of the logarithm of x .

Take, for example, $2.52^{1.42} = 3.72$ (see fig. 84). Here we add a distance represented by 1.42 on scale C to the distance representing 2.52 on scale E, and the answer (3.72) is read off from the log-log scale E. To perform the operation, set 1 on the left-hand side of C scale, opposite to 2.52 on the E scale, as shown in the diagram, and read the answer on the E scale corresponding with 1.42 on the C scale. This is one of the simplest operations to which the log-log rule can be put, and one of the most useful.

Examples :—

4^2	=	16
$4^{2.5}$	=	32
4^8	=	64
$4^{8.5}$	=	128
2.75^2	=	7.56
$2.75^{2.62}$	=	38.9
$2.75^{2.31}$	=	10.35
$55^{0.45}$	=	6.07
$302^{0.37}$	=	8.27.

The extreme right-hand end of scale C will have to be used

in working the last two examples, as the powers are less than unity. It will be noted that the log-log scale extends only from 2 to 1000, so that for numbers less than 2, which may have to be raised to any given power, it is necessary to introduce factors or take reciprocals. For example, 1.76^3 cannot be found in the usual way, as 1.76 is not on the E scale, but by multiplying and dividing it by 2 we get numbers that come on the scale; thus:—

$$1.76^3 = \left(\frac{3.52}{2}\right)^3 = \frac{3.52^3}{2^3} = \frac{43.6}{8} = 5.45.$$

Similarly, $(1.5)^4 = \left(\frac{3}{2}\right)^4 = \frac{3^4}{2^4} = \frac{81}{16} = 5.06.$

Sometimes the number required is outside the range of the C scale, in which case factors must be adopted; thus:—

$$\begin{aligned} 11^3 &= (5.5 \times 2)^3 = 5.5^3 \times 2^3 = 166.4 \times 8 = 1331 \\ 15^{3.5} &= 5^{3.5} \times 3^{3.5} = 280 \times 46.8 = 13,100 \text{ nearly} \\ 8.89^5 &= 8.89^{3+2} = 8.89^3 \times 8.89^2 = 704 \times 79 = 55,500. \end{aligned}$$

When the number is less than unity, it is advisable to work by reciprocals, thus:—

$$\begin{aligned} (0.2)^4 &= \left(\frac{1}{5}\right)^4 = \frac{1^4}{5^4} = \frac{1}{625} = 0.0016 \\ (0.526)^{\frac{17}{16}} &= \left(\frac{1}{1.9}\right)^{\frac{17}{16}} = \frac{1}{(1.9)^{\frac{17}{16}}} = \frac{1}{\frac{(3.8)^{\frac{17}{16}}}{2^{\frac{17}{16}}}} = \frac{2.088}{4.125} = 0.505. \end{aligned}$$

To find the root of any given number, say the r^{th} root of a number n .

Symbolically, $\sqrt[r]{n} = x$

or $n^{\frac{1}{r}} = x;$

taking logs of both sides

$$\frac{\log n}{r} = \log x;$$

then taking the logs of the derived equation $\log\text{-}\log n - \log r = \log\text{-}\log x$, i.e. by subtracting the logarithm of r on scale C from the $\log\text{-}\log n$ on scale E, the $\log\text{-}\log$ of the answer x is given on scale E.

Take for example $\sqrt[1.5]{4}$ (see fig. 84). Set 1.5 on scale C against 4 on scale E, and read off the answer (2.52) opposite the 1 on scale C. The operations are exactly the reverse of those required when raising a number to any given power.

Exercises for practice :—

$$\begin{aligned}\sqrt[3]{27} &= 3. & \sqrt[3]{20} &= 2.71. & \sqrt[3]{16} &= 2.52. \\ \sqrt[4]{81} &= 3. & \sqrt[4]{50} &= 2.66. & \sqrt[4]{21} &= 2.14. \\ 2.8 \sqrt[3]{9.1} &= 2.2. & 3.5 \sqrt[3]{91} &= 3.62. \\ 1.414 \sqrt[3]{20} &= 8.31. & 0.55 \sqrt[3]{3.5} &= 9.76.\end{aligned}$$

Use the right-hand end of scale C for the last two exercises. Here again it may be necessary to introduce factors where the limit of the rule is reached, as, for example,

$$\begin{aligned}\sqrt[3]{6} &= \sqrt{\frac{\sqrt[3]{60}}{10}} = \frac{\sqrt[3]{60}}{\sqrt[3]{10}} = \frac{3.91}{2.154} = 1.82 \\ \sqrt[2.5]{5} &= \frac{\sqrt[2.5]{50}}{\sqrt[2.5]{10}} = \frac{4.78}{2.51} = 1.90,\end{aligned}$$

or for numbers less than units, reciprocals may be required, as in

$$\sqrt[4]{0.25} = \sqrt{\frac{\sqrt[4]{1}}{4}} = \frac{\sqrt[4]{1}}{\sqrt[4]{4}} = \frac{1}{\sqrt[4]{80}} = \frac{2.115}{2.99} = 0.707.$$

To find the power of the root of any number.—This exercise is a combination of the two operations just described and illustrated. Thus, to find the p^{th} power of the r^{th} root of any number n ,

$$n^{\frac{p}{r}} = x, \quad \log n \times \frac{p}{r} = \log x,$$

and $\log\text{-}\log n - \log r + \log p = \log\text{-}\log x$.

For example, find the value of

$$5 \cdot 24^{\frac{1.5}{2.1}} = 3.26.$$

Set 2.1 on scale C opposite 5.24 on scale E and read the answer off scale E opposite 1.5 on scale C.

Examples :—

$$14.7^{\frac{2}{3}} = 6. \quad 16^{\frac{1.5}{2.5}} = 5.28. \quad 5.5^{\frac{4.13}{1.96}} = 44.$$

To find the value of the negative power of a number, say $n^{-p} = x$. To convert the negative power into a positive one, it will be necessary to take the reciprocal of the expression, thus :—

$$n^{-p} = \frac{1}{n^p} = x = (\text{say}) \frac{1}{y},$$

then $n^p = y$, and by finding the value of y and taking its reciprocal, we obtain the value of x , thus $n^p = y$; taking logs

of both sides twice $\log\text{-}\log n + \log p = \log\text{-}\log y$, and having found y , $x = \frac{1}{y}$.

For example,

$$2 \cdot 52^{-1 \cdot 77} = x.$$

$$\text{Let } y = \frac{1}{x},$$

then

$$2 \cdot 52^{+1 \cdot 77} = y \\ = 5 \cdot 143$$

$$\therefore x = \frac{1}{5 \cdot 143} \\ = 0 \cdot 195.$$

To find the logarithm of any number n to any base b.

Let $\log_b n = x$,

then

$$b^x = n;$$

or, taking logs of both sides,

$$x \log b = \log n \\ \text{or } x = \frac{\log n}{\log b}$$

taking logs of both sides,

$$\log x = \log\text{-}\log n - \log\text{-}\log b.$$

For example,

$$\log_{10} 2 = x,$$

i.e.

$$10^x = 2;$$

or

$$\log x = \log\text{-}\log 2 - \log\text{-}\log 10.$$

Set the extreme right-hand 1 on scale C to 10 on scale E, and read the answer from scale C opposite to 2 on scale E.

Examples:—

$$\begin{array}{ll} \log_{10} 2 = 0 \cdot 301. & \log_{10} 3 \cdot 5 = 0 \cdot 544. \\ \log_{10} 3 \cdot 1416 = 0 \cdot 497. & \log_{10} 9 \cdot 1 = 0 \cdot 959. \end{array}$$

If the logarithms of numbers less than 2 or greater than 10 are required, use the *left*-hand 1 on scale C set opposite to 10 on scale E, and read the answer from scale C opposite to the number on scale E, thus:—

$$\log_{10} 15 = 1 \cdot 176. \quad \log_{10} 1 \cdot 5 = 0 \cdot 176. \quad \log_{10} 1 \cdot 8 = 0 \cdot 255.$$

It will be noted in the last two examples that the characteristic of the logarithm has to be altered to suit the number of digits in the integral part of the number.

If the base of the system of logarithms considered be other than 10, say base for the Napierian or hyperbolic logarithms which has a value of 2.718 (approx.), the 1 on scale C has to be set to 2.718 on scale E and the answer read off from scale C opposite the number on scale E.

For example, $\log 4 = 1.386$, i.e. $2.718^{1.386} = 4$.

The suffix "e" after the word log denotes the base of the Napierian or hyperbolic logarithms, and the division 2.718 on scale e is marked "e" on the rule.

Examples:—

$$\log_e 3 = 1.098. \quad \log_e 10 = 2.3. \quad \log_e 17.5 = 2.86.$$

Similarly, the logarithms to any other base can be found by setting the 1 on C opposite the given base, thus:—

$$\begin{array}{ll} \log_2 2 = 0.431. & \log_5 10 = 1.431. \\ \log_{2.5} 10 = 1.388. & \log_{6.5} 25 = 1.72. \end{array}$$

Given the logarithm of a certain number to a known base, to find the number.—This is simply the reverse operation to that just described. For example, given that $\log_{10} x = 0.699$, to find x : Set the extreme right-hand 1 on scale C to 10 on scale E, and read the answer from scale E opposite to 699 on scale C. Thus with the slide C set in any one position for a given base, the logs and antilogs of various numbers can be read off without moving the slide.

Exercises for practice:—

$$\begin{array}{lll} \log_{10} x = 0.753, & \therefore x = 5.66, & \\ \text{or, } \text{antilog}_{10} 0.753 = 5.66. & \text{antilog}_{10} 0.7 = 5.01. & \\ \text{,, } 0.4 = 2.51. & \text{,, } 1.15 = 14.13. & \\ \text{,, } 2.5 = 316.2. & \text{,, } 3 = 1000. & \end{array}$$

The same rules also apply regarding the use of factors and reciprocals where the expression is outside the limits of the rule. For instance,

$$\begin{aligned} \log_e 1.8 &= \log_e \left(\frac{9}{5} \right) = \log_e 9 - \log_e 5 \\ &= 2.197 - 1.609 = 0.588. \end{aligned}$$

Practical Application of the Log-Log Scale to Engineering Problems.—Many examples occur in engineering practice where it is found that a certain quantity or result varies according to the power of another quantity.

For instance, the flow of water over rectangular weirs or bays is found to vary as the $(\frac{3}{2})$ th power of the depth of water over

the sill of the weir, so that for bays or weirs of a given width the quantity of water flowing over per minute or per second will vary directly as the $\sqrt[3]{h^3}$, where h represents the depth from the level of free water to the sill of the weir. By the aid of the log-log scale a table can readily be constructed from which a curve can be plotted on squared paper which will show at a glance the quantity of water at various depths for a given size of weir.

Take, for example, a bay or weir 6 inches wide, and assuming a coefficient of discharge of 0.62, the formula giving the quantity of water becomes

$$Q = 5\frac{1}{3} \times 0.62 \times 0.5 \sqrt[3]{h^3},$$

where Q = quantity of water in cubic feet per second.

h = depth of water from sill of weir to free level, in feet.

Reducing the quantity to gallons per minute, and combining all the numerical quantities together, we have

$$G = 618 \sqrt[3]{h^3}, *$$

where G = quantity of water in gallons per minute.

Table V. has been constructed from this formula by the aid of the log-log scale giving the quantity of water for various depths, and the results should be plotted on squared paper showing intermediate values.

TABLE V.—DISCHARGE OVER 6-INCH TUMBLING BAY OR WEIR, IN GALLONS PER MINUTE, FOR DEPTHS FROM 1 INCH TO 6 INCHES.

Depth or Head h .		$\sqrt[3]{h^3}$	G. Gallons per Minute.
Inches.	Feet.		
1.0	0.083	0.0241	14.8
1.5	0.125	0.0442	27.3
2.0	0.16	0.0682	42.1
2.5	0.2083	0.0952	58.8
3.0	0.25	0.125	77.1
3.5	0.2916	0.157	97
4.0	0.3	0.192	118.5
4.5	0.375	0.2291	141.5
5.0	0.416	0.2685	166
5.5	0.4583	0.314	194
6.0	0.5	0.358	218

* 1 cubic foot = 6.235 gallons.

The student should calculate similar tables for bays of other widths, say 3 inches and 9 inches, and plot the results on squared paper.

Another use to which the log-log scale can be put is that of determining the ratio between the tight and slack sides of a belt or rope used in the transmission of power. An investigation of the theory of this subject would be out of place in this work, but we may state the result.

It is found that the ratio referred to varies according to the angle of contact with the smaller pulley, and the coefficient of friction between the leather or rope and the pulley. The law is expressed by the formula

$$\frac{T}{t} = e^{\mu\theta},$$

where T = tension or tight side of belt or rope.

t = tension on slack side of belt or rope.

e = base of Napierian logarithms (2.718).

μ = coefficient of friction between belt and pulley.

θ = angle of contact or embrace in circular measure.

So that, given the coefficient of friction and the angle of contact, the ratio between the tight and slack sides of a belt or rope can be calculated.

Take, for example, the value of $\mu = 0.3$, then the ratio of $\frac{T}{t}$ for values of θ from 90° to 270° may be found as follows:—

$$\frac{T}{t} = e^{\mu\theta}$$

$$\log_e \frac{T}{t} = \log_e e^{\mu\theta}$$

$$= \mu\theta \log_e e$$

$$= \mu\theta$$

$$\text{or} \quad \frac{T}{t} = \text{antilog}_e \mu\theta.$$

Hence by finding the values of μ and θ and taking the corresponding antilogs to the Napierian base (2.718), we obtain the values of the ratios $\frac{T}{t}$. The following table (VI.) for example,

gives the value for $\frac{T}{t}$ for $\mu = 0.3$ at angles of 90° , 120° , 150° , 180° , 210° , 240° , and 270° , obtained in this way, and all these results

should be checked over, and finally plotted out on squared paper between degrees and $\frac{T}{t}$ (as in fig. 85, p. 128).

TABLE VI.

Angle θ .		$\mu\theta$	$\frac{T}{t}$
Degrees.	Radians.		
90	1.57	0.471	1.60
120	2.09	0.628	1.86
150	2.61	0.782	2.18
180	3.14	0.941	2.56
210	3.66	1.1	3.00
240	4.19	1.255	3.51
270	4.71	1.415	4.11

Use may be made of the log-log scale to calculate the size of shafting required to transmit a given power at a given number of revolutions. The usual formula connecting power transmitted with shearing stress produced by torsion only (neglecting bending) is—

$$T_m = f_s \frac{\pi}{16} d^3,$$

where T_m = twisting moment of the shaft, in lb.-inches.
 f_s = shearing stress produced in lbs. per square inch.
 d = diameter of shaft in inches.

As the power transmitted in terms of the twisting moment and speed is given by

$$\frac{\text{H.P.}}{n} = \frac{16T_m}{63024},$$

where n = revolutions per minute, the above formula can be modified by combination, thus:—

$$\frac{\text{H.P.}}{n} = f_s \frac{\pi}{63024} d^3;$$

or, taking f_s at 8000 lbs. per square inch (an average safe value for mild steel), and combining the numerical quantities, we get,

$$\frac{\text{H.P.}}{n} = 0.0249 d^3;$$

so that, given the power and speed, the size can be calculated from

$$d^3 = \frac{\text{H.P.}}{n} \times 40 \text{ (approximately).}$$

For various values of $\frac{\text{H.P.}}{n}$, therefore, the size of shaft can be

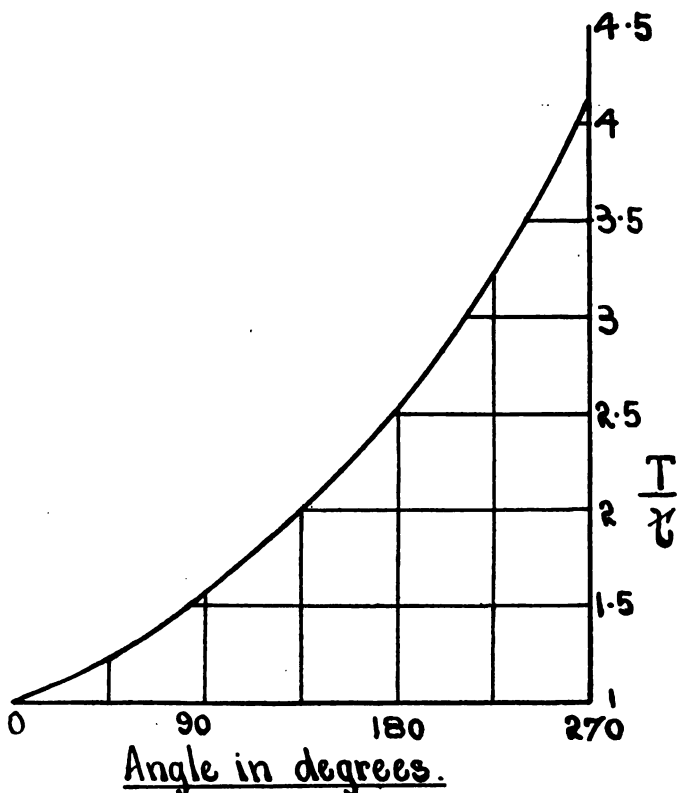


FIG. 85.

found (using the log-log scale) by taking the cube root of the product of $\frac{\text{H.P.}}{n} \times 40$. Table VII. gives, by way of examples, the diameters of shafting required for various values of $\frac{\text{H.P.}}{n}$ from

TABLE VII.—SIZE OF MILD-STEEL SHAFT FOR TORSION ONLY.

$\frac{\text{H.P.}}{n}$	d^3	d (Inches).
0.25	10	2.15
0.50	20	2.71
0.75	30	3.11
1.00	40	3.42
1.25	50	3.68
1.50	60	3.91
1.75	70	4.12
2.00	80	4.31
2.25	90	4.48
2.50	100	4.64
2.75	110	4.79
3.00	120	4.98

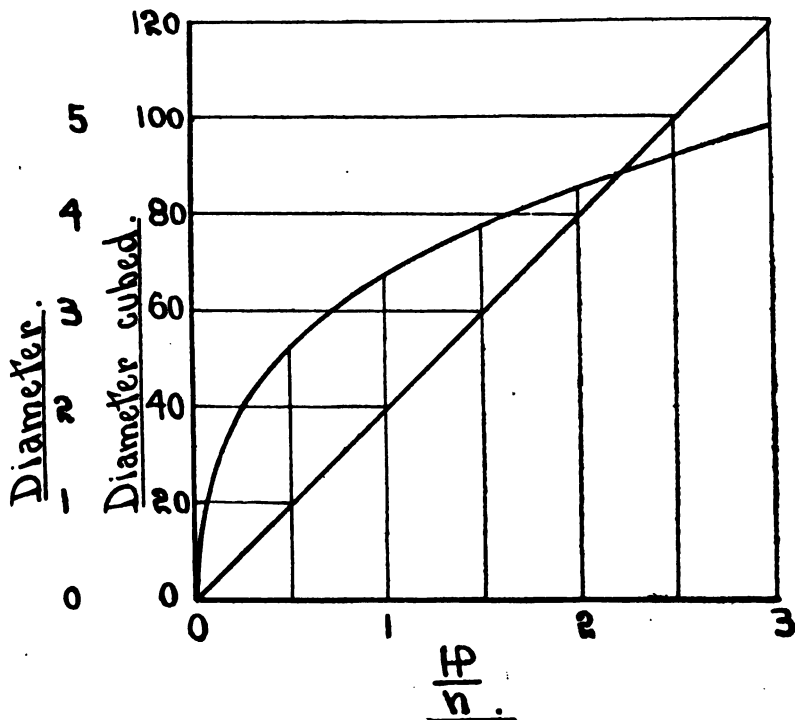


FIG. 86.

0.25 to 3, but the student must remember that this size of shaft is applicable only to cases where there is no bending, that is, where the shaft is subject to *pure torsion* only.

The results as plotted on squared paper are shown in fig. 86, so that intermediate sizes can be read off as required.

Another useful exercise is to calculate the increased size of shaft required when *combined bending and torsion* are allowed for. Thus, assuming the maximum bending moment in lb.-inches = B and the maximum twisting moment in lb.-inches = T , and the ratio of $\frac{B}{T} = k$, it can be shown that a combination of bending and torsional stresses B and T respectively produces (what is usually termed) an equivalent twisting moment (T_e) equal to

$$T_e = T(k + \sqrt{k^2 + 1}).$$

Thus, for shafts subject to combined bending and twisting the stress is greater than that caused by twisting alone by the factor shown inside the brackets, say $x = (k + \sqrt{k^2 + 1})$. The following values of k and x are useful as showing the ratio between the equivalent twisting moment (T_e) caused by the combined stress compared with the twisting moment (T) caused by torsional stress only:—

$$\begin{array}{l} k = \frac{1}{4}, \quad \frac{1}{2}, \quad 1, \quad 1\frac{1}{2}, \quad 2, \quad 3. \\ x = 1.28, \quad 1.62, \quad 2.41, \quad 3.3, \quad 4.24, \quad 6.16. \end{array}$$

To allow for this increase in the equivalent twisting moment, the diameter of shaft will have to be increased in the proportion of the cube root of the above figures x , as shown by the accompanying Table VIII.

The curves representing these results are plotted in fig. 87, p. 131, and show even more forcibly than the figures the comparatively large effect caused by bending in addition to torsional stresses.

The log-log scale very considerably facilitates the calculations necessary for drawing expansion and compression curves for gases. It is well known that the adiabatic expansion or compression of a perfect gas follows the law

$$p \times v^\gamma = \text{a constant},$$

where γ is equal to the ratio of the specific heats of the gas at constant pressure and constant volume. The value of γ for air is 1.408, so that in order to draw the adiabatic curve for air, we first require to find numerical values of p and v connected by the equation $p \times v^{1.408} = \text{a constant}$. Take, for example, the adiabatic expansion of air from an initial pressure of 100 lbs. per square inch absolute, and an initial volume of 1. By denoting

TABLE VIII.—SIZE OF MILD-STEEL SHAFTS TO WITHSTAND COMBINED BENDING AND TORSIONAL STRESSES.

$\frac{\text{H.P.}}{n}$	Diameter of Shaft for Twisting only.	Diameter of Shaft for combined B and T at the following Values of k .					
		$\frac{1}{4}$	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	3
0.25	2.15	2.34	2.53	2.88	3.20	3.49	3.94
0.50	2.71	2.95	3.19	3.63	4.03	4.40	4.96
0.75	3.11	3.39	3.66	4.16	4.62	5.03	5.69
1.00	3.42	3.72	4.02	4.58	5.08	5.52	6.26
1.25	3.68	4.00	4.33	4.92	5.47	5.97	6.72
1.50	3.91	4.25	4.60	5.22	5.82	6.33	7.16
1.75	4.12	4.48	4.85	5.52	6.12	6.68	7.53
2.00	4.31	4.69	5.08	5.78	6.41	6.99	7.88
2.25	4.48	4.88	5.28	6.01	6.67	7.26	8.18
2.50	4.64	5.04	5.45	6.21	6.90	7.51	8.48
2.75	4.79	5.21	5.62	6.42	7.12	7.76	8.77
3.00	4.93	5.37	5.80	6.60	7.33	7.98	9.00

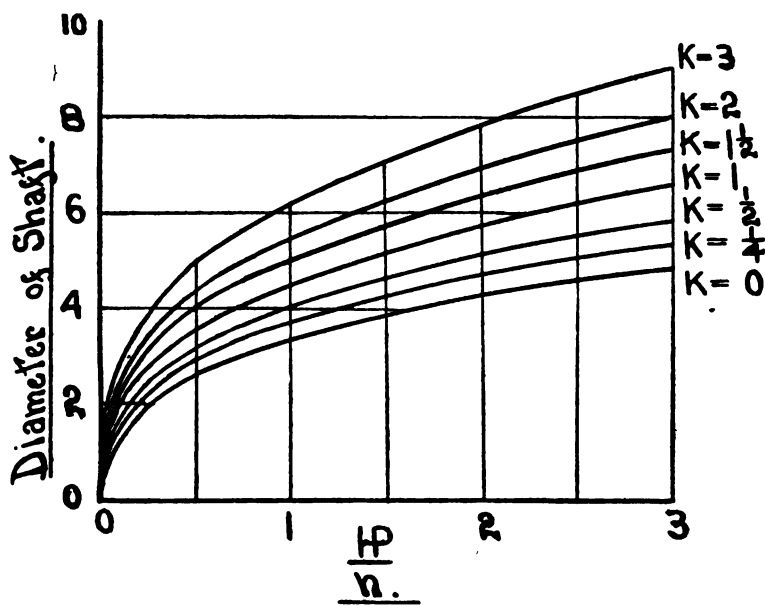


FIG. 87.

successive stages in the expansion by the suffixes 2, 3, 4, etc., we have the formulæ

$$p_1 \times V_1^{1.408} = p_2 \times V_2^{1.408} = p_3 \times V_3^{1.408} = \text{a constant.}$$

But $p_1 \times V_1^{1.408} = 100 \times 1^{1.408} = 100 \times 1 = 100,$

$$\therefore p_2 \times V_2^{1.408} = 100,$$

or $p_2 = \frac{100}{V_2^{1.408}} \quad \text{and} \quad p_3 = \frac{100}{V_3^{1.408}}, \text{ etc.}$

Let $V_1, V_2, V_3, \dots$ represent 1, 2, 3, $\dots$ cubic feet; then the pressures at these points will be found by dividing 100 (the constant) by $2^{1.408}, 3^{1.408}, 4^{1.408}, \text{ etc.}$ The following table, No. IX., gives the values of $p_2, p_3, \text{ etc.}$, for volumes V from 1 to 10 cubic feet, and these results are plotted in fig. 88, p. 133.

TABLE IX.—ADIABATIC EXPANSION OF AIR,
BY FORMULA $pv^{1.408} = \text{A CONSTANT.}$

V Cubic Feet.	$V^{1.408}$	p lbs.
1	1.00	100
2	2.65	37.7
3	4.69	21.3
4	7.08	14.2
5	9.62	10.4
6	12.42	8.04
7	15.45	6.48
8	18.6	5.38
9	22.0	4.54
10	25.6	3.90

Similar curves should be constructed by the student for the adiabatic expansion of superheated steam.

The log-log scale is also useful for finding the value of the exponent n in the equation $pv^n = \text{a constant}$, for an actual expansion or compression curve obtained from any engine. Take, for example, the tests of a Crossley-Otto gas engine, where the mean indicator diagram furnished the following particulars of pressures and volumes:—

p_a (beginning of compression)	13.8	lbs.	$v_a = 0.8377$	cubic ft.
p_b (end " ")	67.8	"	$v_b = 0.2467$	"
p_d (beginning of expansion)	240	"	$v_d = 0.2617$	"
p_e (end " ")	48.71	"	$v_e = 0.8377$	"

Then, for the compression stroke, since

$$\begin{aligned}
 p_a \times v_a^n &= p_b \times v_b^n \\
 \log p_a + n \log v_a &= \log p_b + n \log v_b \\
 n (\log v_a - \log v_b) &= \log p_b - \log p_a \\
 \therefore n &= \frac{\log p_b - \log p_a}{\log v_a - \log v_b} \\
 &= \frac{1.83 - 1.14}{1.923 - 1.392} = \frac{0.69}{.531} = 1.30;
 \end{aligned}$$

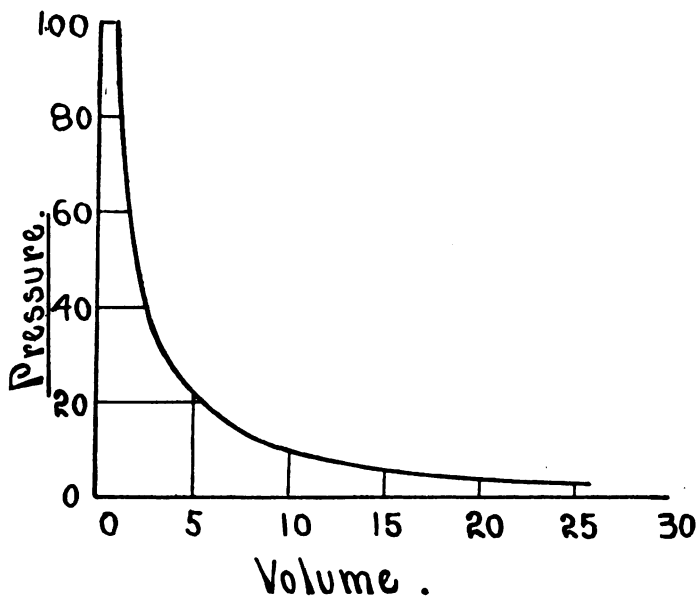


FIG. 88.

and similarly, for the expansion,

$$\begin{aligned}
 n &= \frac{\log p_a - \log p_e}{\log v_e - \log v_a} \\
 &= \frac{2.38 - 1.688}{1.923 - 1.417} = \frac{0.692}{0.506} = 1.37.
 \end{aligned}$$

The value of n in the expansion curves for steam engines can be found in a similar manner. Take, for example, the large compound pumping engine at the Southwark and Vauxhall Waterworks,

Hampton, tested by the Steam Jacket Research Committee of the Institution of Mechanical Engineers in 1894.*

In the first test, with all jackets on, the following pressures were recorded at cut-off and release respectively in the high-pressure cylinder :—

$$\begin{aligned}
 p_1 &= 79.37 \text{ lbs.} & v_1 &= 11.619 \text{ cubic ft.} \\
 p_2 &= 26.12 \text{ ,,} & v_2 &= 38.37 \text{ ,,} \\
 n &= \frac{\log p_1 - \log p_2}{\log v_2 - \log v_1} \\
 &= \frac{1.899 - 1.416}{1.584 - 1.065} = \frac{0.483}{0.519} = 0.93.
 \end{aligned}$$

The same engine, tested with the jackets on, gave the following pressures at cut-off and release in high-pressure cylinder :—

$$\begin{aligned}
 p_1 &= 81.7 \text{ lbs.} & v_1 &= 12.31 \text{ cubic ft.} \\
 p_2 &= 28.63 \text{ ,,} & v_2 &= 38.37 \text{ ,,} \\
 n &= 0.92.
 \end{aligned}$$

The mean value for the exponent n from the point of cut-off in high-pressure cylinder to the end of expansion in low-pressure cylinder is as follows :—

$$\begin{aligned}
 \text{1st test, } p_1 &= 79.37 \text{ lbs.} & v_1 &= 11.619 \text{ cubic ft.} \\
 p_2 &= 8.0 \text{ ,,} & v_2 &= 101.74 \text{ ,,} \\
 n &= 1.06. \\
 \text{2nd test, } p_1 &= 81.7 \text{ lbs.} & v_1 &= 12.31 \text{ cubic ft.} \\
 p_2 &= 7.52 \text{ ,,} & v_2 &= 101.74 \text{ ,,} \\
 n &= 1.129.
 \end{aligned}$$

Showing that the re-evaporation due to the jackets was effected in the low-pressure cylinder only. This was again proved by a third test in which the high-pressure cylinder only was jacketed, when the pressures were

$$\begin{aligned}
 p_1 &= 81.4 \text{ lbs.} & v_1 &= 12.617 \text{ cubic ft.} \\
 p_2 &= 7.65 \text{ ,,} & v_2 &= 101.74 \text{ ,,} \\
 n &= 1.133.
 \end{aligned}$$

Another useful purpose to which this form of slide rule can be applied is the estimation of the horse-power transmitted by spur gearing at various speeds. With cast-iron wheels, assuming a maximum safe-working stress of 1500 lbs. per square inch, and allowing for the whole pressure to be carried by any one tooth at

* *Proceedings Institution of Mechanical Engineers.*

a time, it is found that the maximum safe-working pressure allowable is

$$P = 300 p^2$$

where P = working pressure on teeth in lbs.
 p = pitch of gear in inches.

The power transmitted, therefore, will vary as the square of the pitch, and directly as the velocity of the rim of the wheel, and can be represented by the formula—

$$\text{H.P.} = \frac{p^2 \times V}{110}.$$

Calculate the results in the following Table (X.), which gives the H.P. transmitted by C. I. unshrouded wheels from 1-inch to 3-inch pitch, at velocities varying from 100 to 500 feet per minute.

TABLE X.—H.P. TRANSMITTED BY C.I. UNSHROUDED SPUR WHEELS.

Velocity Feet per Minute, V.	Pitch of Teeth in Inches, p .				
	1 in.	1½ in.	2 in.	2½ in.	3 in.
100	0·91	2·0	3·6	5·7	8·2
150	1·36	3·0	5·4	8·5	12·2
200	1·82	4·0	7·2	11·3	16·8
250	2·27	5·0	9·1	14·2	20·4
300	2·72	6·0	10·9	17·0	24·5
350	3·18	7·0	12·7	19·9	28·6
400	3·63	8·0	14·5	22·7	32·7
450	4·09	9·0	16·3	25·6	36·8
500	4·54	10·0	18·2	28·4	40·9

The student should extend the table by calculating the power transmitted by gears of ¾-inch, 1¼-inch, 1½-inch, 2¼-inch, 3½-inch, and 4-inch pitch, and up to speeds of 1000 feet per minute. The results should then be plotted on squared paper, as shown in fig. 89, p. 136. An additional exercise could be made by calculating similar results for shrouded wheels, adding 18 per cent to the powers, thus:—

$$\text{H.P.} = \frac{p^2 \times V}{95};$$

and a further extension by calculating the power transmitted by steel wheels from the formulæ $H.P. = \frac{p^2 \times V}{70}$. It should be borne in mind that the above values are only suitable for wheels where the width of face of teeth is not less than three times the pitch. If the width of face be less than this amount, the power will be reduced accordingly.

Another example of use to which the log-log scale lends itself very readily is in calculating the power absorbed by

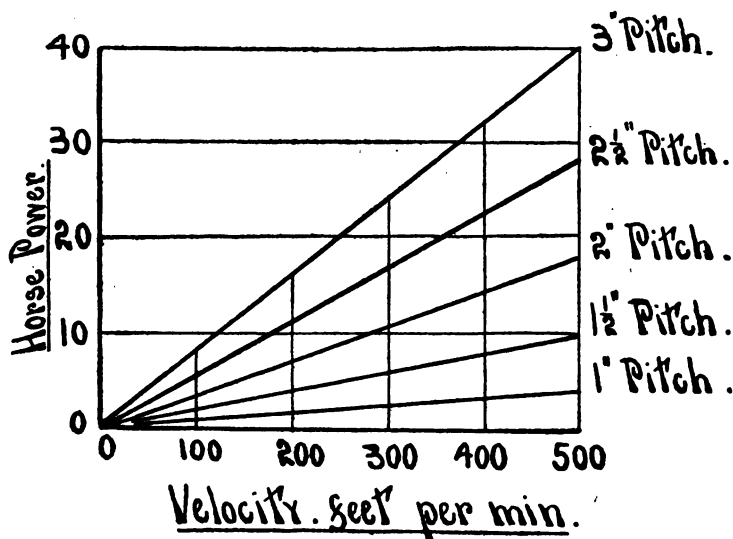


FIG. 89.

water dynamometers of the Froude type. With this brake it is found that the power absorbed varies directly as the square of the speed and as the 5th power of the diameter of the revolving disc or vanes. It is also found that an 18-inch dynamometer will absorb about 6 H.P. at 100 revolutions per minute; consequently at 200 revolutions it will absorb four times as much, or 24 H.P.; whereas a dynamometer 3 feet in diameter will absorb $2^5 = 32$ times the power of the 18-inch one at the same speed. Table XI. has been calculated to give the power absorbed by various sizes from 12 inches to 36 inches in diameter, at speeds from 100 to 500 revolutions per minute.

TABLE XI.—POWER ABSORBED BY FROUDE WATER DYNAMOMETER.

Diameter in Inches.	Revolutions per Minute.				
	100	200	300	400	500
12	0·8	3·1	7·1	12·6	20
18	6	24	54	96	150
24	25	101	228	406	633
30	77	307	690	1230	1920
36	192	770	1730	3070	4800

The student should plot these results on squared paper, and extend the table by calculating the power absorbed by other sizes, such as 15 in., 21 in., 27 in., 33 in., 40 in. The formula to use is :—

$$\text{H.P.} = \frac{d^5 \times n^2}{3,150,000,000}$$

where d = diameter of rotating disc in inches.
 n = revolutions per minute.

The log-log scale can be used for calculating the extension or compression of helical springs of various sizes under given loads, where G , the rigidity modulus, is known. For round section steel springs it is found that the extension varies as the 3rd power of the diameter from centre to centre of coil, and inversely as the 4th power of the diameter of the wire. The extension for any load W can be represented by the formula :—

$$\delta = \frac{8W \cdot D^3 \cdot n}{d^4 \times G}$$

where δ = extension or compression in inches.
 W = axial load in lbs.
 D = diameter of coil from centre to centre.
 n = number of coils.
 d = diameter of wire in inches.
 G = modulus of rigidity of material in lbs.

Taking G at 13×10^6 lbs. (an average value for spring steel) the extension per 1-lb. load and per coil is given by

$$\delta = \frac{8}{13 \times 10^6} \times \frac{D^3}{d^4}$$

Table XII., p. 138, has been calculated from this formula, and gives the extension or compression in inches for each 1 lb. of

load and per coil. That is, the given extension must be multiplied by the load or pressure in lbs. and by the number of coils, to give the required extension or compression in each particular instance.

The student should use the log-log scale to verify these results.

TABLE XII.—EXTENSION AND COMPRESSION OF STEEL HELICAL SPRINGS PER 1 LB. LOAD AND PER COIL, IN INCHES.

Diameter of Wire. Inch.	Diameter of Coil from Centre to Centre.						
	1 in.	1½ in.	2 in.	2½ in.	3 in.	3½ in.	4 in.
0·10	0·00616	0·0207	0·0493	0·0962	...	...	...
0·125	0·0025	0·00852	0·0202	0·0395	0·0682	...	...
0·15	0·00122	0·00412	0·0097	0·0191	0·0329	0·0521	...
0·175		0·00221	0·00522	0·0102	0·0177	0·0380	...
0·20		0·00130	0·00308	0·00602	0·0104	0·0165	0·0246
0·225		0·00081	0·00192	0·00375	0·00648	0·0102	0·0154
0·25 = ¼		0·00053	0·00126	0·00246	0·00426	0·00675	0·0101
0·281 = ⅙			0·00087	0·00171	0·00296	0·00468	0·0070
0·312 = ⅕			0·00051	0·00101	0·00174	0·00276	0·00413
0·344 = ⅙			0·00035	0·00069	0·00118	0·00188	0·00281
0·375 = ⅙			0·00025	0·00048	0·00084	0·00133	0·00199
0·406 = ⅙				0·00035	0·00060	0·00096	0·00143
0·437 = ⅙				0·000262	0·000453	0·000718	0·00107
0·469 = ⅙				0·000201	0·000346	0·000548	0·000821
0·50 = ½				0·000154	0·000266	0·000421	0·00063

EXERCISES.

Special exercises as such on the use of the ordinary slide rule are not required, as the student will, no doubt, solve the majority of the *practical engineering problems* given in this book by its assistance.

1. With the V notch or triangular weir, the quantity of water discharged is given approximately by the formula $Q = 12·8bh^{\frac{3}{2}}$, where Q = quantity of water in cubic feet per second, h = depth in feet from bottom of notch to free level, and b = breadth in feet for the notch at the free surface.

Calculate by aid of the log-log scale the quantities in gallons per

minute for five different values of h , with corresponding values of b , and then plot the results between h and Q .

2. Calculate the values of $\frac{T}{t}$ for a belt, having given the following values for the angle of contact, taking in each case ($\mu = 0.35$):— 160° , 170° , 180° , 200° , 210° .

3. Extend the results of Table VII., page 129, from $\frac{H.P.}{n} = 3$ to $\frac{H.P.}{n} = 10$, and then plot curves for the entire range—

(a) showing relationship between d and $\frac{H.P.}{n}$,

(b) „ „ „ „ d^3 and $\frac{H.P.}{n}$.

4. Assuming the relationship between p and v for dry saturated steam is given by $pv^{\frac{17}{16}} = k$, find five values of v corresponding to any five values of p , having given that 1 lb. of steam at 14.7 lbs. absolute pressure occupies 26.39 cubic feet. Compare the results obtained with those given in the Steam Table XXII., page 290.

5. Work out the value of n in the formula $pv^n = k$ for the following two tests made on a set of triple-expansion engines* :—

1st test, jackets on—

$$\begin{array}{ll} p_1 = 127 \text{ lbs.} & v_1 = 1.193 \text{ cubic feet.} \\ p_4 = 8.93 \text{ „} & v_4 = 14.315 \text{ „} \end{array}$$

2nd test, jackets off—

$$\begin{array}{ll} p_1 = 129.5 \text{ lbs.} & v_1 = 1.622 \text{ cubic feet.} \\ p_4 = 8.06 \text{ „} & v_4 = 14.315 \text{ „} \end{array}$$

Ans. The values of n are 1.068 and 1.275 respectively, the results showing increased condensation without jackets.

6. Extend Table XII. by calculating the extensions for other sizes of springs, such as 0.1875 in. ($= \frac{3}{16}$ in.), 0.30 in., 0.35 in., etc., and also for intermediate sizes of coils, say $1\frac{1}{4}$ in., $1\frac{3}{4}$ in., $2\frac{1}{4}$ in., etc. Then plot out the extensions of the series on squared paper, first for each spring separately to a base of diameter of coil, and afterwards to a base of diameter of wire. The series would then be complete, and would give a working table of practical value.

* *Institution of Mechanical Engineers*, Third Report of Research Committee on the Value of the Steam Jacket.

CHAPTER V.

THE VALUE OF SQUARED PAPER.

THE value of squared paper to the engineer cannot be over-estimated. By plotting off to scale along two lines at right angles, the various values of two quantities which have been found to be mutually dependent, a picture, as it were, is obtained showing at a glance the kind of variation which has taken place. Where the variation is regular one is very often enabled to obtain a formula connecting the two variables. By working through the exercises in this book, the student will notice the practically unlimited application of squared paper to experimental, practical, and commercial work. In the present chapter, therefore, it will be sufficient to summarise briefly some of the principal advantages of squared paper, and to show how the results, when regular, may be symbolically related.

Summary of Advantages.—(a) Presents a picture by which the mind's eye can comprehend at a glance the nature of the relationship or variation between two variable quantities which are mutually dependent upon one another.

(b) Corrects for errors of observation, experiment, and calculation.

(c) Is of great time-saving value in many ways which will be obvious to the student who works the various exercises given in this work.

(d) When the variations as plotted follow some regular law, empirical or rational formulæ can be determined connecting the variables by means of which, when one quantity is stated, the other may be calculated.

(e) Enables one to take off from the curve showing the relationship between the variables' intermediate values. The value of one variable being found when the other is given.

(f) Enables one to detect any unusual variation of one quantity with another.

Kinds of Squared Paper.—For the purpose of plotting and

interpreting numerical results, accurately divided paper is used, and may be obtained in various sizes. Specimen sheets are shown at the end of this book. Squared paper may be obtained—

(1) Divided horizontally and vertically into inches and tenths of an inch. See specimen sheets.

(2) Divided horizontally and vertically into centimetres and millimetres. See specimen.

(3) With logarithmic divisions in either one direction or both. See specimen sheets.

Exercise books of squared paper are also commonly used and are convenient for keeping results together.

Both paper and books may readily be obtained from any large stationer.

The student should note (1) that almost all the plotted results in this work were originally set out on ordinary squared paper divided into inches and tenths of an inch, but they are shown in the figures in the text on a reduced scale, the smaller or tenth-of-an-inch divisions being left out. To reproduce the original diagrams would be far too costly, and really quite unnecessary.

(2) That it is not sufficient for him simply to read about squared paper and its uses. *He should take each of the exercises given in the several chapters and plot out the results either on the loose sheets or in an exercise book, and, as far as practicable, the formulæ should be derived.*

We shall first explain, by means of a few examples, how squared paper may be used to interpret the relationship between two quantities which vary dependently, and then we shall afterwards treat of the subject in general terms. If the student masters this preliminary part of the work, he will find that his knowledge is extended sufficiently for most practical purposes by the further study of the numerous examples given in this work.

Example 1.—Weights were suspended from a helical spring, and the stretch of the spring for each load measured. The results are given in the table below. Represent graphically the relation-

TABLE XIII.

Load in lbs. . .	5	10	15	20	25
Stretch in inches .	0.21	0.39	0.64	0.85	1.04

ship between load and extension, and find a formula which will give the stretch for a given load.

Taking a sheet of squared paper, plot out the loads to some convenient scale along a base line (as OX in fig. 90), and along vertical lines from the points obtained the stretch (parallel to OY as in the fig.), also to some convenient scale (though not necessarily the same scale). Then it will be seen that the points thus obtained lie approximately along a straight line passing through O (as OA), which should be drawn, showing that the extension of a helical spring is directly proportional to the load applied.

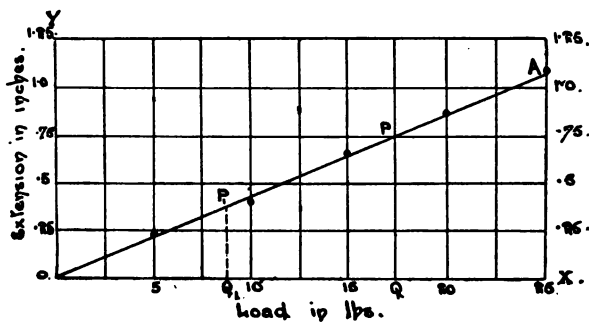


FIG. 90.

Taking any two points P, P₁ on the line, we have, since the triangles OPQ and OP₁Q₁ are similar,

$$PQ : P_1Q_1 :: OQ : OQ_1 \text{ (see fig. 90),}$$

i.e. in a helical spring the stretch or extension is proportional to the load.

To find a formula connecting stretch with load, take any point P on OA and find the value of $\frac{PQ}{OQ}$. Now PQ represents 0.75

extension, and OQ denotes 17.5 lbs. load. Hence $\frac{PQ}{OQ} = \frac{0.75}{17.5} = 0.04286$ of an inch extension for each 1 lb. weight.

$$\therefore \text{Extension} = 0.04286 \times \text{load.}$$

Example 2.—In some tests on the compression of long spiral springs, three of square section and three of circular section wire, the compressions under varying loads when plotted to a load base gave the results for δ tabulated below (see fig. 91, p. 143).

TABLE XIV.

Spring Number.	Square Section.			Round Section.		
	I.	II.	III.	IV.	V.	VI.
Diameter to centre of coil in inches	2.56	2.50	2.50	3.50	2.50	2.56
Size of steel in inches	0.297	0.157	0.20	0.254	0.252	0.314
Total length of spring in inches	90 in.	87 in.	87 in.	88 in.	90 in.	89 in.
Compression "K" when $K = \delta W$ lbs.	0.0094	0.022	0.0625	0.0505	0.0302	0.016
G (modulus of rigidity of material in lbs.)	14.3×10^6	7.2×10^6	9.7×10^6	13×10^6	11.8×10^6	9.9×10^6

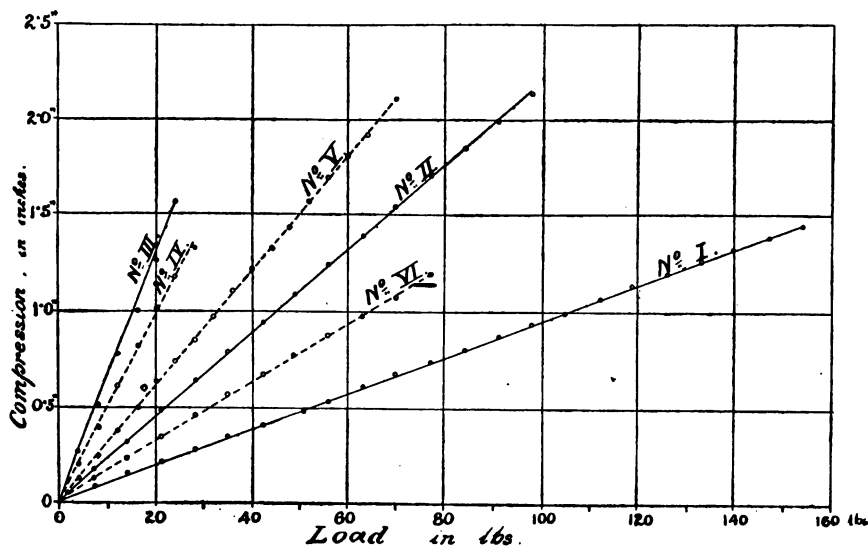


FIG. 91.

The actual compressions and loads not given in the table were plotted on squared paper, and the results are shown in fig. 91 on a reduced scale. From the lines obtained and numbered I. to VI. the value of δ in the equation

$$K = \delta W$$

was obtained. We give the diagram and the table to enable the student to form some idea of the ultimate value of testing springs and plotting the results.

Example 3.—A firm of engineers making a *speciality of a given type of engine in different sizes* found, on plotting the costs to scale along a horse-power base, that the points obtained approximated

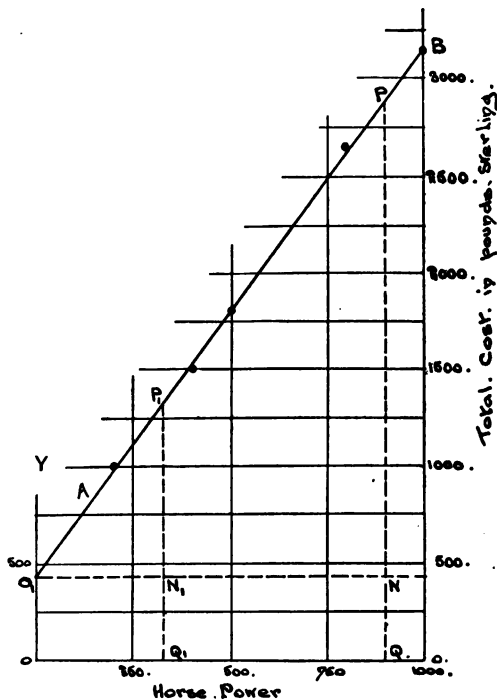


FIG. 92.

to a straight line. Find a formula connecting horse-power (H.P.) and cost (C.), having given the costs for 200, 400, 500, 800, and 1000 H.P. engines as £1000, £1505, £1815, £2650, and £3150 respectively.

First plotting the horse-power along the base with the corresponding costs along the verticals, we obtain five points which enable us to draw the straight line AB, fig. 92, approximately to these figures. This line indicates, as we shall see immediately, that *the cost increases by an equal amount for*

each unit increase in the power and that the formula is of the form,

$$\text{Cost} = \text{some constant} + \text{H.P.} \times \text{some constant.}$$

The student will no doubt notice that the line AB produced intersects the axis OY in a point O_1 , some distance above the point of origin, O, giving apparently the cost OO_1 when the power of the engine is *nil*. We shall explain this difficulty later, but it is sufficient for the present to point out that the linear law ceases to be true when a certain minimum power is reached.

To find a formula for cost in terms of H.P., take any convenient point P on the line AB and drop a perpendicular PQ to the base OQ. Then PQ represents the cost of an engine whose power is denoted by OQ.

$$\text{But} \quad PQ = PN + NQ = PN + OO_1,$$

and OO_1 actually measures 0.88 of a unit, and therefore represents 0.88 of £500 = £440.

$$\text{Further,} \quad \frac{\text{PN expressed as money in pounds}}{\text{O}_1\text{N expressed as units of power}}$$

gives the value in pounds sterling for each unit increase in power, and this ratio has always the same value for any point on the line. Thus if another point P_1 be taken, $\frac{P_1N_1}{O_1N_1} = \frac{PN}{O_1N}$, because the triangles $O_1P_1N_1$ and O_1PN being similar, their sides are proportional.

PN measures 4.95 units in length and represents £4.95 × 500 = £2475, and O_1N is 3.6 units long and denotes 900,

$$\therefore \frac{PN}{O_1N} = \frac{2475}{900} = 2.75;$$

i.e. each unit increase in the power costs £2.75, consequently, since $\frac{PN}{O_1N} = 2.75$

$$\begin{aligned} PN &= O_1N \times 2.75 \\ &= 2.75 \text{ H.P.} \end{aligned}$$

Finally, by repeating our original statement, and substituting for OO_1 and PN, we obtain

$$\begin{aligned} \text{Cost (PQ)} &= OO_1 + PN \\ \text{,, (C)} &= \text{£440} + 2.75 \text{ H.P.} \end{aligned}$$

Example 4.—The firm who manufactured the engines in Example 3 supply a 600 H.P. engine of the same type. What would be its approximate cost?

Using the equation just obtained we have

$$\begin{aligned}\text{Approximate cost} &= \text{£}440 + 2.75 \times 600 \\ &= \text{£}2090 \text{ sterling.}\end{aligned}$$

Example 5.—A number of indications were made on a small

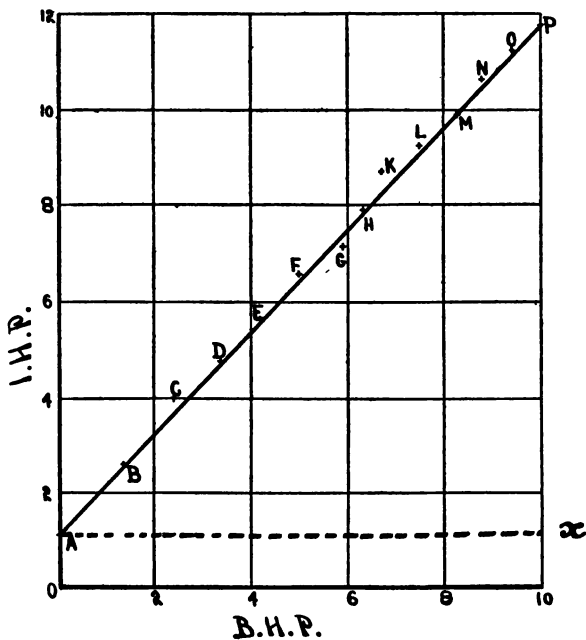


FIG. 93.

steam engine with various brake loads with the following results:—

TABLE XV.

Trial.	A.	B.	C.	D.	E.	F.	G.	H.	K.	L.	M.	N.	O.	P.
B.H.P.	0	1.4	2.4	3.4	4.2	5.0	5.9	6.3	6.7	7.5	8.3	8.8	9.4	10.0
I.H.P.	1.15	2.62	4.0	4.75	5.6	6.55	7.1	7.9	8.7	9.2	9.9	10.6	11.2	11.75

The results were plotted on squared paper as in fig. 93, and a linear law was found to connect I.H.P. with B.H.P. It was seen

that points for all the tests excepting C, G, and K came fairly on the line. When these three different tests were repeated, the corrected figures were

TABLE XVI.

Trial.	C.	G.	K.
B.H.P. .	2.3	5.9	6.8
I.H.P. .	3.65	7.4	8.5

which brought the record almost exactly on the straight line. The law for the variation between I.H.P. and B.H.P. was then determined by measuring the mean slope of the line AP; thus, by drawing a horizontal line from A to x to meet the perpendicular from P, the height xP , divided by the base Ax , gives the *rate of increase of I.H.P. per unit increase in B.H.P.* Thus in the present case $\frac{xP}{Ax} = \frac{10.6}{10} = 1.06$, and the law for the curve is found to be

$$\text{I.H.P.} = 1.06 \text{ B.H.P.} + 1.2.$$

Example 6.—The values of the latent heat (L) of steam at different temperatures (t) are given in Table XVII. Find a formula connecting L and t , and from it calculate the latent heat of steam at 250° F.

TABLE XVII.

t °F.	212°	281°	328°	358°	382°	401°
L in B.Th.U.	966	917	884	862	845	831

First, the results between L and t should be plotted on squared paper as in fig. 94, when it will be seen that the latent heat diminishes regularly by a constant amount for equal intervals or rise in temperature.

To find a formula, take any convenient point P on the line AB and draw PY_1 and PQ horizontally and vertically respectively. Then, since the triangles APY_1 and PBQ and AOB are similar, their sides are proportional.

$$\therefore \frac{AY_1}{PY_1} = \frac{PQ}{BQ} = \frac{OA}{OB} = \text{a constant} = 0.71.$$

In other words, the latent heat of steam decreases by 0.71 of a B.Th.U. for each degree rise in temperature.

Further, since $\frac{OA - OY_1}{PY_1}$ or $\frac{a - PQ}{OQ} = \frac{a}{OB} = 0.71$,

$$a - PQ = 0.71 \times OQ,$$

$$166 - PQ = 0.71 \times OQ.$$

or

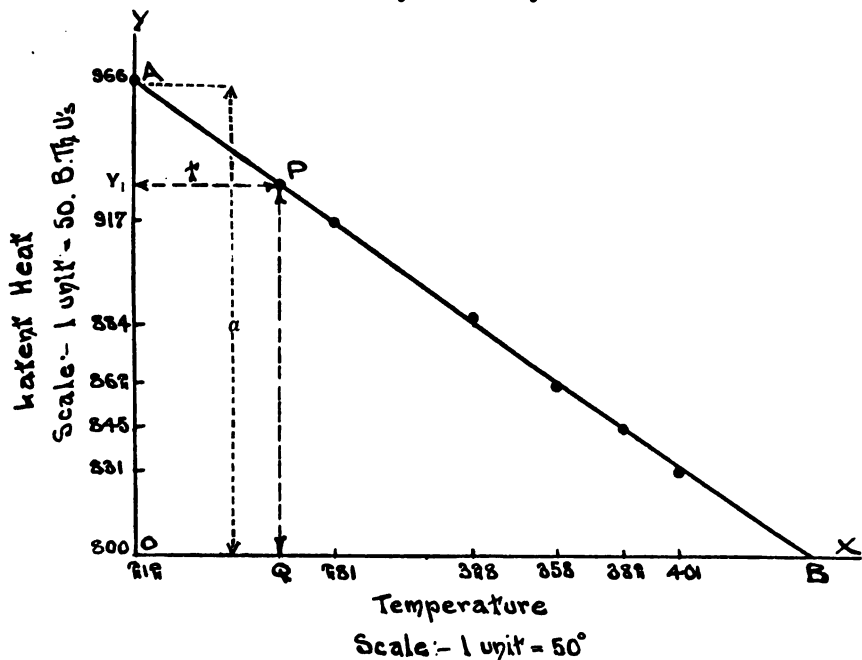


FIG. 94.

But $PQ = L - 800$ and $OQ = t - 212$.

Hence $166 - (L - 800) = 0.71(t - 212),$
 $\therefore L = 966 - 0.71(t - 212),$

and by clearing of brackets and simplifying,

$$L = 1114 - 0.71t.$$

The latent heat of steam at 250° F. is found by substituting for t in either of the formulæ just derived, hence

$$L = 1114 - 0.71 \times 250 = 1114 - 177.5 = 936.5 \text{ B.Th.U.}$$

Example 7. Discharge of water through circular orifices.—In some recent steam-engine tests (made by the author) the quantity of circulating water was measured in tanks in the usual way and then passed into a settling tank and allowed to flow through two different sizes of circular holes in brass plates. The arrangement thus afforded a ready means of checking the tank readings by

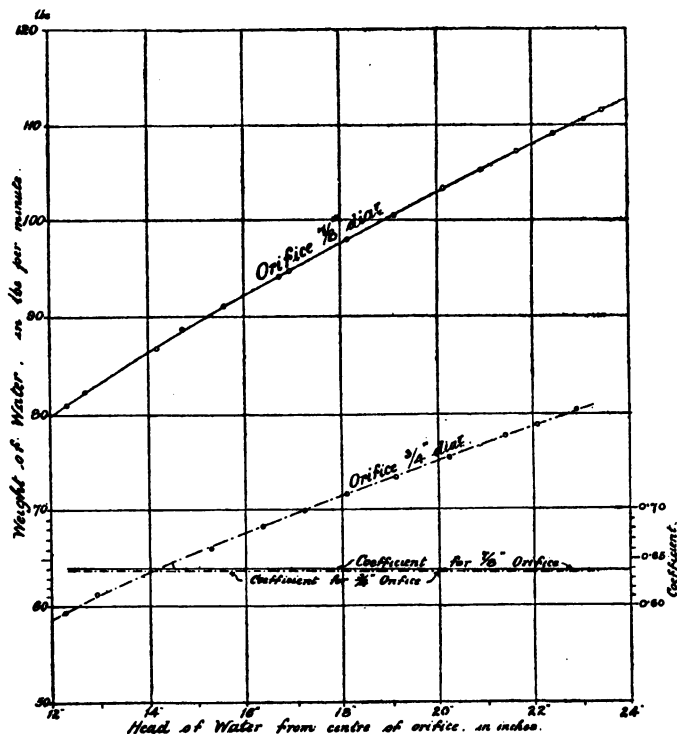


FIG. 95.

observing the head of the water flowing through the orifice. In calculating the coefficient of discharge through the orifice it was discovered that the *coefficient varied sensibly with the head of water*, being slightly greater with the low heads and decreasing as the head got greater. The coefficients for both sizes of orifice ($\frac{3}{4}$ inch and $\frac{7}{8}$ inch diameter) are shown in fig. 95, plotted to a base of head of water, as also the curves for the actual weight of water discharged per minute for each of the two sizes.

The calculation for the theoretical weight of water being calculated from the formula

$$W_t = \sqrt{H} \times 481.5 \times A \times w,$$

where W_t = theoretical weight of water in lbs. per min.,

H = head of water from centre of orifice in feet,

A = area of orifice in square feet,

w = weight of 1 cubic foot of water in lbs.,

$$\text{the coefficient} = \frac{\text{actual discharge}}{\text{theoretical discharge}}$$

over a given period of time (say per minute).

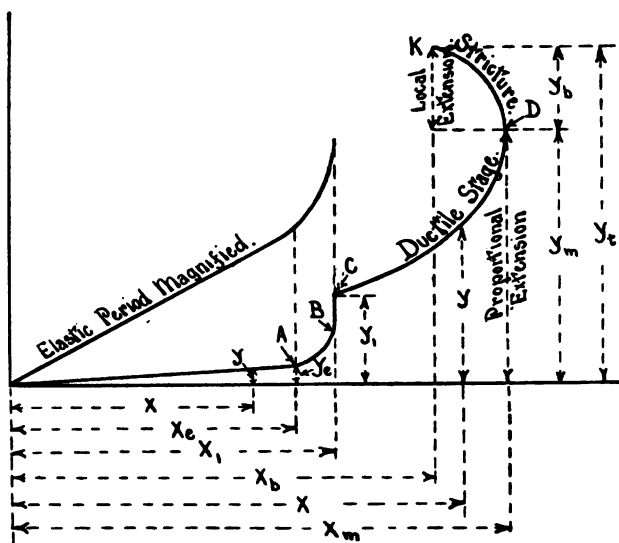


FIG. 96.

Example 8.—If a specimen piece of wrought iron, mild steel, or other good **ductile material** of uniform cross section be held firmly at each end, and **pulled longitudinally** by an increasing force or load, the specimen stretches continuously until breaking takes place. It is usual to measure the actual stretch under each load as the test goes on, and then to plot the results obtained between load and extension or stretch. Fig. 96 shows the type of diagram that would be obtained, and a study of this diagram furnishes a good deal of valuable information regarding the behaviour of a ductile material tested to destruction.

Referring to the typical diagram, we notice (1) that during a certain period and up to a certain load (x_e) the extensions (y) are proportional to the loads (x). This period is called the **elastic period**, owing to the fact that if the load be removed the specimen returns to practically its original length. The value of the load or stress (x_e) at the **elastic limit for wrought iron and mild steel** usually lies somewhere between 11 and 18 tons per square inch, and is called the **limit of elasticity**. The stretch during this elastic period is of the order of $\frac{1}{1000}$ of the length of the piece tested in the case of wrought iron or mild steel.

(2) When the limit of elasticity is reached the line begins to curve as at A, until quite suddenly, for some load (x_1) just a little above x_e , the specimen stretches without increase in load. This stretch for iron and steel is from $\frac{1}{50}$ to $\frac{1}{100}$ of the length of the specimen tested. The total stretch up to point C is denoted by y_1 in the figure. The point B in the diagram and the load (x_1) corresponding to it are called the **yield point**. For commercial purposes the **yield point** and limit of elasticity are taken as coincident.

(3) Having reached a definite point C, the extensions (y) begin to take place more slowly for increases in load (see the curve), afterwards increasing rapidly with small increases in load, till what is known as the **maximum load** (x_m) is reached—the extension being represented by y_m . Up to the point of maximum load the extensions or stretches are more or less uniform along the entire length of the specimen tested, so that up to this point (marked D in the diagram) we have what is known as the period of proportional elongation or extension.

(4) From the point of maximum load the stretch increases without any addition of load; in fact, the load is gradually taken off as the extension proceeds, until the specimen breaks with a load represented by x_b ; $y_b + y_m$ giving the final extension. The increase in extensions after the point D are the extensions due to local elongation near the breaking point, for the specimen is reduced very considerably in sectional area, with a good deal of stretch near the fracture. This **breaking load of wrought iron and mild steel varies from 18 to 30 tons per square inch cross section**. The total extension from start to finish is from 10 to 30 per cent. on a 10-inch length, and the reduction in area from as low as 7 to as much as 50 per cent. from the original area.

The student should now plot out the results given in Example 7, p. 159.

Having illustrated by means of a few examples some of the uses to which squared paper can be put, we now proceed to deal with the subject in more general terms.

Referring to fig. 97, let AB be a line obtained from the plotting

of two mutually dependent variables, and let P, P_1 , and P_2 , etc., denote points on this line. Draw the horizontals and verticals shown. The lines of reference OX, OY are termed axes, and O is called the point of origin. Thus OX, OY in Example 1 are the axes of load and extension respectively; in Example 3, axes of power and cost; in Example 6, axes of temperature and heat units; and in Example 8, axes of stress and strain.

Distances marked off along the axis OX are called abscissæ (singular, abscissa) and are often denoted by x , while distances marked off along the verticals and parallel to OY are named

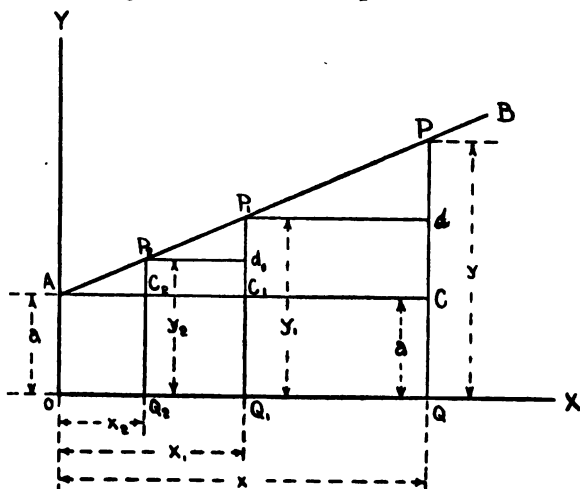


FIG. 97.

ordinates. These ordinates are conveniently referred to by the letter y . Now x and y are the mutually dependent variables, and the variation between them is represented by the straight line. From this line it is an easy matter to find an equation connecting the variables. This equation, when found, is called the **equation to the straight line**.

We now proceed to find this equation. The triangles APC , AP_2C_2 , P_1Pd , $P_2P_1d_1$, are similar triangles, and therefore their sides are proportional.

$$\text{Hence} \quad \frac{PC}{AC} = \frac{Pd}{P_1d} = \frac{P_1d_1}{P_2d_1} = \text{constant} = b \text{ (say),}$$

$$\text{using the symbols} \quad \frac{y-a}{x} = \frac{y-y_1}{x-x_1} = \frac{y_1-y_2}{x_1-x_2} = b;$$

taking the first term and the last,

$$\frac{y-a}{x} = b; \therefore y-a = bx,$$

and

$$y = a + bx.$$

Notice that this equation has the same form as the one derived in Examples 3 and 5, pp. 144, 146.

If AB intersects the axis OY produced below OX instead of above, then the value of a is taken as negative.

If the line AB passes through the origin O as in Examples 1 and 2, pp. 142, 143, the constant $a=0$, and the equation becomes

$$y = bx,$$

or, referring to fig. 98,

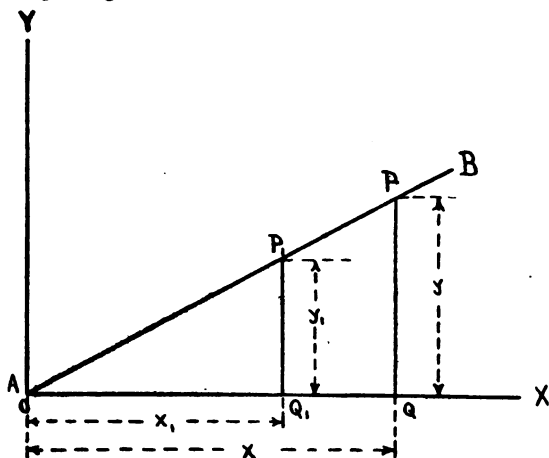


FIG. 98.

$$\frac{y}{x} = \frac{y_1}{x_1} = \text{constant} = b,$$

and $\therefore$

$$y = bx.$$

Compare this equation with the one derived in Example 1, p. 142. If the line slopes down to the right towards OX, as shown in fig. 99, the equation is—

$$y = a - bx,$$

for

$$\frac{a-y}{x} = \frac{y_1-y}{x-x_1} = \text{constant} = b,$$

from which we obtain $a-y = bx$ or $y = a - bx$.

Notice that this equation is identical in form with that derived from Example 6, p. 148.

Example 9.—Table XVIII. gives the volume in cubic feet occupied by 1 lb. of steam at the given pressures. Find the curves showing graphically the relationship between p and V and

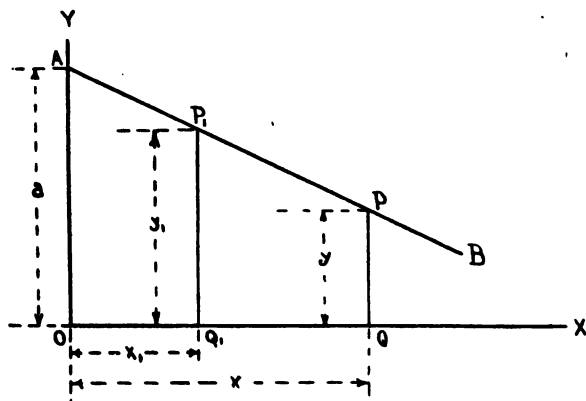


FIG. 99.

log p and log V , and then determine an equation connecting p with V , and also the logarithms of these two variables.

TABLE XVIII.

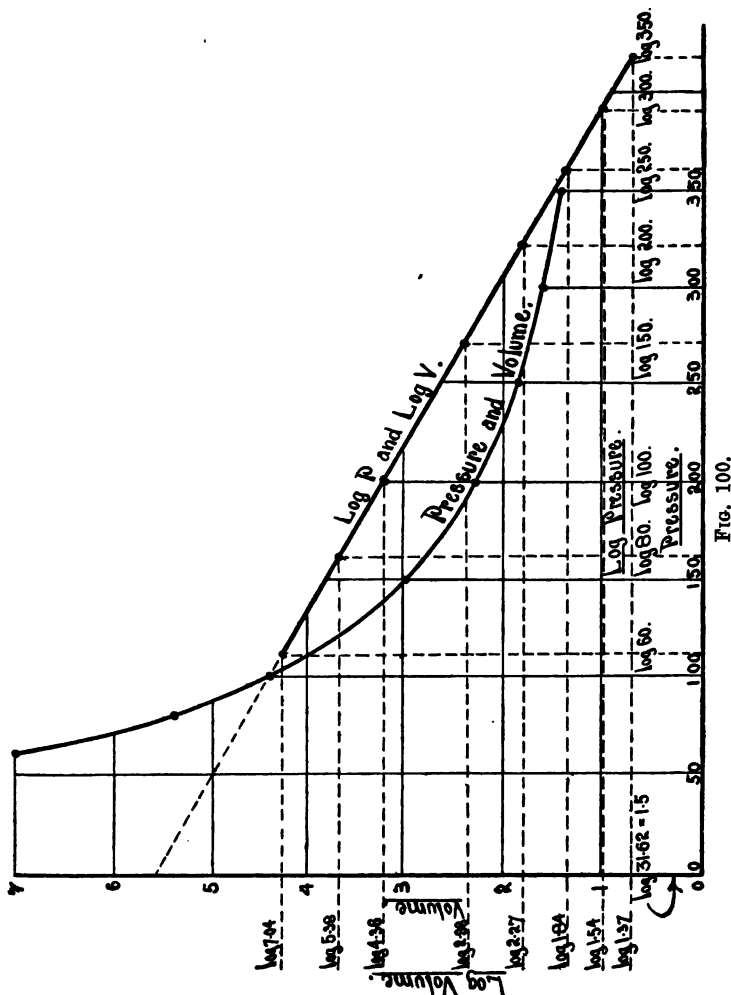
Pressure	60	80	100	150	200	250	300	350
Volume	7.04	5.38	4.36	2.98	2.27	1.84	1.54	1.33
Log p	1.7782	1.9031	2.0000	2.1761	2.3010	2.3979	2.4771	2.5441
Log V	0.8476	0.7308	0.6395	0.4742	0.3560	0.2648	0.1875	0.1367

It will be noticed that pressures and volumes are taken from the steam tables, p. 290, and the logs from the table of common logarithms given on pp. 78 to 80.

(a) First plotting the results between p and v , we obtain the curve in fig. 100. The relation between p and v is not therefore a linear or straight-line relation. Nevertheless the curve is an interesting one, in that it shows very clearly the kind of variation of v with p . We see at a glance that in the lower ranges of pressure a small increase in the pressure corresponds to a large drop in

the volume, while in the higher ranges increase in pressure produces only a small change in volume (vide Table XXII., pp. 290, 291).

(b) To find a formula connecting p and v it will be necessary



for us, first of all, to tell the student that these two variables are connected by an equation of the form

$$pv^n = \text{a constant} = k \text{ (say),}$$

when n and k are constants whose values are to be determined. That such a formula does hold, can readily be shown, as follows:—

Taking logs of both sides, we have

$$\log p + n \log v = \log k.$$

Now this result is the equation to a straight line which is similar in form to

$$y = a - bx;$$

for by transposing $\log p$ and dividing by n we get

$$\log v = \frac{1}{n} \log k - \frac{1}{n} \log p,$$

$\log v$ and $\log p$ corresponding to y and x respectively, $\frac{1}{n} \log k$

taking the place of a , and $\frac{1}{n}$ the place of b . We have next to show that the result of plotting the several values of $\log p$ and $\log v$ along rectangular co-ordinates gives points approximating to a straight line.

Plotting the values given in the table, we see (fig. 100) that they do give the linear law, and therefore the equation to this line is of the logarithmic form, as given above, and the formula required is $pv^n = k$, when n and k are to be determined.

To determine the values of n and k , we fix any two convenient points on the straight line showing the relationship between $\log p$ and $\log v$, and measure off the ordinates and abscissæ in terms of $\log p$ and $\log v$ respectively.

Thus, since the points for 60 and 250 lbs. pressure are approximately on the line, we may use the values of the logs given in table, and substitute as under in the two logarithmic equations, obtaining two equations from which we may determine both n and k .

$$\begin{aligned}\log 60 + n \log 7.04 &= \log k \\ \log 250 + n \log 1.84 &= \log k,\end{aligned}$$

or

$$\begin{aligned}1.7782 + n \times 0.8476 &= \log k \\ 2.3979 + n \times 0.2648 &= \log k.\end{aligned}$$

$$1.7782 + n \times 0.8476 = 2.3979 + n \times 0.2648,$$

i.e.

$$n(0.8476 - 0.2648) = 2.3979 - 1.7782$$

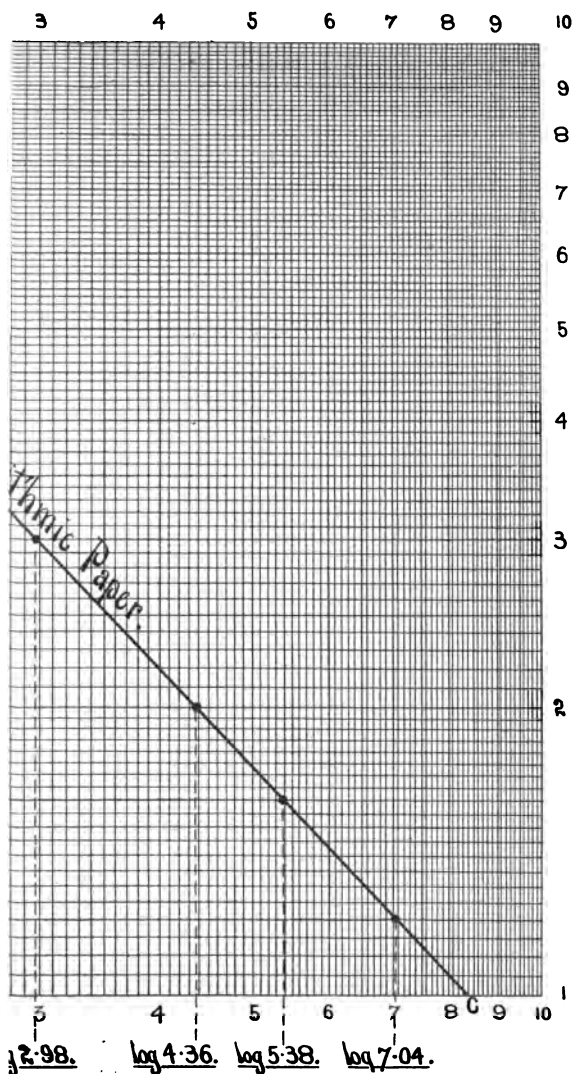
$$0.5828 n = 0.6197$$

$$\therefore n = \frac{0.6197}{0.5828} = 1.06.$$

With a little more accuracy in the plotting of $\log p$ and $\log v$,



[To face p. 157.



2.98. log 4.36. log 5.38. log 7.04.

with larger scales, and taking two sets of values for $\log p$ and $\log v$ directly from the line, the index n can be determined to four places of decimals quite readily.

Substituting the value of n in one of the equations, we have

$$1.7782 + 1.06 \times 0.8476 = \log k,$$

or

$$\log k = 2.6766 \text{ approx.}$$

Then from the table of antilogs, p. 82, we find that $k = 474.9$. We must also point out that the value of k would be more accurately determined with more accurate values of n .

Our equation connecting p and v is

$$pv^{1.06} = 475 \text{ (approx.)}.$$

Logarithmic Paper.—Results showing the variations between two quantities can often be more quickly interpreted by plotting their values directly on logarithmic paper instead of on the ordinary inches and tenth-of-an-inch paper. The specimen sheet, fig. 101, gives some idea of the **double-log paper**. It is shown on a reduced scale. In its two directions its construction and divisions are identical with those on the ordinary slide rule, the actual numbers, as it were, being figured, instead of the logs, against the points whose positions really define lengths corresponding to the logs of the numbers.

Taking the exercise just given, we first fix our scales for convenience in plotting.

Thus, plot $\frac{p}{50}$ as ordinates, and v as abscissæ, as shown in fig.

101. Points are obtained indicating the straight-line law connecting $\log p$ and $\log v$.

Otherwise the student may plot p and v directly without altering the scale, but a much smaller diagram would be the result.

Now, the line obtained has an equation of the form $y = a - bx$. $\log \frac{p}{50}$ takes the place of y , $\log v$ the place of x , and a is represented by AB in the figure.

But
$$AB = \log 9.5 \text{ or } \log \frac{475}{50}.$$

Hence we may rewrite this equation to the line thus:

$$\log \frac{p}{50} = \log 9.5 - b \log v.$$

The value of b is found as already explained.

Thus, $b = \frac{AB}{AC}$ or $\frac{\log 9.5}{\log 8.4} = 1.06,$

so that we may rewrite the logarithmic equation

$$\log \frac{p}{50} = \log 9.5 - 1.06 \log v;$$

from which we obtain

$$\log \frac{p}{50} + 1.06 \log v = \log 9.5,$$

or $\log \left[\left(\frac{p}{50} \right) \times (v^{1.06}) \right] = \log 9.5,$

i.e. $\log \frac{pv^{1.06}}{50} = \log 9.5;$

so that $\frac{pv^{1.06}}{50} = 9.5,$

and $pv^{1.06} = 9.499 \times 50 = 474.9,$

the equation derived by the previous method with the plotting on ordinary squared paper.

EXERCISES.

1. An indiarubber rod 0.627 in. diameter and 55 centimetres long was suspended from one end and loaded by increasing loads from the other end, and the total extension for each load was measured. The following are the results:—

Load (lbs.),	2	4	6	8	10	12	14
Extension (cm.),	0.4	0.95	1.55	2.10	2.75	3.45	4.11

Plot these results, find the average extension for each 2-lb. load, and obtain an equation connecting load and extension.

Ans. 0.55 cm., $\delta = 0.275 W.$

2. The following results were obtained from experiments on a crane:—

Load lifted,	5.06 lbs.	13.25 lbs.	26.00 lbs.	35.25 lbs.
Effort exerted,	14 lbs.	42 lbs.	84 lbs.	112 lbs.

The effort was exerted from the circumference of a pulley taking the place of the ordinary handle-bar.

Find the law connecting effort and load.

3. The weights of some small oil engines were as follows:—

B.H.P.,	2.5	5.0	10,	20
Weight (cwts.),	17	27	52	105

Find the approximate weight of engines of $7\frac{1}{2}$, 12, and 15 B.H.P.

Ans. 40·5, 64, 79·4 cwt.

4. A series of blast fans gave the following results:—

Diar. of fan,	12 in.	16 in.	20 in.	24 in.
Revs. per min.,	2·500	2·100	1·750	1·600
Cub. ft. air per min.,	1·100	1·650	2·150	2·500

Give sizes of fans and speeds for the following outputs—

(a) 3·000 cub. ft. per min., (b) 800, (c) 2·750 cub. ft. per min.

5. A small and uneconomical steam engine used the following quantities of steam at the different loads:—

B.H.P.,	2·8	4·3	5·15	6·2	8·0
Total steam in lbs. used per hour,	200	275	320	370	464

Plot the following curves to a base of B.H.P.

(a) Total steam used per hour

(b) Steam used per B.H.P. per hour

and find the law of curve (a)

6. If the law connecting B.H.P. and I.H.P. of the above engine be $I.H.P. = 1·1 \text{ B.H.P.} + 0·6$, find the I.H.P. of the engine at each of the above loads and plot a curve showing steam used per I.H.P. per hour to a base of B.H.P.

7. In a tensile test of a piece of mild steel bar the following results were obtained:—

Load on bar (tons),	3·62	7·68	9·04	10·85	11·75	12·2	10·27
Stretch (total, inches),	0·0041	0·18	0·27	0·58	1·05	1·6	2·45

Plot the results for this test, showing graphically the relationship between stress and strain.

8. A piece of wrought iron 0·72 in. diameter and $4\frac{1}{8}$ in. long was twisted to destruction, and the angles of twist corresponding to the twisting moment or torque are given below. Plot a curve showing graphically the relationship between twist and moment and obtain an equation to the curve.

Hint.—The equation is of the form $y = ae^{bx}$, where y and x represent the twist and torque respectively and a and b are constants to be determined—

Torque in lbs.	Twist.	Torque in lbs.	Twist.	Torque in lbs.	Twist.
1000	2°	2600	69°	3400	181
1800	16°	2800	89	3800	288
2000	25°	3000	116	4100	412
2400	51	3200	150	4300	556

* Paper by C. E. Larard, *Proceedings Institution of Mechanical Engineers*, 1907.

9. Plot curves to the following equations :—

$$\begin{array}{lll}
 (1) & y = 2.3x & (2) \quad y = 3 + 2\frac{1}{2}x \quad y = 15 - 2x \\
 (3) & y^2 = 8x & (4) \quad xy = 20 \\
 (5) & xy^2 = 100 & (6) \quad y = ae^{bx}
 \end{array}$$

Hint.—Take a series of values for x in each equation and calculate the corresponding values of y , then plot the two sets of numbers along rectangular axes.

SECTION II.

CHAPTER VI.

PULLEYS, WHEELS IN TRAIN, CHANGE WHEELS, SPEED OF SHAFTING, TOOLS, Etc.

Speed Ratio of Pulleys.—Where a number of shafts have to be driven from some main shaft, sometimes at different speeds, the usual means of transmitting the power are from belt or rope pulleys. In Chapter IV. we found a method of expressing the power transmitted by belts, ropes, and spur gearing, when run at

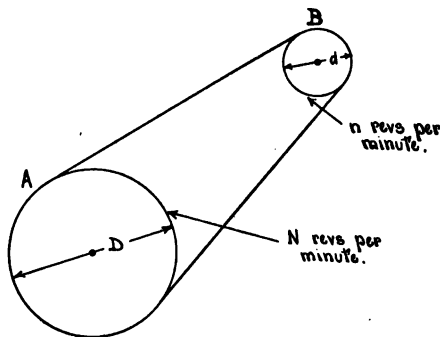


FIG. 102.

different speeds, and we propose now to deal with the methods of fixing the size of pulleys according to the speed desired.

A combination of two or more gear wheels (or pulleys) is spoken of as a **train**, the first wheel in the series, that is, the one which gives off the power, being known as the **driver**; and the last wheel, which receives the power, being termed the **follower**. Intermediate wheels have sometimes to be interposed between driver and follower to give required directions of rotation or speed.

With a single belt or rope drive, as shown in fig. 102, the rule connecting diameters of pulleys with speeds of rotation is a very

simple one. Let A be the driver and B the follower. Then if D and d represent their diameters, and N and n the number of revolutions that each makes per minute, assuming no slip, we have—

$$D \times N = d \times n,$$

$$\text{or} \quad D = \frac{d \times n}{N},$$

$$\text{or} \quad d = \frac{D \times N}{n}.$$

The two diameters D and d must both be expressed in terms of the same unit—*i.e.* if d be given in inches, D must also be taken in inches; or if D be given in feet, then d must be in feet. Except for very large pulleys, it will generally be found most convenient to express both D and d in inches.

If we have to include a given amount of slip in the belt, let x represent the percentage of slip, then $\frac{100-x}{100}$ will represent the ratio between the actual speed obtained and the speed calculated assuming no slip. The number of revolutions n of the follower will therefore be smaller than that given by the above rule, in the ratio of $\frac{100-x}{100}$, or, to give the required speed after allowing for slip, the diameter D of the driver must be greater in the ratio of $\frac{100}{100-x}$.

For example, if $D = 3$ feet, $d = 12$ inches and $N = 100$. Then, assuming no slip,

$$n = \frac{D \times N}{d} = \frac{36 \times 100}{12} = 300 \text{ revolutions per minute};$$

or, assuming 5 per cent. slip,

$$n = \frac{D \times N}{d} \times \frac{95}{100} = 285 \text{ revolutions per minute.}$$

If we have to find D , the diameter of the driver, given $d = 10$ inches, $n = 288$ revolutions per minute, and $N = 60$ revolutions per minute. Then, neglecting slip of belt,

$$D = \frac{d \times n}{N} = \frac{10 \times 288}{60} = 48 \text{ inches diameter};$$

or, assuming 4 per cent. slip of belt,

$$D = \frac{d \times n}{N} \times \frac{100}{100-4} = 50 \text{ inches diameter.}$$

The same rules apply whether the belt be an open one or a crossed one, the only difference being that, if the belt be crossed, the direction of rotation of the follower is the reverse of that of the driver. In applying the above rules for rope drives, the diameters of the pulleys should be measured to the centre of the ropes.

Compound or Double Belt Drives.—Where the value of " e " * exceeds 8, or is less than $\frac{1}{8}$, the comparative sizes of the pulleys are too large for the difference in speed to be obtained by a single belt, and an intermediate shaft and pair of pulleys have to be used, necessitating the use of two, and sometimes three belts. Take, for example, the combination of pulleys and belts shown in fig. 103, which represents diagrammatically a compound or double drive from a small engine to a centrifugal fan. Here the pulley A on the engine shaft is the first driver, and the pulley D on the fan spindle the last follower. As the speed required by

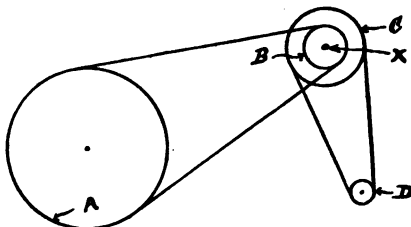


FIG. 103.

the fan (2000 revolutions per minute) is too high to be obtained by a single drive from the engine shaft, a countershaft has to be interposed between the engine and fan, as shown at X, with two pulleys B and C mounted upon it, the former to receive the belt from A, and the latter to drive the fan at D. Thus B is the "follower" of the first drive, and C the "driver" of the second drive. In proportioning the sizes of the various pulleys to be used, we call the ratio of the speed of the last follower, divided by the speed of the first driver, the value of the train. Let this be represented by e . Then if

N_a = revolutions per minute of engine (pulley A)

N_d = revolutions per minute of fan (pulley D), and

D_a, D_b, D_c, D_d = the diameters of A, B, C, and D respectively
(all in inches),

we have

$$e = \text{value of train} = \frac{N_d}{N_a}.$$

* " e " denotes the speed ratio of the train or value of the train.

Let $N_a = 100$ revolutions per minute ; then if the fan has to make 2000 revolutions per minute,

$$e = \frac{2000}{100} = 20.$$

In any system of belt pulleys, the value of e is found by dividing the continued product of the diameters of all the "drivers" by the continued product of the diameters of all the "followers." In the present case, therefore,

$$e = \frac{D_a \times D_c \text{ (drivers)}}{D_b \times D_d \text{ (followers)}}$$

and
$$N_d = N_a \times e \quad \text{or} \quad = N_a \times \frac{D_a \times D_c}{D_b \times D_d}.$$

Thus, whatever diameters of pulleys are fixed upon, they must be such that the value of e as given above shall be 20. We have therefore to split up the value e into two ratios, $\frac{D_a}{D_b}$ and $\frac{D_c}{D_d}$, which, when multiplied together, shall give us 20. If we make both ratios equal, they will be equal to $\sqrt{20}$, or 4.47 ; but as this is a surd quantity, the diameters resulting from such a ratio would lead to fractions of an inch, which are undesirable. Naturally, therefore, we choose any two factors which, multiplied together, make 20, and so we choose 4×5 ; that is, make one ratio 4 and the other 5. The question of which ratio should be used for either drive is not of very much importance. Generally it is preferable to make the drive which has the larger speed the one with the smaller ratio. To fix the actual sizes of pulleys required, assuming that they are not given, we have to proportion the pulleys so that they will give a reasonable speed for the belt—say between 800 and 1600 feet per minute. If we fix upon the lower value for A, we find $D_a = 2$ ft. 6 in., and $D_b = \frac{30}{5} = 6$ in. That is, the countershaft will make 500 revolutions per minute. If, therefore, we put D_c at 12 in. and $D_d = \frac{12}{4} = 3$ in., we get belt speeds of 780 feet and 1570 feet per minute respectively. The widths of belt can now be fixed as explained in Chapter X. All the above calculations assume no slip in the belts. In actual practice this may amount to 5 per cent. of the speed, and if it be required to allow for this, the proportions of the pulleys should be calculated to give a theoretical speed at D of 5 per cent. more than that required, in this case 2100 revolutions per minute.

The value of the train would then be designed for e equal to $\frac{2100}{100} = 21$ instead of 20.

Example 1.—An engine has to drive a countershaft at 180 revolutions per minute. The size of flywheel on the engine is 7 ft. 0 in. diameter, and its speed 80 revolutions per minute. Find the size of pulley required on the countershaft, (a) assuming no slip of belt; (b) assuming 4 per cent. slip of belt.

Neglecting slip,
$$e = \frac{180}{80} = 2\frac{1}{4} \text{ to } 1.$$

Diameter of pulley required $= \frac{84}{2\frac{1}{4}} = \frac{84 \times 4}{9} = 37\frac{1}{3}$ in. (say 37 in.).

Including slip,
$$e = \frac{180}{80 \times 0.96} = 2.34.$$

Diameter of pulley required $= \frac{84}{2.34} = 35.8$ in.
 $= (\text{say}) 36$ in.

Example 2.—A lathe countershaft has to be driven at 200 revolutions per minute from a line shaft running at 120 revolutions per minute. If the size of the fast and loose pulleys on the countershaft be 14 inches diameter, find the size of drum required to drive it from the line shafting, assuming a slip of belt of 5 per cent.

Here
$$e = \frac{200}{120 \times 0.95} = 1.75.$$

Diameter required $= 14 \times 1.75 = 24\frac{1}{2}$ in. = (say) 25 in. diameter.

Example 3.—A centrifugal pump has to run at 1600 revolutions per minute, and is provided with a 4-inch pulley on its spindle. Find suitable sizes of pulleys for a compound drive from a line shaft making 110 revolutions per minute, assuming 4 per cent. and 6 per cent. slips in belts.

Here
$$e = \frac{1600}{110 \times 0.96 \times 0.94} = 16.1.$$

For this value of e two drives, each with a speed ratio of 4, could be arranged, or, preferably, such factors as 5×3.2 or $4.8 \times 3\frac{1}{3}$. Assume the former; then if D_a, D_b, D_c, D_d represent the diameters, as in fig. 103, p. 163,

$$D_c = 4 \times 3.2 = 12.8 = (\text{say}) 13 \text{ in.}$$

$$\frac{D_b}{D_a} = \frac{1}{5}, \text{ and let } D_a = 30 \text{ in., then } D_b = \frac{30}{5} = 6 \text{ in.}$$

This gives approximate speeds of belts of 860 and 1800 feet per minute respectively, and a final speed of pump equal to

$$110 \times \frac{30}{6} \times 0.96 \times \frac{13}{4} \times 0.94 = 1610 \text{ revolutions per minute (approx.).}$$

Speed Cones or Step Pulleys.—When the speed of the driven shaft is subject to frequent changes, as in most machine tools, speed cones or step pulleys are employed, as in fig. 104.

Let D_1, D_2, D_3 represent the successive diameters of the cone pulleys on the driving shaft, and d_1, d_2, d_3 those on the driven or machine shaft. Then there are three different values for e , the

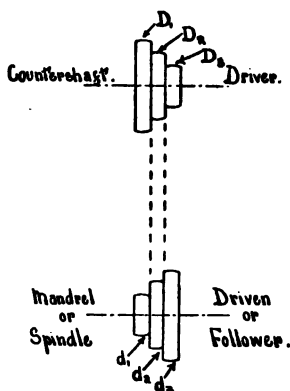


FIG. 104.

value of the train, depending upon which pair of pulleys the belt is on. Let e_1, e_2 , and e_3 represent these three values for the train.

Then
$$e_1 = \frac{D_1}{d_1}, \quad e_2 = \frac{D_2}{d_2}, \quad e_3 = \frac{D_3}{d_3}.$$

Then, if N represents the revolutions per minute of the driver, and n_1, n_2, n_3 the revolutions of the follower,

$$\begin{aligned} n_1 &= N \times e_1 = N \times \frac{D_1}{d_1} \\ n_2 &= N \times e_2 = N \times \frac{D_2}{d_2} \\ n_3 &= N \times e_3 = N \times \frac{D_3}{d_3}. \end{aligned}$$

Then the total ratio of variation in speed is $\frac{n_1}{n_3} = \frac{D_1}{d_1} \times \frac{d_3}{D_3}$, and

the middle speed ratio (with belt on central pulleys) is $\frac{n_2}{N} = e_2$.

Given the sizes of the pulleys, and the speed of the driving shaft, the various speeds of the driven shaft can be calculated by the above formulæ.

Example.—A countershaft with speed cone 13 in., 15 in., and 17 in. diameter makes 80 revolutions per minute, and drives a radial drill with a speed cone 10 in., 8 in., and 6 in. diameter respectively. Find the various speeds of the machine shaft.

Here $N = 80$, and $D_1, D_2, D_3 = 17$ in., 15 in., and 13 in.

Also d_1, d_2 , and $d_3 = 6$ in., 8 in., and 10 in. respectively.

Therefore

$$n_1 = 80 \times \frac{17}{6} = 226 \text{ revolutions per minute.}$$

$$n_2 = 80 \times \frac{15}{8} = 150 \quad \text{,,} \quad \text{,,}$$

$$n_3 = 80 \times \frac{13}{10} = 104 \quad \text{,,} \quad \text{,,}$$

The above are all calculated neglecting slip of the belt. If this is to be allowed for, multiply each of the speeds obtained as above by the fraction $\frac{(100 - x)}{100}$, where x represents the percentage of slip.

To be absolutely accurate, the thickness of the belt should also be allowed for. That is to say, the effective diameter of the pulley should be measured to the centre of the belt or band; but in practice this degree of accuracy is seldom required.

Exercises :—

(1) A shaft fitted with a cone pulley of 12 in., 14 in., and 16 in. diameters makes 80 revolutions per minute, and drives a second shaft on which is keyed a cone pulley whose diameters are 10 in., 8 in., and 6 in. respectively. Find the speeds of the latter shaft, neglecting the slip of the belt and its thickness.

Ans. 96, 140, and 213 revolutions per minute.*

(2) A lathe mandril has a four-speed step cone of 5 in., $6\frac{1}{2}$ in., 8 in., and $9\frac{1}{2}$ in. diameter mounted on it, and is driven from a similar cone (reversed) on the countershaft which makes 120 revolutions per minute. Find the various speeds of mandril obtained without using the back gear, neglecting slip and thickness of belt.

Ans. 63, $97\frac{1}{2}$, 148, and 228 revolutions per minute.

* *Note.*—All answers are approximate only, obtained by 10-inch slide rule.

Gear Wheels.—The maximum variation of speed obtainable by the use of pulleys is limited by practical considerations. With speed cones, a variation of 4 to 1 cannot be exceeded without the use of pulleys whose relative sizes become so large that the amount of contact of the belt on the small pulley is not sufficient to prevent excessive slipping. Where a larger ratio is required, therefore, we must have recourse to spur or other gearing, the relative velocities of which we will now consider.

If two spur-wheels gear together, as shown by the pitch circles A, B in fig. 105, the relative velocities are inversely as the numbers of teeth in them.

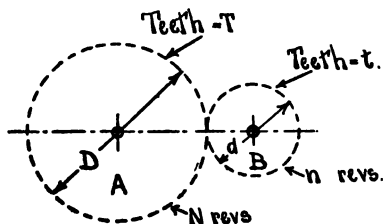


FIG. 105.

If D be the diameter of the pitch circle of A in inches, and

d „ „ „ B „

N be the revolutions per minute made by A, and

n " " " B.

T be the number of teeth in A, and

t " " B

then, as the linear velocities of the pitch circles must be equal,

$$\frac{N}{n} = \frac{t}{T} \quad \text{or} \quad N = \frac{n \times t}{T} \quad \text{or} \quad n = \frac{N \times T}{t}.$$

So that, given the speeds of both shafts and the number of teeth in either wheel, we can find the number of teeth required for the other wheel. Or, given the sizes of the two wheels and the speed of one of the shafts, we can find the speed of the other shaft.

Example 1.—A spur-wheel of 60 teeth on the driving shaft gears with a wheel of 40 teeth on the follower. If the driver makes 50 revolutions per minute, find the speed of the follower.

Here $N = 50$ revs., $T = 60$ teeth, and $t = 40$ teeth.

Therefore $n = \frac{N \times T}{t} = \frac{50 \times 60}{40} = 75$ revolutions per minute.

Example 2.—Two shafts are to be connected by gearing so that the driver shall make 30 revolutions, whilst the follower makes 25. If the wheel on the driving shaft has 50 teeth, find the number of teeth required for the wheel on the follower.

Here $N = 30$ revs., $n = 25$ revs., $T = 50$ teeth.

$$\therefore t = \frac{T \times N}{n} = \frac{50 \times 30}{25} = 60 \text{ teeth. } \text{Ans.}$$

Where the pitch diameters are given instead of the numbers of teeth, the same rules hold good, except that the pitch diameters take the place of the numbers of teeth.

Example 3.—A spur-wheel 2 feet in diameter gears with another of 15 in. diameter. If the first make 30 revolutions per minute, find the speed of the smaller wheel.

Here $D = 2$ ft. = 24 in., $d = 15$ in., $N = 30$.

$$\therefore n = \frac{N \times D}{d} = \frac{30 \times 24}{15} = 48 \text{ revolutions per minute. } \text{Ans.}$$

Sometimes the actual numbers of teeth in the wheels are not known, but the distance apart of the shafts from centre to centre is given, as also the relative speeds of the two shafts. The sizes of the wheels can then be calculated by remembering that the distance apart of the shafts is equal to the sum of the radii of each of the wheels.

Example 4.—Two parallel shafts A and B are about 2 feet apart from centre to centre, and are to be connected by spur gearing so that, whilst A makes 63 revolutions per minute, B shall make 90 revolutions per minute. Find suitable wheels of $1\frac{1}{2}$ in. pitch, and calculate the exact centres of the shafts.

Here $N = 63$, $n = 90$, and $\frac{D+d}{2}$ = about 24 in.

$$\text{But } \frac{D}{d} = \frac{n}{N} = \frac{90}{63}, \quad \therefore d = \frac{D \times 63}{90}.$$

$$\text{Therefore } D \left(1 + \frac{63}{90}\right) = 2 \times 24 = 48 \text{ in.}$$

$$\text{or } D = \frac{48 \times 90}{153} = \frac{480}{17} = 28.2 \text{ in.,}$$

$$\therefore N = \frac{3\frac{1}{2} \times 28.2}{1\frac{1}{2}} = 59.1 \text{ teeth.}$$

$$\text{Say } N = 60 \text{ teeth, then } t = \frac{60 \times 63}{90} = 42 \text{ teeth.}$$

Then
$$\frac{D + d}{2} = \frac{(60 + 42) \times 1\frac{1}{2}}{2 \times \pi} = 24.4 \text{ in.}$$

Ans. 60 teeth and 42 teeth.

Exact centres = 24.4 in.

Trains of Wheels.—Two or more wheels geared up together form a train. The velocity ratio of the train is the speed of the

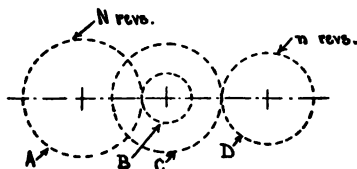


FIG. 106.

last wheel divided by the speed of the first wheel in the train, and is usually represented by e .

Let A, B, C, D, fig. 106, represent a train of four wheels in which the first wheel A drives B, and mounted on the axis of B, and rotating with it, is a third wheel C, gearing into the last follower D. Then if A, B, C, and D respectively represent the numbers of teeth in the four wheels A, B, C, and D, and A makes N revolutions per minute, whilst D makes n revolutions per minute, we have

$$e = \text{velocity ratio of train} = \frac{A}{B} \times \frac{C}{D}$$

and
$$\frac{n}{N} = e = \frac{A \times C}{B \times D} \quad \text{or} \quad n = N \times \frac{A \times C}{B \times D}.$$

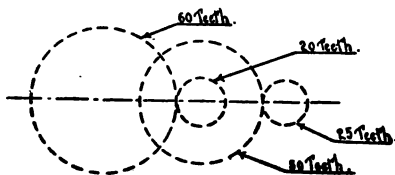


FIG. 107.

Thus the velocity ratio of any train of wheels of this description,

$$e = \frac{\text{product of numbers of teeth in all the drivers}}{\text{product of numbers of teeth in all the followers}}.$$

This rule holds good no matter how many wheels there may be in the train, so long as they are arranged in pairs as shown.

Example 1.—A train of wheels of 60, 20, 50, and 25 teeth are arranged as shown in fig. 107. If A makes 20 revolutions per minute, find e , the value of the train, and the speed of D.

$$\text{Ans. } e = \frac{60 \times 50}{20 \times 25} = 6.$$

Revolutions of D = $20 \times 6 = 120$ revolutions per minute.

The value of the train is unaltered, if the *order of the wheels* be altered, so long as the drivers are still kept as drivers, and the followers as followers. For example, if A gears into D, and C gears into B,

$$e = \frac{A}{D} \times \frac{C}{B} = \frac{60}{25} \times \frac{50}{20} = 6.$$

Also, the *actual numbers of teeth in the wheels may be altered*, and the speed of the last follower remain unaltered, so long as the velocity ratio or value of the train is unaltered.

Thus, if A, B, C, D were wheels of 40, 15, 45, and 20 teeth respectively, the last wheel D would still make the same speed as $e = \frac{A \times C}{B \times D} = \frac{40 \times 45}{15 \times 20} = 6$, as before.

This fact is made use of very extensively for working out the change wheels required for screw-cutting in the lathe, which we will now pass on to consider.

Change Wheels for Screw-cutting.—In arranging change wheels for the lathe, the object sought is to so regulate the speed of the leading screw, as compared with the revolutions of the mandril, that the saddle shall advance along the bed of the lathe a distance equal to the pitch of the screw to be cut, whilst the mandril makes one revolution. For example, to cut a screw of n threads per inch with a leading screw of m threads per inch, the saddle must move a distance of $\frac{1}{n}$ th of an inch whilst the mandril makes one revolution. But for the saddle to move $\frac{1}{n}$ th of an inch, the leading screw must make $\frac{m}{n}$ revolutions.

The velocity ratio or value of the train, therefore, must be $\frac{m}{n}$.

Consider the case of a train to cut out 8 threads per inch, with a leading screw 2 per inch. Whilst the leading screw makes two revolutions, the saddle travels 1 in. But the mandril or job must make eight revolutions whilst the saddle travels 1 in., to cut out a

thread of 8 to the inch. Thus $n=8$, $m=2$, and $e=\frac{m}{n}=\frac{2}{8}$ or $\frac{1}{4}$.

Having fixed the value of the train, to determine the wheels we note that any combination will do, so long as they give the required ratio of $\frac{1}{4}$. Thus a pinion of 20 on the mandril gearing into a wheel of 80 on the leading screw would effect this result. Or a pinion of 25 gearing into a wheel of 100, or a pinion of 30 gearing into a wheel of 120, would do equally well.

The **hand** of the screw to be cut up is determined by the relative directions of rotation of the leading screw and the mandril. If the leading screw be right-handed, and the screw to be cut is also right-handed, then the leading screw must revolve in the *same* direction as the mandril. If the leading screw be right-handed, and the screw to be cut is *left*-handed, then the leading screw must revolve in the *opposite* direction to that

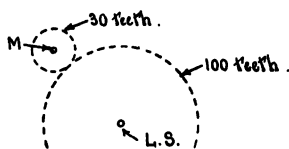


FIG. 108.

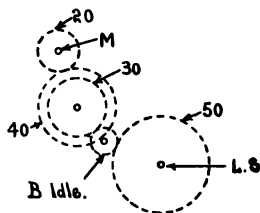


FIG. 109.

of the mandril. With a train of three or four wheels, such as that shown in fig. 110 for the latter, the leading screw revolves in the same direction as the mandril, and the screw cut-up is the same hand as the leading screw. With a train of two or five wheels, such as that shown in figs. 108 and 109, the leading screw revolves in the opposite direction to that of the mandril, and the thread cut up is the opposite hand to that on the leading screw. The small wheel B shown in fig. 109 is an *idle wheel*, as it does not alter the value of the train, but only reverses the direction of rotation.

Example 1.—Given a screw-cutting lathe with a right-handed leading screw of 3 per inch, find the wheels required to cut a left-handed thread of 10 per inch.

Here $m=3$, and $n=10$, $e=\frac{m}{n}=\frac{3}{10}$.

A pinion of 30, therefore, gearing with a wheel of 100, if it can be coupled direct, would do, as the direction of rotation is the reverse of the mandril. Very probably, however, the pinion

would not gear direct into the wheel, owing to the centres of mandril and leading screw being far apart, and a compound train of four wheels, with an idle wheel to reverse the direction, as shown in fig. 109, would be required. To find the sizes for a compound train, proceed thus. Draw a vertical line, and write 3 on one side and 10 on the other, thus:—

Drivers.	Followers.
3	10
30	100
300	1000
600	2000
20×30	40×50

First, multiply each side by 10, producing 30 and 100. This, we have seen, is probably inconvenient. Again multiply each side by any convenient number, say 10, making 300 and 1000. Then see if the products of the numbers of teeth in any two wheels will divide exactly into these numbers. For 1000 we have wheels of 50×20 or 40×25 which would do, but for 300 we have not two wheels small enough. We have therefore to go a step further. Again multiply each side by any number, say 2, making 600 and 2000. Here we see that $600 = 20 \times 30$; and $2000 = 40 \times 50$. The former will therefore do as drivers, with the latter as followers.

They could be arranged as shown in fig. 109, where a pinion of 20 on the mandril gears with a wheel of 40, and a second wheel of 30 on the same stud gears through an idle wheel B, of any diameter, into the wheel of 50 on the leading screw.

Example 2.—Assign suitable numbers to a train of wheels to cut out a right-handed screw of 19 threads per inch with a leading screw $\frac{1}{2}$ -in. pitch.

Here $m = 2$, $n = 19$, $e = \frac{2}{19}$.

Proceeding as explained above:

Drivers.	Followers.
2	19
$\times 10 = 20$	190
$\times 10 = 200$	1900
$\times 5 = 1000$	9500
20×50	95×100

Ans. Drivers = 20 and 50,
Followers = 95 „ 100,
arranged as in fig. 110.

Example 3.—Find the sizes of suitable change wheels for cutting out a left-handed thread $1\frac{1}{8}$ -in. pitch with a leading screw of 4 per inch.

$$\text{Here } m=4, n=\frac{1}{1\frac{1}{8}}=\frac{8}{9}, e=\frac{m}{n}=\frac{4}{\frac{8}{9}}=\frac{36}{8}.$$

Drivers.	Followers.
36	8
$\times 10 = 360$	80
$\times 10 = 3600$	800
$= 40 \times 90$	$= 20 \times 40$

Ans. Drivers = 40 and 90,
Followers = 20 „ 40,
with 1 idle wheel.

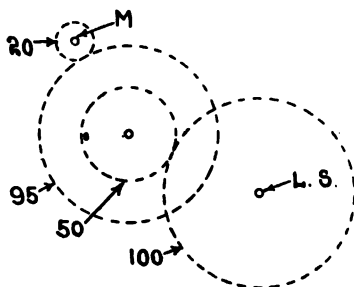


FIG. 110.

Double and Treble Gear for Lathes, etc.—In order to accommodate different sizes of work, lathes, drills, milling machines, etc., must run at widely varying speeds to suit the size of job. For example, a 12-in. lathe may, by means of the gap in the bed, be capable of chucking a job, say, 3 ft. in diameter, and at the same time it should give a speed fast enough for boring a small hole or turning a spindle, say, 1 in. in diameter. If the cutting speed is to be kept the same for these two different sizes, the mandril must be arranged for a total variation in speed of 36 to 1. Cone pulleys are practically unable to give a greater variation than about 6 to 1, so that the remaining speed must be obtained from the back gear, either double or treble. Double-gear machines are those in which a pinion or small wheel fastened to or cast with the speed cone on the machine spindle gears into a larger wheel on an intermediate shaft. The latter has also keyed upon it a second pinion gearing with a large wheel on the mandril.

The back shaft is arranged for throwing in or out of gear as required by means of eccentric bearings. In this way the speed ratio of the cone pulleys can be supplemented or increased by the speed ratio of the gearing. Thus, for example, a lathe with back gear consisting of two pinions, each of 20 teeth, gearing into two wheels, each of 55 teeth, will have a speed ratio due to the gearing alone of $\frac{20}{55} \times \frac{20}{55} = \frac{16}{121}$, or $7\frac{1}{2}$ to 1 approximately. If such a lathe

be fitted with speed cones giving a total variation in speed of 5 to 1, the combined effect of the cones and gearing will be a maximum variation in speed of $7\frac{1}{2} \times 5 = 37\frac{1}{2}$ to 1.

With treble-gearred machines, such as large face lathes, or "break" lathes, as they are termed, specially designed to take wheels, covers, and other work of large diameter, the spur pinion on the intermediate shaft is arranged so that it can either gear direct on to the mandril, or gear into a third wheel or a second intermediate shaft, and back through another pinion on to the wheel keyed to the mandril. Such lathes may have a ratio of, say, 4 to 1 on each gear, making a total variation in speed (with cone pulleys giving, say, 5 to 1) of $4 \times 4 \times 4 \times 5 = 320$ to 1. Thus they are capable of turning the outside of a 40-ft. flywheel with the same cutting speed as they can bore a hole $1\frac{1}{2}$ in. in diameter.

Another consideration in working out speeds for machines is the fact that different materials have to be cut at different speeds according to their hardness. Thus the ordinary *cutting speeds for different metals may be*—

Steel,	20 to 30 feet per minute.
Wrought iron,	25 " " "
Cast iron,	30 to 40 feet per minute.
Gun-metal,	50 feet per minute.
Brass,	60 " "

So that the cutting speed for a job of the same size should be capable of varying from, say, 1 to 3 according to the nature of the metal to be cut.

Example 1.—Arrange suitable back gear for a lathe, to give the same cutting speed on work 30 in. in diameter as on work 1 in. in diameter, assuming a varying cutting speed of from 20 to 50 ft. per minute according to the material, with a speed cone giving a variation of 6 to 1.

Here the total variation of speed required is

$$\frac{30}{1} \times \frac{50}{20}, \text{ or } \frac{1500}{20}, \text{ or } 75 \text{ to } 1.$$

As the cone pulleys give a variation of 6 to 1, the gearing must be arranged to give $\frac{75}{6} = 12\frac{1}{2}$ to 1.

If the two pinions and two wheels are to have the same numbers of teeth respectively, then the ratio of each pair of gears must be

$$\sqrt{12\frac{1}{2}} \text{ to } 1 = (\text{say}) 3.54 \text{ to } 1.$$

By choosing any number t to represent the number of teeth in each pinion, each wheel must have $(3.54 \times t)$ teeth. For practical reasons, it is inadvisable to have pinions of less than 15 or 16 teeth, as they give great unevenness in the motion. The following values of t and T would therefore satisfy our requirement.

$$\begin{array}{ll} t = 15 & T = 53 \\ t = 16 & T = 56 \\ t = 17 & T = 60. \end{array}$$

By determining the pitch of the teeth according to the power required to be transmitted (see Chapter X.), and calculating the sizes of the wheels, a better estimate could be made as to which pair of wheels should be used.

Example 2.—A large face lathe is to be capable of boring a 2-in. hole with the same cutting speed as when turning a wheel 30 ft. in diameter. Find suitable sizes of treble gear, assuming the largest and smallest sizes on the five-step cone pulleys to be each 24 in. and 10 in. diameter respectively. Find also the speed of the countershaft, if the cutting speed for the extreme limits is to be 30 ft. per minute, and find all possible speeds of the mandril in revolutions per minute.

Let N represent maximum speed of mandril in revs. per min.
 n „ minimum „ „ „

Then $\pi \times 30 \text{ ft.} \times n = \text{cutting speed on large diameters.}$
 and $\pi \times \frac{1}{8} \text{ ft.} \times N = \text{„ „ small diameter.}$

But these are both to equal 30 feet per minute.

$$n = \frac{30}{30 \times \pi} = 0.318 \text{ rev. per minute.}$$

$$N = \frac{30}{\frac{1}{8} \times \pi} = \frac{30 \times 6}{\pi} = 57.2 \text{ revs. per minute.}$$

$$e = \text{total variation in speed} = \frac{57.2}{0.318} = 180 \text{ to } 1.$$

The ratio given by the cone pulleys will be

$$\frac{24 \times 24}{10 \times 10} = 5.76,$$

and ratio required by gearing, therefore, $= \frac{180}{5.76} = 31.2$ to 1. If each of the three sets of gear have the same ratio $\left(\frac{T}{t}\right)$ for number of teeth in wheel divided by number of teeth in pinion, then

$$\frac{T}{t} = \sqrt[3]{31.2} = 3.15,$$

or, say,

$$t = 20, \text{ and } T = 63.$$

(It may be desirable to have gears with unequal ratios, depending upon the relative position of the shafts. In that case the total ratio of the three sets of gear should be 31.2. Say the three ratios are $3\frac{1}{2} \times 3\frac{1}{2} \times 2.55$, giving a lower ratio for the last pair.)

Speed of Countershaft.—As the highest speed of the lathe is 57.2 revolutions per minute, and this is obtained by running ungeared, driving from a 24-in. to a 10-in. pulley,

$$\text{Revs. of countershaft} = \frac{57.2 \times 10}{24} = 24 \text{ revs. per minute.}$$

Proof.—Slowest speed $= 24 \times \frac{10}{24} \times \frac{20}{63} \times \frac{20}{63} \times \frac{20}{63} = 0.32$ rev. per minute.

$$\text{Highest speed (ungeared)} = \frac{24 \times 24}{10} = 57.6 \text{ revs. per minute.}$$

$$\text{Cone speeds (ungeared)} = 24 \times \frac{20\frac{1}{2}}{13\frac{1}{2}} = 36.5$$

$$24 \times \frac{17}{17} = 24.0$$

$$24 \times \frac{13\frac{1}{2}}{20\frac{1}{2}} = 15.8$$

$$24 \times \frac{10}{24} = 10.$$

Cone speeds (double gear only) $\left\{ \begin{array}{l} \text{(Above speeds } \div \frac{63}{20} \times \frac{63}{20} = 9.9 \text{ to } 1) \\ 5.8, 3.7, 2.4, 1.6, 1.0 \text{ revs.} \\ \text{per minute.} \end{array} \right.$

Cone speeds (treble gear) $\left\{ \begin{array}{l} \text{Ungeared speeds } \div \frac{63}{20} \times \frac{63}{20} \times \frac{63}{20} = 31.3. \\ 1.8, 1.16, 0.76, 0.55, 0.32 \\ \text{revs. per minute.} \end{array} \right.$

EXERCISES.

1. An engine making 100 revolutions per minute is required to drive a dynamo at 350 revolutions per minute. If the size of pulley on the engine shaft is 7 ft. diameter, find the size of pulley on dynamo shaft (a) assuming no slip of belt, (b) assuming 4 per cent. slip. *Ans.* (a) 2 ft. 0 in., (b) 2 ft. 1 in.

2. A small compound engine, when running at 200 revolutions per minute, is required to drive a dynamo at 500 revolutions per minute. If the engine fly-wheel is 6 ft. 0 in., calculate the diameter of the dynamo pulley, allowing for 5 per cent. slip between the belt and the pulley.

3. Show that the revolutions per minute of a lathe mandril are given by $1.91 \times \text{cutting speed in feet per minute} \div \text{radius in inches of the piece turned}$.

4. Calculate the diameter of the driving pulley for the following planer:—

Speed of line shafting, 120 revolutions per minute; cutting speed of planer, 20 ft. per minute; diameter of pulley on planer, $14\frac{1}{2}$ in.; movements of table of planer on cutting stroke and return for 1 revolution of pulley on planer are $1\frac{1}{2}$ in. and $6\frac{3}{8}$ in. respectively. *Ans.* 19.3 in., say $19\frac{1}{2}$ in.

5. A circular saw 2 ft. in diameter has a cutting speed of 7000 ft. per minute. There is a 5-in. pulley on the saw spindle, driven from a countershaft, which is driven from a line shaft making 120 revolutions per minute. Sketch the arrangement and find suitable diameters for the pulleys, assuming a slip of 5 per cent. in each belt.

6. A radial drill is provided with 10-in. fast-and-loose pulley, and is to run at 120 revolutions per minute. The speed of the

driving shaft is 108 revolutions per minute. Find the size of pulley required to drive the machine, assuming 7 per cent. slip.

Ans. = 12 in.

7. An emery wheel 12 in. diameter is to have a grinding speed of 5000 ft. per minute. It is provided with a 3-in. pulley on the machine spindle, 8-in. fast-and-loose pulleys, and a 12-in. $\times$ 3-in. pulley on the countershaft. Find the size of pulley required on the line shaft, which makes 150 revolutions per minute, assuming 4 per cent. and 8 per cent. slip of belts respectively. *Ans.* = 24 in.

8. Design a suitable compound belt drive for a centrifugal pump which has a 6-in. pulley on its spindle and requires to run at 1500 revolutions per minute, to be driven from a line shaft making 80 revolutions per minute.

9. The leading screw of a lathe has a pitch of $\frac{1}{2}$ in. Sketch an arrangement of wheels for cutting a screw of $\frac{3}{8}$ -in. pitch.

10. Assign suitable wheels for cutting a screw $3\frac{1}{2}$ threads per inch, left hand, with a right-handed leading screw $\frac{1}{4}$ -in. pitch.

11. It is desired to cut a screw of $\frac{5}{8}$ -in. pitch in a lathe with a leading screw of 4 threads per inch. If both screws be right-handed, what wheels would you employ?

12. A lathe has a back gear consisting of two pinions each of 18 teeth gearing into wheels of 40 teeth. The cone pulleys on countershaft and mandril each have 4 steps of 8 in., 10 in., 12 in., and 14 in. diameter. If the countershaft make 80 revolutions per minute, find all the speeds possible of the lathe mandril, neglecting any slip of the belt.

Ans. 140, 96, 67, and 46 revs. per min.

and (with gear) 28.3, 19.4, 13.6, and 9.3 revs. per min.

13. Find suitable numbers of teeth for the back gear of a lathe, consisting of two pinions and two wheels, if the total variation in speed required is 30 to 1 and the cone pulleys give a speed ratio of 4 to 1.

Ans. 15, 16, 17, 18 teeth in pinions.

41, 44, 47, 49 ,, wheels.

14. A geared lathe is to be capable of turning work 3 ft. diameter at 25 ft. per minute, or work 1 in. diameter at 50 ft. per minute. The speed cones give a total variation in speed of 6 to 1. Find suitable numbers of teeth for the pinions (2) and wheels (2) forming the back gear.

Ans. 15 and 52, 17 and 59, 19 and 66.

16 and 55, 18 and 62, 20 and 69.

15. Find suitable numbers of teeth for the wheels of a triple-gear lathe, to give a total variation suitable for turning a maximum of 40 ft. diameter and a minimum of 6-in. bore with a cutting speed of 22 ft. per minute, with speed cones whose smallest and largest diameters are 8 in. and 20 in. respectively.

Note.—Unless otherwise specified, do not use pinions of less than 15 teeth, nor wheels of more than 80 teeth.

CHAPTER VII.

EXAMPLES ON THE PRINCIPLES OF MOMENTS

MANY calculations on various mechanical and other contrivances can be solved by the application of one of two principles or rules well known in applied mechanics, called :

- (1) *The Principle of Moments*, and
- (2) *The Principle of Work* ;

and in this chapter we shall give a few useful examples on the application of the first of these principles to practical problems. Several applications of the principle of work will be dealt with in Chapter IX.

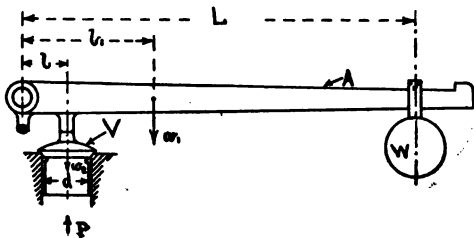


FIG. 111.

Moment of a Force.—The moment of any force with respect to a given point is measured by the product of the intensity of the force multiplied by the perpendicular distance from the point to the line of action of the force. Take, for example, the simple lever-weighted safety valve, as shown diagrammatically in fig. 111, and used on stationary steam boilers to limit the maximum steam pressure. Here the valve V is held down against the elastic force of the steam underneath it by the effect of the weight W suspended from the free end of the lever L. If the moment of the steam pressure on the valve exceed that of the weight, the valve will lift and allow some of the steam to escape ; and conversely, as long as the steam pressure is not sufficient to overcome the moment of

the weight, the valve remains closed and the steam cannot escape. To calculate the weight required for a given size of valve and pressure of steam,

let p = steam pressure in lbs. per square inch above atmosphere.

d = diameter of valve, in inches.

L = length of lever from fulcrum to point of suspension of weight, in inches.

l = length from fulcrum of lever to centre of valve, in inches.

W = weight hung at end of lever, in lbs.

By the principle of moments, if the lever be in equilibrium, then, neglecting the weight of the lever and the weight of the valve,

Moment of steam pressure = Moment of weight,

$$p \times \frac{\pi d^2}{4} \times l = W \times L,$$

$$\text{or} \quad W = \frac{p \times \frac{\pi d^2}{4} \times l}{L}, \quad \dots \dots \dots (1)$$

$$\text{or} \quad L = \frac{p \times \frac{\pi d^2}{4} \times l}{W} \quad \dots \dots \dots (2)$$

So that, given the size of the valve, distance from fulcrum of lever to centre of valve, and steam pressure at which the valve is to lift, we calculate W by (1) or L by (2).

Examples:—

(1) In a lever-weighted safety valve, 4 in. diameter, with lever 28 in. long, and distance from centre of valve to fulcrum $3\frac{1}{2}$ in., calculate the weight required at end of lever so that valve may blow off at 100 lbs. pressure.

Here $L = 28$ in., $l = 3\frac{1}{2}$ in., $d = 4$ in., $p = 100$ lbs.

$$\begin{aligned} \text{By (1)} \quad W &= \frac{100 \times \frac{\pi}{4} \times 4^2 \times 3\frac{1}{2}}{28 \text{ in.}} \\ &= \frac{100 \times 11 \times 16 \times 7}{14 \times 2 \times 28} = \frac{1100}{7} = 157\frac{1}{7} \text{ lbs.} \quad \text{Ans.} \end{aligned}$$

(2) Find the length of lever required for a 3-in. safety valve to blow off at 60 lbs. pressure, if the weight at end of lever be 75 lbs., and the distance from centre of valve to fulcrum be $2\frac{1}{2}$ in.

Here $W = 75$ lbs., $l = 2\frac{1}{2}$ in., $d = 3$ in., $p = 60$ lbs.

$$\begin{aligned}\text{Then, by (2)} \quad L &= \frac{60 \times \frac{\pi}{4} \times 3^2 \times 2\frac{1}{2}}{75} \\ &= \frac{60 \times 11 \times 9 \times 5}{14 \times 2 \times 75} = 99 = 14\frac{1}{7} \text{ in. from fulcrum of} \\ &\quad \text{lever to weight.}\end{aligned}$$

(3) In a $3\frac{1}{2}$ in. diameter safety valve, the length of lever from fulcrum to end is 24 in., the weight is 100 lbs., and distance from fulcrum of lever to centre of valve is 3 in. Find the lowest steam pressure which will open the valve, neglecting the weight of the valve and lever.

$$\text{Here} \quad 100 \times 24 = p \times \frac{\pi}{4} \times (3\frac{1}{2})^2 \times 3,$$

$$\text{or} \quad p = \frac{100 \times 24}{\frac{\pi}{4} \times (3\frac{1}{2})^2 \times 3}.$$

$$p = \frac{100 \times 24 \times 14 \times 2 \times 2}{11 \times 7 \times 7 \times 3} = \frac{6400}{77} = 83.1 \text{ lbs. (approx.).} \quad \text{Ans.}$$

For exact calculation, the weight of the lever, when known, should be taken into account by assuming it to act at the centre of gravity of the lever. The weight of the valve should also be taken into account for accurate calculation. For example, let us find the steam pressure required to open a 3-in. valve, having given weight of lever = 7 lbs., distance of centre of gravity of lever to fulcrum = 8 in., and weight of valve = 4 lbs., distance from fulcrum to centre of valve = $2\frac{3}{4}$ ins., weight at end of lever = 84 lbs., and distance from fulcrum to weight = 2 ft. 0 in.

Here, the moment of the steam is $p \times \frac{\pi}{4} \times 3^2 \times 2\frac{3}{4}$, and the clockwise moments, tending to keep the valve shut, are :

- (1) The weight $W = 84$ (lbs.) $\times 24$ (in.) = 2016 inch-lbs.
- (2) The lever itself = 7 (lbs.) $\times 8$ (in.) = 56 ,,
- (3) The valve itself = 4 (lbs.) $\times 2\frac{3}{4}$ (in.) = 11 ,,

$$\text{Total clockwise moment} = 2083 \quad ,,$$

$$\therefore p \times \frac{\pi}{4} \times 3^2 \times 2\frac{3}{4} = 2083 \text{ inch-lbs.,}$$

$$\text{or} \quad p = \frac{2083 \times 14 \times 4}{11 \times 9 \times 11} = 107.1 \text{ lbs. per sq. in.} \quad \text{Ans.}$$

Symbolically, using the rotation of fig. 111,

$$\left[\left(p \times \frac{\pi d^2}{4} \right) - W_2 \right] \times l = W \times L + w_1 \times l_1.$$

The principle of moments can be applied to find the magnitude of the reactions at the supports in the case of beams supporting one or more loads placed symmetrically or otherwise with respect to the middle. If the beam be loaded with one central load, the reaction at each support is equal to one-half the load plus one-half

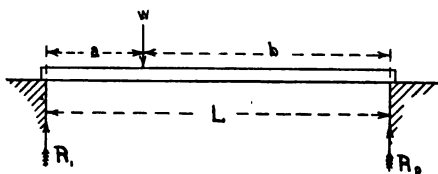


FIG. 112.

the weight of the beam. If, however, the load be away from the centre of the beam, as shown in fig. 112, where R_1 and R_2 are the reactions, we have, by the principle of moments, and taking moments about the right-hand support,

$$R_1 \times L = W \times b \quad \text{or} \quad R_1 = \frac{W \times b}{L};$$

and taking moments about the left-hand support,

$$R_2 \times L = W \times a \quad \text{or} \quad R_2 = \frac{W \times a}{L};$$

or as $R_1 + R_2 = W$, then $R_2 = W - R_1$,

and

$$R_1 = W - R_2.$$

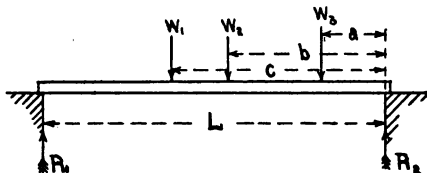


FIG. 113.

For a number of loads, W_1, W_2, W_3 , as shown in fig. 113, taking moments about the right-hand support, we have

Clockwise moment = Anti-clockwise moment,

$$\text{or} \quad R_1 \times L = \begin{cases} W_1 \times c \\ + W_2 \times b \\ + W_3 \times a, \end{cases}$$

$$\text{or} \quad R_1 = \frac{cW_1 + bW_2 + aW_3}{L},$$

$$\text{and} \quad R_2 = (W_1 + W_2 + W_3) - R_1.$$

When the weight of the beam is known, add one-half its weight to each reaction, if it is of uniform section. If the cross section is not uniform, find the centre of gravity of the beam, and take the weight of the beam as another load acting through this centre. The student will notice that the sum of the vertical forces acting upward must be equal to the sum of the vertical forces acting downward, for the beam to be in equilibrium.

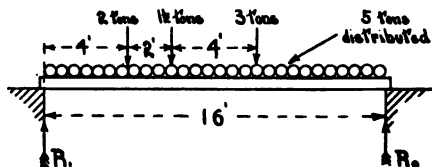


FIG. 114.

Example 1.—A beam of 16 feet span carries the following loads:—

- (1) A distributed load of 5 tons.
- (2) A load of 2 tons acting at 4 feet from the left-hand support.
- (3) A load of $1\frac{1}{2}$ tons acting at 6 feet " " "
- (4) A load of 3 tons acting at 10 feet " " "

Find the reaction at each support. The beam and its loading are represented diagrammatically in fig. 114. Taking moments about the left-hand support, we have,

Clockwise Moments.	Anti-clockwise Moments.
2 (tons) $\times$ 4 (ft.) = 8 ton-feet.	R_2 (tons) $\times$ 16 (ft.).
$1\frac{1}{2}$ " $\times$ 6 " = 9 "	
3 " $\times$ 10 " = 30 "	
5 " $\times$ 8 " = 40 "	
Total = 87 "	

$$\therefore R_2 = \frac{87}{16} = 5.43 \text{ tons.}$$

$$\therefore R_1 = (2 + 1\frac{1}{2} + 3 + 5) - 5.43 = 6.07 \text{ tons.}$$

Ans. Left-hand support = 5.43 tons.

Right-hand „ = 6.07 „

The same principle can be used when the forces are not parallel. Take, for example, the bell-crank lever shown in fig. 115. Here

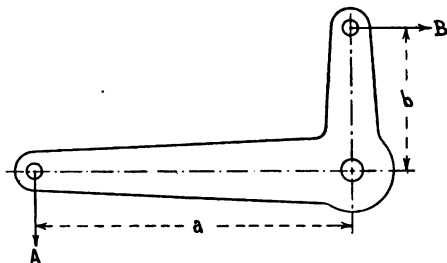


FIG. 115.

the two forces acting at the ends of the bell crank are at right angles to each other, and for equilibrium we have

$$A \times a = B \times b,$$

or

$$A = \frac{B \cdot b}{a}.$$

If the two forces are inclined to each other at any angle, the same relationship holds good, provided the distances a and b are

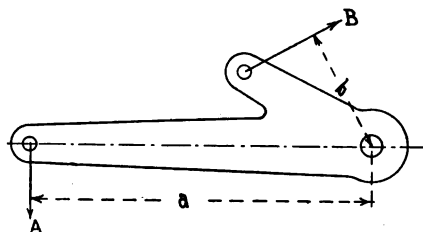


FIG. 116.

measured at right angles to the line of action of the forces (fig. 116).

In the case of the simple derrick crane shown in outline in fig.

117, the pull T on the tie-rod can be found from the simple equation

$$T \times b = W \times a,$$

or

$$T = \frac{W a}{b}.$$

If b is not known, it can be calculated, when l , l_1 , and l_2 are known, from the proportion.

As $l : (l_1 + l_2) :: (l_1 - l_2) : (x - y)$.

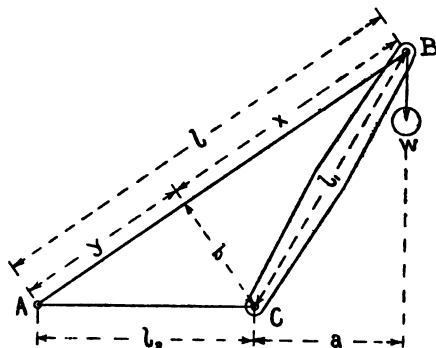


FIG. 117.

Hence $(x - y)$ can be obtained, and $(x + y)$ is already known, being equal to l , so that x and y can be found. The length of b can then be found from

$$l_0^2 = y^2 + b^2 \quad \text{or} \quad b = \sqrt{l_0^2 - y^2}.$$

Or, the length of b can be found by calculating the area of the triangle ABC, knowing the lengths of the three sides ; thus

$$\text{Area} = \sqrt{s(s-l)(s-l_1)(s-l_2)},$$

where s = semi-perimeter ; and as $\text{area} = \frac{l \times b}{2}$,

$$b = \frac{\text{area} \times 2}{l}.$$

To find the balance weight required for an overhung crane,

such as that shown in fig. 118, we have, by taking moments about x ,

$$W \times b = P \times a,$$

or

$$W = \frac{P \times a}{b}.$$

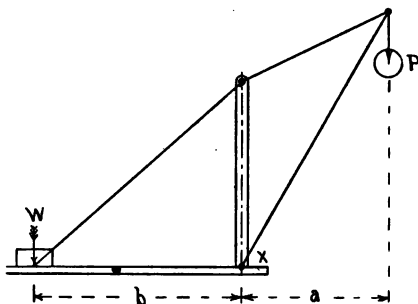


FIG. 118.

EXERCISES.

1. A safety valve is $2\frac{1}{2}$ in. diameter and the lever 18 in. long over all. The distance from fulcrum of lever to centre of valve is 3 in. What weight must be hung at the end of the lever so that steam may blow off at 100 lbs. per sq. in.? *Ans.* = 82 lbs.

2. If the lever in the above example is a uniform bar weighing 10 lbs., and weighs 3 lbs., find the weight required.

Ans. 76.5 lbs.

3. What must be the length of the lever of a safety valve whose area is 10 sq. in., if the weight is 180 lbs., steam pressure 120 lbs. per sq. in., and distance from centre of valve to fulcrum = $3\frac{1}{4}$ in.?

Ans. 23.3 in.

4. Find the blowing-off pressure for a safety valve having given weight = 150 lbs., length of lever = 17.5 in., weight of lever = 20 lbs., distance from centre of gravity of lever to fulcrum = $6\frac{1}{4}$ in., and from fulcrum to valve $3\frac{3}{4}$ in. Diameter of valve = 4 in., and weight of valve = 7 lbs.

5. A wooden beam 15 ft. long and weighing 400 lbs. supports a load of 1 ton at a distance of 5 ft. from one support. Find the pressure on each support.

Ans. 946.6 and 1193.3 lbs.

6. An iron girder 20 ft. span, weighing 40 lbs. per ft. run, carries a distributed load of 1200 lbs., and two concentrated loads of 1000 lbs. each, at 7 ft. and 12 ft. from one support. Calculate the pressure on each support.

7. A wrought-iron tube 4 in. outside diameter, $\frac{1}{4}$ in. thick, and 20 ft. long, is carried on the shoulders of two men 16 ft. apart, each man being 2 ft. from the end. Find the load carried by each man.

Note.—Wrought iron weighs 0.285 lb. per cub. in.

Ans. 105 lbs. each.

8. If one of the men in the above exercise was 1 ft. from one end of the tube, and the other 3 ft. from the other end, find the weight supported by each.

Ans. 88 and 113 lbs.

9. An air pump is worked by a bell-crank lever, whose arms are at right angles, and the forces act at right angles to the arms. The pump rod is worked from the longer end, which is 3 ft. long, and a coupling rod drives the shorter end, which is 2 ft. long, from the tail rod of the engine. If the pull in the pump rod is 6000 lbs., find the pull in the coupling rod.

Ans. 9000 lbs.

10. A derrick crane is arranged as shown in fig. 117, the lengths l , l_1 , and l_2 being 15 ft., 12 ft., and 8 ft. respectively. Find the tension in the tie rod when the crane is lifting 5 tons.

11. A steel-joist girder, whose effective depth is 10 in., carries a load of 5 tons distributed over a span of 12 ft. and a concentrated load of 2 tons in the centre. Find the area of top and bottom flange required, taking $f_t = f_c = 5$ tons per sq. in. Use the formula—

Bending moment at centre = Area $\times f_t$ or $f_c \times D$ where D denotes the depth.

Ans. 3.2 sq. in.

12. Find the maximum concentrated load which a built-up girder can safely carry in the centre, given the following particulars:—Length = 50 ft., effective depth = 30 in., top and bottom plates each 12 in. $\times \frac{7}{8}$ in., and each provided with two angle irons 4 in. $\times$ 4 in. $\times$ 1 in. It already carries a distributed load of 10 tons (including its own weight). Take $f_t = f_c = 6$ tons per sq. in.

Ans. 40.6 tons.

13. Find the stress in the top boom of an open-latticed girder, the effective depth being 7 ft. 6 in., span = 150 ft., distributed load = 100 tons, maximum concentrated load (in centre) = 100 tons, the top boom being 5 plates thick in the centre of the span, and each plate being 2 ft. 0 in. wide $\times$ $1\frac{1}{4}$ in. thick.

Ans. 5 tons per sq. in.

CHAPTER VIII.

WORK, POWER, AND ENERGY.

Work.—When a body, or system of bodies, has been moved by force, mechanical work is said to have been performed. It is measured by the product of the magnitude of the force into the displacement of the centre of mass of the body in the direction in which the force acts.

Thus, work is done when a weight is lifted in opposition to the force of gravity, whether by muscular effort or by some mechanical contrivance. The resistance may be overcome, however, either by animal or machine effort, in other ways than by opposing the force of gravity. In the steam engine, for example, work is done in the engine cylinder by the expansive force of the steam setting in motion the piston and the engine parts against various resistances, including friction. In the crab winch force is exerted to turn the handles, and in this way work is applied to a machine. In an ordinary machine tool—a lathe, for example—work is put into the machine by the difference of the tensions of the two sides of the belt effecting a continual rotary displacement, and useful work is given out by resistance overcome at the point of the tool. We might here note that in any machine the useful work given out is always less than the work put into it or applied to it—the ratio of the former quantity to the latter being known as the mechanical efficiency of the machine. Numerous illustrations might be quoted, but for the present these will be sufficient, as we shall presently deal with calculations from a practical point of view.

The equation of work may be restated as under :—

Work done = force $\times$ displacement (in direction of force).

It is here necessary to define the units in terms of which work may be measured.

The British engineer commonly uses the foot-pound (ft.-lb.) or the foot-ton (ft.-ton), both of these being gravitation units. The foot-pound is the work done when a force equivalent to a 1-pound

weight effects displacement of 1 foot. The foot-ton is 2240 times greater than the foot-pound, and is the measure of the work when the force is equivalent to a ton weight, and the displacement 1 foot. The inch-pound (in.-lb.) is also used.

The **British absolute unit of work**, called the **foot-poundal**, is largely used for academical calculations, and is not used by practical engineers. The foot-poundal is the measure of the work done when the force is 1 poundal and the distance 1 foot. Since the weight of 1 pound is equivalent to g poundals or absolute units of force, it follows that the foot-pound is g times the magnitude of the foot-poundal = 32.2 foot-poundals.

For electrical and scientific work the **C.G.S. (centimetre, gramme, and second)** system of measurement is adopted. In the C.G.S. system the unit of work is the **Erg**. This is an absolute unit of work, and is the work done when the unit of force is 1 dyne, and the displacement in the direction of the force 1 centi-

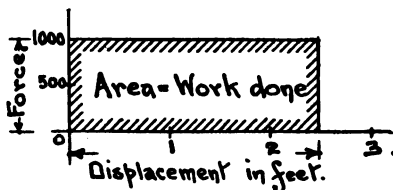


FIG. 119.

metre. Since the erg is an exceedingly small unit of work (the dyne being rather less than the weight of 1 cubic millimetre of water), the **Joule**, which is $10,000,000$ ergs = 10^7 ergs, is taken for practical calculations.

1 joule is equivalent to 0.7373 ft.-lb. (approx.), and 1 ft.-lb. to 1.3562 joules or 1.3562×10^7 ergs. In the C.G.S. system, a gravitation unit of work equal to **centimetre-gramme** is also used, and represents the work done when a force equivalent to the weight of 1 gramme effects a displacement of 1 centimetre. Since the weight of 1 gramme is equivalent to g dynes, it follows that 1 centimetre-gramme unit of work = g ergs = 981 ergs.

Graphic Representation of Work.—The amount of work done can be represented by the area of a diagram, to suitable units. Thus the work done in lifting a body weighing 1000 lbs. against the force of gravity through a vertical height of $2\frac{1}{2}$ feet can be represented by a rectangle, as in fig. 119, the vertical (or ordinate) of which represents to some suitable scale the magnitude of the force exerted, and the horizontal distance (or abscissa) the vertical displacement of the body in feet.

The rectangular diagram represents as many units of area as there are units of work done.

$$\begin{aligned}\text{Area of diagram} &= 1000 \text{ (units of length)} \times 2\frac{1}{2} \text{ (units of length)} \\ &= 2500 \text{ units of area.}\end{aligned}$$

$$\begin{aligned}\text{Work done} &= 1000 \text{ (units of force)} \times 2\frac{1}{2} \text{ (feet)} \\ &= 2500 \text{ ft.-lbs.}\end{aligned}$$

Many examples for calculation dealing with the amount of work done will no doubt readily occur to the student. The **draw-bar pull exerted by a locomotive** in hauling a train is a measure of the force required to overcome the frictional and air resistance to motion, and when the force is expressed in lbs., and multiplied by the mean speed in feet per minute, the product will give the number of ft.-lbs. of mechanical work done per minute by the locomotive. Similarly, a **pump raising water** from a mine or well performs work, the amount of which is equal to the weight of the water lifted, multiplied by the vertical height through which the water has been raised. If **water is pumped into mains against a known resistance or pressure**, the work done can be found by multiplying the weight of the water pumped by the height or head in feet equivalent to the given pressure in the mains (a pressure of 1 lb. per square inch is equivalent to a head of 2.3 feet).

Work done by a Varying Force.—Engineering problems often present themselves for solution, in which the magnitude of the force exerted is not constant, but varies from moment to moment. If the rate of variation in the force be constant, *i.e.* the magnitude of the force either increases or decreases in direct proportion to the displacement, the amount of work done can be calculated by taking the product of the mean force exerted into the displacement. For example, suppose a coil of rope or chain lying on the ground to be picked up by one end, and gradually lifted in a vertical direction until the other end just touches the ground. Then the amount of work done will be the product of the mean force exerted into the height raised, or length of chain at the moment just before commencing to lift the chain. The pull exerted at first is *nil*, and gradually increases until, when the full length of the chain is suspended, it is equal to the weight of the chain. For intermediate stages in the operation the pull is proportional to the actual length suspended. Fig. 120 represents by its area the work done, the base AC representing the total length of chain, and the vertical height the maximum pull or weight of the chain, W.

Work done = mean force $\times$ displacement

$$= \frac{W}{2} \times D.$$

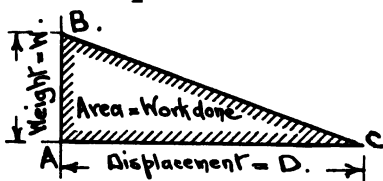


FIG. 120.

If the chain be 100 fathoms long, and weighs 16 lbs. per fathom, the work done will be

$$\begin{aligned} & \frac{1600 + 0}{2} \times 600 \\ &= 800 \times 600 \\ &= 480,000 \text{ ft.-lbs.} \end{aligned}$$

N.B.—A fathom = 6 feet.

A similar calculation is made for finding the work done in raising water from a well, the level of which gets lower and lower as the pumping continues. In this case, the displacement is a variable one, and the force exerted is constant for equal quantities of water.

The calculation of work done in stretching an iron rod within the limits of elasticity by means of a gradually applied load is a further instance of a varying force, the work done being represented by the triangle ABC (fig. 121), the maximum load applied, P , being represented by the base BC , and the amount of stretch by the length l .

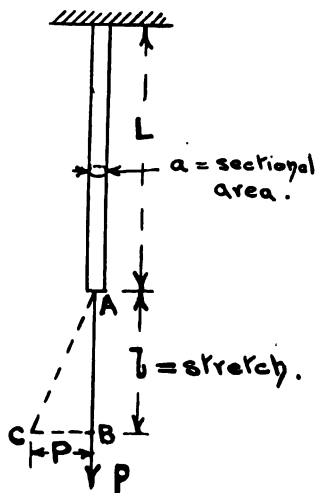


FIG. 121.

Work done = average force $\times$ elongation

$$= \frac{P}{2} \times l.$$

If a represents the cross-sectional area of the rod in square inches, and E the modulus of elasticity of the material in lbs., then the force or stress produced per square inch is $\frac{P}{a}$ lbs.

But $\frac{l}{L}$ is the strain or ratio of the elongation to the original length of the bar, and the ratio of stress to strain is the modulus of elasticity, E .

$$\therefore E = \frac{P}{a} \div \frac{l}{L} = \frac{P \times L}{a \times l},$$

or
$$l = \frac{P \times L}{E \times a}.$$

The work done, therefore, is—

$$\frac{P}{2} \times l \quad \text{or} \quad \frac{P}{2} \times \frac{P \times L}{E \times a} = \frac{1}{2} \frac{P^2 \cdot L}{E \cdot a},$$

and as $P = fa$, where f denotes the stress per square inch.

$$\begin{aligned} \text{Work done} &= \frac{1}{2} a \cdot L \frac{f^2}{E} \\ &= \frac{1}{2} (\text{volume of bar}) \times \frac{f^2}{E}. \end{aligned}$$

If f be the maximum stress at the elastic limit, the work done in stretching the rod is called the Resilience of the bar.

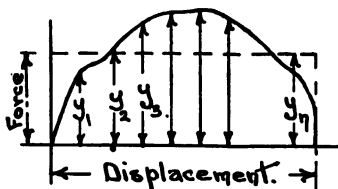


FIG. 122.

If the magnitude of force varies from moment to moment as represented in fig. 122 by the ordinates Y_1, Y_2, Y_3 , etc., to find the area, divide the diagram into any number of convenient parts, and measure the mean height of each, as Y_1, Y_2 , etc. By adding together these heights, and dividing by their number, we obtain the height of the mean ordinate (Y_n), representing the magnitude of the force. This, multiplied by distance or displacement, will give the amount of work done.

Power.—Power is the rate of doing work. In calculating in the previous exercises the amount of work done, no mention whatever was made of the time taken to do the work. Accepting the above definition for power, it is evident that the actual power developed by any prime mover or machine must include the time element.

The unit of power adopted in this country is the horse-power, and is equivalent to work at the rate of 33,000 ft.-lbs. per minute, or 550 ft.-lbs. per second. This quantity was originally introduced by Watt, and he estimated the performance of a good horse of average strength at 22,000 ft.-lbs. per minute, allowing 50 per cent. over, in order that the steam engines which were then supplanting horse-work should have a little margin. The unit of power is now fixed at 33,000 ft.-lbs. per minute, without any idea of comparison with horse-work, although the name horse-power (written H.P. for short) is still retained.

The H.P. of any prime mover, or machine, therefore, is given by the expression

$$\text{H.P.} = \text{Work done per minute in ft.-lbs.} \div (33,000),$$

$$\text{or } \text{H.P.} = \text{Work done per second in ft.-lbs.} \div (550).$$

In electric current working the unit of power is called the Watt, and a current has this unit power when doing work at the rate of 1 joule or 10^7 ergs per second. One watt is equivalent to 0.7373 ft.-lbs. per second, and 1 ft.-lb. per second is equivalent to 1.3562 watts. A larger unit of power is used for electrical work, and is known as the Board of Trade unit, and called the kilowatt hour.¹

A current has this unit power when working for an hour at the rate of 100 watts per second.

One horse-power is equivalent to $\frac{550}{0.7373} = 746$ watts, so that

1 kilowatt, or 1000 watts, is equivalent to $\frac{1000}{746} = 1.343$ H.P. (approximately $1\frac{1}{3}$), and 1 H.P. = 0.746 kilowatt.

Engineers frequently speak of horse-power hours, by which is meant the product of the rate of working in H.P. (i.e. for one minute) into the time in hours. Similarly, kilowatt hours is the rate of working in kilowatts (per sec.) $\times$ time in hours.

As the methods of finding the horse-power of an engine or other prime movers, and the calculations for the determination of the dimensions of the cylinder of an engine to develop a given horse-power, are explained in Chapter XV., we shall here consider a few

of the more elementary and theoretical results leading up to this chapter. In the heat engine, whether steam, gas, or oil, mechanical work is obtained from the heat energy of the working substance, giving a pressure, and effecting a mass displacement (either of piston or turbine rotors) by means of which a shaft is rotated, and useful work done.

Work and Power.—First let us consider the simple case in which steam is admitted into a cylinder and maintained at constant pressure throughout the stroke.

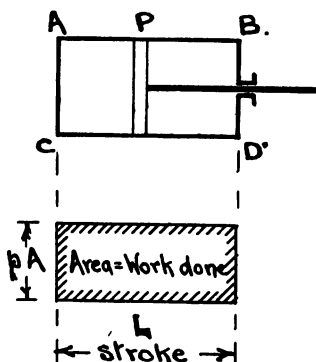


FIG. 123.

Let ABCD (fig. 123) represent a cylinder, and P a movable disc or piston sliding inside it. If A be the area of the piston in square inches, and p the pressure of the steam in lbs. per square inch above the atmosphere, then the total pressure on one side (say the left-hand side) of the piston will be $(p \times A)$ lbs. If the steam pressure forces the piston from one end of the cylinder to the other, traversing a distance equal to L feet, then the work done can be represented by the rectangle (fig. 123), where the

ordinate of force is $(p \times A)$ lbs., and the displacement L feet, and

$$\text{Work done} = p \times A \times L \text{ (ft.-lbs.)} = 144 \text{ pv ft.-lbs.,}$$

where v = volume swept out by piston in cubic feet.

Now, considering the other side of the piston on the return stroke, the steam can only act upon the area of the face of the piston, i.e. the area A square inches minus the area (a) of the cross section of the piston rod. The net area is therefore $(A - a)$ square inches, and the force producing motion $p(A - a)$ lbs. The work done on the return stroke is therefore $p(A - a) \times L$ (ft.-lbs.), or the total work done during a double stroke or one revolution

$$= pAL + p(A - a)L = p \times 2L \left(A - \frac{a}{2} \right).$$

Example.—Let the cylinder be 12 in. in diameter, the piston rod 2 in. in diameter, and let $p = 60$ lbs. and $L = 1\frac{1}{2}$ ft. Then the work done during the forward stroke (i.e. at the “head” end of the cylinder) will be

$$60 \times \frac{\pi}{4} \times 12^2 \times 1.5 = 60 \times 113 \times 1.5 = 10170 \text{ ft.-lbs.,}$$

and the work done during the return stroke (*i.e.* at the crank end of the cylinder) will be

$$60 \times \frac{\pi}{4} (12^2 - 2^2) \times 1.5 = 60 \times 110 \times 1.5 = 9900 \text{ ft.-lbs.}$$

The total work done in 1 revolution, therefore, is :

$$10170 + 9900 = 20070 \text{ ft.-lbs.}$$

If the engine makes 100 revolutions per minute, the work done per minute will be

$$20070 \times 100 = 2007000 \text{ ft.-lbs.,}$$

and the
$$\text{H.P.} = \frac{2007000}{33000} = 60.6 \text{ H.P. (approx.).}$$

Now consider the method of calculating the work done by gas in expanding according to some definite law, such as that known as Boyle's Law, in which the pressure during expansion is inversely proportional to the volume, the temperature of the working substance being kept constant. Such an expansion is called an **Isothermal expansion**.

Symbolically,

$$p \times v = \text{a constant quantity.}$$

Assume a cylinder whose cross-sectional area is 1 square foot to contain v_1 cubic feet of a perfect gas (see fig. 124). If the gas expands in doing work on the piston from a volume v_1 to a volume v_2 cubic feet, the pressure will fall as represented by a curve (see fig. 124) such that at any moment its value is inversely proportional to the volume occupied by gas. The work done during expansion, therefore, will be represented by the shaded area in the diagram; and it can be proved (see p. 44) that this

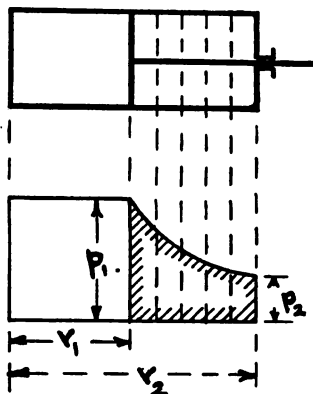


FIG. 124.

$$\text{Work done} = p_1 v_1 \log_e \frac{v_2}{v_1} = p_1 v_1 \log_e r,$$

where r is the ratio $\frac{v_2}{v_1}$, or **ratio of expansion**, as it is termed. It should be noted that the logarithms here referred to are the Napierian or hyperbolic logarithms, and are equal to the common

log multiplied by 2.3026. (A table of Naperian logs is given on p. 85.) There are two methods of finding points on the curve representing the varying pressures—(a) the geometrical method, and (b) the mathematical. First taking the former method, draw two lines OX, OY (fig. 125) at right angles to each other, and plot the initial pressure and volume p_1 and v_1 , forming the rectangle OABC. Take a distance OD on OX such that $OD = v_2$, and erect the perpendicular DF meeting AB produced in F. Between B and F take any number of points, and join to O, and drop perpendiculars to OX. From the intersection of these radial lines

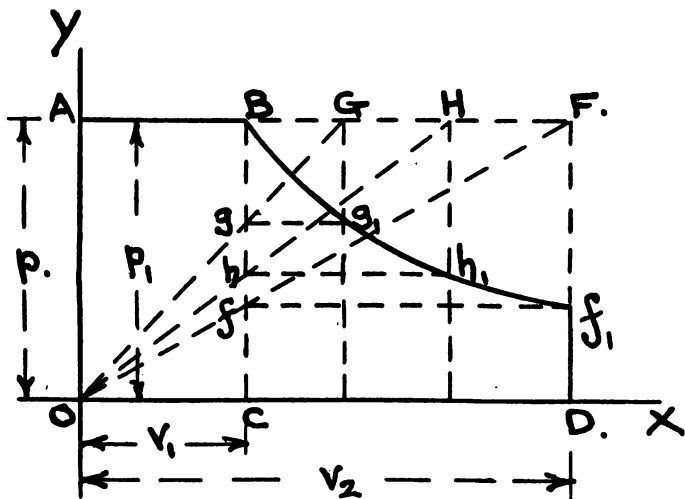


FIG. 125.

with BC, viz. from g , h , and f , draw gg_1 , hh_1 , and ff_1 parallel to OX, to meet the perpendiculars from G, H, and F respectively. These points g_1 , h_1 , and f_1 will be points on the required curve, which is a rectangular hyperbola (see p. 62); the curve can then be drawn in. Next, to calculate the position of the points, find the product of the initial pressure p_1 and volume v_1 , and call this K. The value of any intermediate pressure can then be found by dividing K by the volume for that pressure. Take, for example, the case of a cylinder containing 5 cubic feet of gas at 150 lbs. pressure expanding at constant temperature to 10 cubic feet.

$$\text{Here } p_1 v_1 = k \text{ (a constant)} = 150 \times 5 = 750.$$

$$p_2 \times 10 = 750 \quad \text{or} \quad p_2 = \frac{750}{10} = 75 \text{ lbs.}$$

Let p_6 = pressure when volume = 6 cubic ft.

Then $p_6 \times 6 = 750$ or $p_6 = 125$ lbs.

Similarly, $p_7 = \frac{750}{7} = 107.14$ lbs. for 7 cubic ft.

$$p_8 = \frac{750}{8} = 93.75 \text{ ,, for 8 ,,}$$

and $p_9 = \frac{750}{9} = 83.33 \text{ ,, for 9 ,,}$

By plotting these results to a suitable scale, and drawing the curve through the points so obtained, the mean pressure and the work done during the expansion can be obtained, as explained in Chapter XV., or it may be calculated by the formula

$$\text{Work done} = p_1 v_1 \times \log_e \frac{v_2}{v_1}, \text{ already given.}^*$$

$$= 150 \times 5 \times 0.6931 = 520 \text{ ft.-lbs. (approx.).}$$

Work done during Adiabatic Expansion.—The relationship between the pressure and volume for the adiabatic expansion of a gas, steam or other gas, can be represented by an empirical formula or equation to the expansion curve of the form

$$pv^n = \text{a constant,}$$

where n can be determined; then the work done during the expansion can be found from the formula (see p. 44)

$$\text{Work done} = \frac{p_1 v_1 - p_2 v_2}{n - 1}.$$

The method of finding the pressure ordinates for the curve for a given exponent n is given in Chapter IV.

Example.—Consider, for example, the work done during the expansion of 5 cubic feet of dry saturated steam at a pressure of 150 lbs. absolute to a volume of 10 cubic feet, according to the law $pv^n = \text{a constant}$, where $n = 1.0646$, its value for dry saturated steam. Let p_1, v_1 be the pressure and volume at the commencement of the expansion, and p_2, v_2 similar values at the end.

* *Note.*—The formula gives the work done per square inch of piston, per square foot of piston, or per full area of piston, according as p_1 is expressed in lbs. per square inch, lbs. per square foot, or total pressure on piston respectively.

Then as $pv^n = \text{a constant,}$

$$p_1 v_1^{1.0646} = p_2 v_2^{1.0646},$$

or $150 \times 5^{1.0646} = p_2 \times 10^{1.0646},$

or $p_2 = 71.7 \text{ lbs. (approx.)}$

The work done during expansion is found by direct substitution in above formula.

$$\text{Thus, Work} = \frac{150 \times 5 - 71.7 \times 10}{1.0646 - 1} = \frac{750 - 717}{.0646} = 510 \text{ ft.-lbs.}$$

And dividing this by the displacement $v_2 - v_1$, we obtain the pressure during the expansion as 102 lbs. per square inch. With adiabatic expansion, the work done by the same quantity of steam (initially dry) expanding to the same volume, and taking n at 1.135, would be:

$$p_2 = \frac{150}{2^{1.135}} = 68.3 \text{ lbs. per square inch (approx.)}$$

$$\text{Work done} = \frac{750 - 683}{0.135} = 496 \text{ ft.-lbs. (approx.)}$$

which corresponds with a mean pressure during the expansion only of 99.2 lbs. per square inch. These three cases show the slight difference in work areas obtained by

- (a) The isothermal expansion of steam,
- (b) The expansion of dry saturated steam,
- (c) Adiabatic expansion,

for 5 cubic feet of steam (initially dry), at a pressure of 150 lbs. per square inch, expanding to 10 cubic feet.

Mean Pressure during Expansive Working.*—In the case of the steam engine, where steam is admitted at boiler pressure on the piston, the volumes occupied by the steam can be represented by a diagram, such as that shown in fig. 126, consisting of two distinct parts, one at constant pressure during the admission period, the other at varying pressure during the expansion.

The total work done per stroke is obviously the work done during admission + the work done during expansion, and is represented by the areas shaded in the figure.

* (Assuming hyperbolic expansion.)

Work done during admission = area of rectangle ABCD = $p_1 v_1$.

„ „ expansion = „ figure CEFD = $p_1 v_1 \log_e r$.

$$\therefore \text{Total work done per stroke} = p_1 v_1 + p_1 v_1 \log_e r, \quad \text{or } p_1 v_1 (1 + \log_e r).$$

Let p_m represent the mean or average pressure throughout the stroke, and representing the height of a rectangle AGHF, whose area is equal to the area of the combined areas ABCD and DCEF. Then,

$$p_m \times v_2 = p_1 v_1 (1 + \log_e r),$$

or

$$p_m = \frac{p_1 v_1 (1 + \log_e r)}{v_2}$$

$$= p_1 \frac{1 + \log_e r}{r}.$$

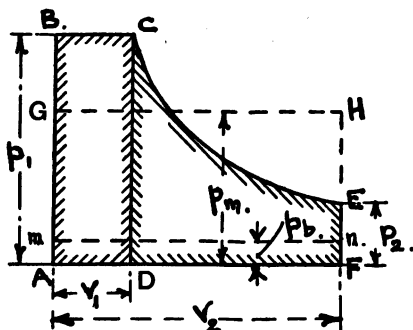


FIG. 126.

This is the general expression used, with a modification explained in Chapter XV., for obtaining the mean pressure of an expansive engine, assuming isothermal expansion. In practice, the actual mean pressure obtained would be less than that given in the above expression, owing to three causes:—

(a) The expansion is not isothermal—the index n having a greater value than unity.

(b) The back pressure on the other side of the piston requires the expenditure of work in overcoming it.

(c) By reason of throttling or wire-drawing during admission and release, the corners of the diagram are rounded instead of sharp as shown.

If p_b be the back pressure in lbs. per square inch absolute

(from 3 to 5 lbs. for condensing engines, and 16 to 18 lbs. for non-condensing engines), mean pressure obtained, called the **mean effective pressure**, is found as follows:—

$$\frac{\left\{ \begin{array}{c} \text{Area of figure} \\ \text{ABCD} \end{array} \right\} + \left\{ \begin{array}{c} \text{Area of figure} \\ \text{CEFD} \end{array} \right\} - \left\{ \begin{array}{c} \text{Area of figure} \\ mnFA \end{array} \right\}}{\text{Final volume } v_2}$$

$$= \frac{p_1 v_1 + p_1 v_1 \log_e \frac{v_2}{v_1} - p_b v_2}{v_2} ;$$

and since $r = \frac{v_2}{v_1}$

$$\therefore v_2 = v_1 r.$$

$$p_m = \frac{p_1 v_1 + p_1 v_1 \log_e r - p_b v_1 r}{v_1 r}$$

$$= \left(\frac{p_1}{r} + \frac{p_1}{r} \log_e r - p_b \right)$$

$$= \left(p_1 \frac{1 + \log_e r}{r} - p_b \right).$$

Exercises on the Calculation of Power.—We will consider the following few typical cases.

Example 1.—A steam engine having a single cylinder 10-in. diameter, 12-in. stroke, is double-acting, and using steam at 60 lbs. gauge pressure, cutting off at $\frac{1}{4}$ stroke, with 3 lbs. back pressure. Neglecting the effects of clearance and wire-drawing, and assuming isothermal expansion ($pv = \text{a constant}$), find

- (a) The work done per stroke during admission ;
- (b) " " " " expansion ;
- (c) The total work done per minute at 160 revolutions per minute ;
- (d) The H.P. of the engine.

(a) The work done during admission will be :

$$\text{Effective pressure} \times \text{area} \times \text{displacement} = (60 + 15 - 3) \times 78.5 \times \frac{1}{4}$$

$$= 1413 \text{ ft. lbs.}$$

(b) The work done during expansion will be :

Positive work – negative work.

The positive work = $144 p_1 v_1 \log_e r$.

The negative work = back pressure on piston $\times$ distance from cut-off to end of stroke.

Now distance from cut-off to end of stroke = $L - l$, where L denotes length of stroke, and l the distance from commencement of stroke to cut-off.

But $\frac{L}{l} = r$ (the expansion ratio).

Hence $l = \frac{L}{r}$, so that $L - l = L - \frac{L}{r} = L\left(1 - \frac{1}{r}\right)$;

so that Negative work = $p_3 \times \frac{\pi}{4} d^2 \times L\left(1 - \frac{1}{r}\right)$;

hence work done during expansion

$$\begin{aligned} &= 144 p_1 v_1 \log_e r - \left\{ p_3 \frac{\pi}{4} d^2 \times L\left(1 - \frac{1}{r}\right) \right\} \\ &= (144 \times 75 \times \frac{78.5}{144} \times \frac{1}{4} \times 1.386) - 3 \times 78.5 \times \frac{3}{4} \\ &= 2040 - 176 = 1864 \text{ ft.-lbs.} \end{aligned}$$

(c) Total work done per minute

$$= (1413 + 1864) \times 2 \times 160 = 1,049,000 \text{ ft.-lbs. (approx.).}$$

(d) The H.P. will be $\frac{1049000}{33000} = 31.8$.

Example 2.—Find the corrected power of the above engine by taking into account the piston rod, $1\frac{1}{4}$ -in. diameter, on one side of the piston only.

$$\text{As } 10^2 : 10^2 - \frac{(1\frac{1}{4})^2}{2} :: 31.8 : x.$$

$$x = \frac{31.8 \times 99.22}{100} = 31.5 \text{ H.P.}$$

Example 3.—Find the power absorbed by the friction of a slide valve, 8 in. $\times$ 12 in. area, pressure 80 lbs. per sq. in., travel 3 in., coefficient of friction 0.1 at 150 revolutions per minute.

$$\begin{aligned} \text{Power absorbed} &= \frac{\text{Force overcome} \times \text{displacement}}{33000} \\ &= \frac{(80 \times 8 \times 12 \times 0.1) \text{ lbs.} \times \frac{1}{4} \times 2 \times 150}{33000} \\ &= 1.74 \text{ H.P.} \end{aligned}$$

Example 4.—Find the power required to raise 1500 gallons of water 80 ft. high in $1\frac{1}{2}$ hours, assuming the combined efficiency of the pump to be 0.5.

$$\begin{aligned}\text{Water H.P.} &= \frac{\text{Weight lifted per minute} \times \text{displacement}}{33000} \\ &= \frac{1500 \times 10}{90} \times 80 \\ &= \frac{33000}{33000} = 0.404 \text{ H.P.}\end{aligned}$$

$$\text{H.P.} = \frac{\text{Water}}{\text{Efficiency}} = \frac{0.404}{0.5} = 0.81 \text{ H.P. (approx.)}$$

Example 5.—Find the power required to pump 500 gallons of water per minute into hydraulic mains at 750 lbs. pressure.

$$\begin{aligned}\text{Water} &= \frac{\text{Weight per minute} \times \text{equivalent displacement}}{33000} \\ &= \frac{5000 \times 750 \times 2.3}{33000} = 261 \text{ H.P.}\end{aligned}$$

Note.—A number of practical examples on Power will be found in Chapter XV.

Example 6.—At the waterworks for supplying a town, $7\frac{1}{2}$ million gallons of water are raised daily to a height of 120 ft. What rate of working is this in horse-power?

$$\begin{aligned}\text{H.P.} &= \frac{\text{Work done per minute}}{33000} \\ &= \frac{(7500000 \times 10) \text{ lbs.} \times 120 \text{ ft.}}{24 \times 60 \times 33000} = 190 \text{ H.P. (approx.)}\end{aligned}$$

Example 7.—A steam crane is observed to lift some machinery weighing 15 tons at the rate of 5 ft. per minute. What is the useful horse-power of the crane during this operation?

$$\text{H.P.} = \frac{15 \times 2240 \times 5}{33000} = 5 \text{ H.P.}$$

Example 8.—A locomotive exerts a pull of 3000 lbs., and draws a train at a speed of 20 miles an hour. Find the H.P.

(*Note.*—One mile per hour = 88 feet per minute.)

$$\text{H.P.} = \frac{3000 \times 20 \times 88}{33000} = 160 \text{ H.P.}$$

Example 9.—A steam crane lifts a weight of 1000 lbs. through 30 ft. in 40 seconds. Assuming that 45 per cent. of the engine-power is absorbed in friction, calculate the actual power of the engine.

$$\begin{aligned}\text{Useful H.P.} &= \frac{\text{Weight lifted} \times \text{displacement (in ft. per minute)}}{33000} \\ &= \frac{1000 \times 30 \times 60}{40 \times 33000} = 1.36 \text{ H.P.}\end{aligned}$$

$$\text{Engine H.P.} = \frac{\text{Useful power}}{\text{Efficiency}} = \frac{1.36}{0.55} = 2.48 \text{ H.P.}$$

Example 10.—A ship is driven at a speed of 10 knots by engines developing 10,000 I.H.P. Assuming that 60 per cent. of this power is used for propelling purposes, calculate the resistance offered by the water to the motion of the ship.

(Note.—1 knot = 6080 feet per hour).

$$\begin{aligned}\text{Useful H.P.} &= \text{I.H.P.} \times \text{efficiency.} \\ &= 10000 \times 0.6 = 6000.\end{aligned}$$

Let R = resistance to motion in lbs.

$$\text{Then} \quad R \times \frac{6080}{60} \times 10 = 6000 \times 33000,$$

$$\text{or} \quad R = 196,000 \text{ lbs. (approx.).}$$

Example 11.—A belt transmits power from one shaft to another, and has a velocity of 20 ft. per second. If the effective tension (or difference of tensions) be 350 lbs., calculate the H.P. transmitted.

$$\begin{aligned}\text{H.P.} &= \frac{\text{Difference in tensions (lbs.)} \times \text{speed (ft. per minute)}}{33000} \\ &= \frac{350 \times 20 \times 60}{33000} = 12.7 \text{ H.P.}\end{aligned}$$

Energy.—The energy possessed by a body is its capacity for doing work, and is measured in ordinary work units. Energy may be either potential or kinetic.

Potential Energy is energy due to the position or state of the body. For example, a coiled spiral spring has stored up in it a certain amount of potential energy which it is capable of giving out on being released. Similarly, water stored in a tank at the top of a house or on the roof of a factory has potential energy which can be converted into actual work by conducting it to a

water-wheel or turbine. In each case the amount of potential energy is always equal to the amount of work expended in placing the body in such a state or position. In the case of **water at an elevation**, the useful potential energy is calculated by taking the product of the weight of the water into the available height or fall.

Kinetic Energy is the energy possessed by a moving mass in virtue of its velocity. A body in motion is capable of doing a definite amount of work before it is brought to rest—that is, it can overcome some force or resistance through a definite distance. The connection between the two forms of energy and the amounts of work stored up may be made clear by a simple illustration. If a ball weighing 2 lbs. be lifted to an elevation of, say, 50 feet, the work done against the force of gravity will be

$$2 \times 50 = 100 \text{ ft.-lbs.}$$

It will therefore possess 100 ft.-lbs. of potential energy with respect to its lower level. If now it be allowed to fall, it will accumulate an increasing amount of kinetic energy, so that by the time it reaches the ground, and at the moment of striking the ground, its kinetic energy will be exactly the same as its potential energy was at the elevation of 50 feet, *i.e.* 100 ft.-lbs. We have therefore this useful connecting-link between potential and kinetic energy—that the work stored up (as potential energy) in a body, by reason of its elevation above any normal level, is W , its weight in lbs., multiplied by H , its available height in feet. Also the work stored up in a moving body (as kinetic energy) moving at a velocity of v feet per second is given by the expression

$$\frac{W \times v^2}{2g},$$

where g is the acceleration due to the force of gravity, or 32.2 feet per second per second. It is proved in elementary works on mechanics that the velocity acquired by a body falling freely under the influence of the force of gravity, where H is the height of the fall in feet and g the acceleration, is given by

$$v^2 = 2gH; \quad \therefore H = \frac{v^2}{2g},$$

and

$$WH = \frac{Wv^2}{2g}.$$

Let us now consider a few practical examples of work done at the expense of stored-up energy.

The hand fly-press for punching or stamping sheet-metal consists of a central screw passing through the nut formed in the bridge of the framework, and carrying two long arms, each with a heavy ball attached to the end. By setting the balls in motion or by giving them a spin about the axis of the screw, kinetic energy is imparted to the punching die, sufficient to shear out or stamp the metal to the desired shape or size. Thus, if W be the weight of the two balls, and v their velocity in feet per second at the moment the punch strikes the plate, the stored-up energy will be $\frac{Wv^2}{2g}$ ft.-lbs.; and if there were no friction between the screw and the nut, this amount of work would be given out in overcoming the resistance of the material to shearing or stamping. Let t = thickness of plate in inches when the plate is sheared through, and R the average force or resistance to shearing.

$$\text{Then} \quad \frac{Wv^2}{2g} = R \times \frac{t}{12}.$$

Next, taking friction into account, and denoting the efficiency of the screw by η , then

$$\text{Work utilised in punching} = \frac{Wv^2}{2g} \eta,$$

$$\text{and this must} = R \frac{t}{12};$$

$$\begin{aligned} \therefore W &= \frac{2}{12} \frac{g}{\eta} \frac{tR}{v^2} \\ &= \frac{1}{6} \frac{g}{\eta} \frac{tR}{v^2}, \end{aligned}$$

so that the weights can be calculated for a given energy of stamping.

The pile driver is an apparatus for driving stakes or piles whereon to rest heavy foundations, and consists of a vertical timber framework forming guides for a tup or monkey, which is raised by hand or steam power, and is released after reaching some elevation, and allowed to fall on to the top of the pile. If W be the weight of the tup in lbs., and H the fall in feet, then each blow gives out WH ft.-lbs. of kinetic energy, which is absorbed in driving the pile through a certain distance— x inches. The average force of the resistance (F) in lbs. therefore will be

$$F = \frac{WH}{\frac{x}{12}}.$$

This calculation assumes that the whole of the stored energy is expended in overcoming the resistance. As a matter of fact, the resistance increases as the pile is driven further and further in, and more and more work is lost in damaging the head of the pile and the striking face of the driver.

The calculations for the **drop-hammer**, used by smiths for drop-forging, are similar to the above.

The ordinary engine **flywheel** is another instance of energy being alternately stored up and given out as required. In all reciprocating engines the turning moment on the crank-shaft varies very considerably with each cycle, the amount of variation depending upon the number of cylinders, position of cranks, ratio of expansion, etc. The inequality in the turning moment on the shaft would produce an inconvenient variation in speed and a jerkiness in running, if it were not for the addition of a heavy wheel to store up the excess energy when the turning moment was greater than the resisting moment, and give it out when the turning moment was less than the resisting moment. Now, the flywheel is made of sufficient weight to prevent variation of speed from this cause from exceeding a certain limit which depends on the type of engine and the purpose for which the engine is required. When the driving torque is greater than the resisting torque, the speed of the engine increases above the normal speed; but by making the wheel heavy enough the excess energy is absorbed by the wheel, with only a small increase in velocity; and, on the other hand, when the driving torque is less than the resisting torque, the flywheel does part of the driving, giving out the excess energy previously stored up, with only a small fall in speed below the normal. Here it is useful to remember that the energy absorbed or given out by the wheel for a given speed fluctuation is directly proportional to the weight of the wheel.

Let W represent the weight of the wheel rim in lbs., v its mean circumferential speed in feet per second, when rotating at its normal speed of, say, N revolutions per minute; and let v_0 and v_1 represent the minimum and maximum circumferential velocities with N_0 and N_1 , the corresponding rotational speeds.

The fluctuation in the number of revolutions, or $N_1 - N_0$, corresponds to the fluctuation in the circumferential velocity, or $v_1 - v_0$; and N_1 is as much above the normal speed N as N_0 is below it. Consequently, v_1 will be as much above the normal velocity v as v_0 is below it, so that

$$\frac{v_1 + v_0}{2} = v \quad \text{or} \quad \frac{N_1 + N_0}{2} = N;$$

$$\therefore v_1 + v_0 = 2v \quad \text{and} \quad N_1 + N_0 = 2N.$$

Now $\frac{Wv^2}{2g}$ gives the kinetic energy stored up in the wheel at its normal speed ;

$\frac{Wv_0^2}{2g}$ its kinetic energy at the lowest speed ;

and $\frac{Wv_1^2}{2g}$ kinetic energy at top speed.

Now, the greatest amount of fluctuation in energy which the wheel is capable of giving out or taking in, without altering the engine speed beyond the limits N_0 and N_1 , or v_0 and v_1 , will be given by

$$Q = \frac{Wv_1^2}{2g} - \frac{Wv_0^2}{2g} = \frac{W}{2g} (v_1^2 - v_0^2) = \frac{W}{2g} (v_1 - v_0)(v_1 + v_0).$$

The speed fluctuation allowed may be expressed as a ratio of the normal speed, thus :

$$\frac{N_1 - N_0}{N} \quad \text{or} \quad \frac{v_1 - v_0}{v} = k \text{ (say),}$$

so that

$$v_1 - v_0 = kv,$$

or

$$N_1 - N_0 = kN.$$

Taking the equation for fluctuation of energy just given, by substituting $v_1 - v_0 = kv$ and $v_1 + v_0 = 2v$, we obtain

$$Q = \frac{W}{2g} \times kv \times 2v = k \frac{W}{g} v^2 ;$$

but

$$v = \frac{2\pi RN}{60} = \frac{\pi RN}{30} ;$$

hence

$$Q = \frac{\pi^2}{(30)^2 g} \times k \times R^2 N^2 W.$$

Now, let q denote the ratio that the excess or deficient energy developed per stroke due to inequality of turning moment is to the work done per stroke on the crank-pin (or pins).

Now, since the expression

$$\frac{33000 \times \text{I.H.P.}}{2N}$$

gives the work per stroke, the fluctuation of energy on each side of the mean speed N is given by

$$\frac{33000q \times \text{I.H.P.}}{2N} ;$$

and this result must be equal to half the total fluctuation Q .

$$\therefore \frac{1}{2} \frac{\pi^2}{900g} k R^2 N^2 W = \frac{33000}{2N} q \times \text{I.H.P.};$$

from which
$$W = \frac{9.7 \times 10^6 q (\text{I.H.P.})}{k N^3 R^2} \text{ lbs.}$$

$$= \frac{43260 q (\text{I.H.P.})}{k N^3 R^2} \text{ tons.}$$

The following values for k may be used:—

Electric lighting engines	0.01 to 0.02
Factory engines	0.02 to 0.05

The values of q vary considerably, depending upon several circumstances which we leave for further study; but a general idea of the order of these values is:—

For single-cylinder engines	0.32
„ two-cylinder engines with cranks at right angles	0.08
„ three-cylinder engines, cranks at 120°	0.035

For single-cylinder gas engines working on the Otto cycle the value of q may vary from 3 to 5 times the work done per stroke; and with two-cylinder engines with cranks at right angles q may have any value between 1.0 and 1.5. These large variations account for the heavy flywheel always used on internal combustion engines as compared with flywheels used for ordinary steam engines of equal power.

Example 1.—Find the weight of flywheel rim necessary for an engine indicating 50 H.P. at 120 revolutions per minute, assuming a radius of gyration of 4 feet, and taking $q=0.32$, for a total fluctuation of speed of 1 per cent.

Here $k=0.01$.

$$W = 43260 \times \frac{0.32 \times 50}{0.01 \times (120)^3 \times 4^2} = 2.5 \text{ tons.}$$

Example 2 (I.C.E., October 1897).—In a gas engine using the Otto cycle, the I.H.P. is 8, and the speed is 264 revolutions per minute. Treating each fourth single stroke as effective, and the resistance as uniform, find the amount of energy which must be stored in the flywheel in order that the speed shall not vary by more than one-fortieth above or below its mean value.

Take $q = 3$ and $k = \frac{1}{40}$ th above + $\frac{1}{40}$ th below = $\frac{2}{40} = 0.05$.

$$\therefore \frac{3 \times 8 \times 33000}{2 \times 264} = \text{K.E.} \times 2k.$$

or
$$\text{K.E.} = \frac{3 \times 8 \times 33000}{2 \times 264 \times 2 \times 0.05} = 15,000 \text{ ft.-lbs.}$$

Example 3.—If a body weighing 5000 lbs. and moving at 5 ft. per second is brought to rest by the action of a force in 6 in., what would be the average magnitude of the force?

$$\text{K.E.} = \frac{Wv^2}{2g} = \frac{5000 \times 5^2}{2 \times 32.2} = 1940 \text{ ft.-lbs.}$$

Let F = average magnitude of force; then

$$F \times \frac{1}{2} \text{ ft.} = 1940, \text{ or } F = 3880 \text{ lbs.}$$

Example 4.—Calculate the work accumulated in a flywheel weighing 5 tons, the radius of gyration being 5 ft., and the revolutions 100 per minute.

$$v = \frac{2\pi RN}{60} = \frac{2 \times \pi \times 5 \times 100}{60} = 52.2 \text{ ft. per second.}$$

$$\text{K.E.} = \frac{Wv^2}{2g} = \frac{5 \times 2240 \times (52.2)^2}{2 \times 32.2} = 475,000 \text{ ft.-lbs.}$$

Example 5.—In a fly-press for punching, the two balls each weigh 25 lbs., and they are fixed at a distance of 30 in. from the axis of the screw. If the weights are given a velocity of 10 ft. per second, find the average force exerted by the punch through a plate $\frac{1}{10}$ in. thick, assuming that 20 per cent. of the energy is lost in friction, and that the weights are brought to rest.

$$\text{K.E.} = \frac{50 \times 10^2}{2 \times 32.2} = 77.6 \text{ ft.-lbs.}$$

As 20 per cent. is lost in friction, energy available for punching

$$= 77.6 \times 0.08 = 62 \text{ ft.-lbs.}$$

Let F = average force exerted by punch.

$$F \times \left(\frac{1}{10} \times \frac{1}{2} \right) = 62 \text{ ft.-lbs. ;}$$

then

$$F = 62 \times 10 \times 2 = 7440 \text{ lbs. (approx.)}$$

Example 6.—The flywheel of a punching machine weighs 1 ton, and has a radius of gyration of 3 ft. 0 in. At the instant of commencing to punch the hole, it was estimated to be rotating at 120 revolutions per minute, and after punching its speed was

only 100 revolutions per minute. Calculate the amount of work spent in punching and overcoming frictional resistances.

$$\text{Loss of kinetic energy} = \frac{W}{2g}(v_1^2 - v_0^2).$$

$$v_1 = \frac{2\pi rN}{60} = \frac{2 \times \pi \times 3 \times 120}{60} = 37.7 \text{ ft. per second.}$$

$$v_0 = \frac{2 \times \pi \times 3 \times 100}{60} = 31.4 \text{ ft. per second.}$$

$$v_1^2 - v_0^2 = (v_1 + v_0)(v_1 - v_0) = 69.1 \times 6.3 = 436 \text{ ft. per second.}$$

$$\therefore \text{Loss of K.E.} = \frac{2240 \times 436}{2 \times 32.2} = 15,100 \text{ ft.-lbs.}$$

Alternative Method :—

$$\text{K.E. at 120 revolutions} = \frac{2240 \times (37.7)^2}{2 \times 32.2} = 49,400 \text{ ft.-lbs.}$$

As K.E. varies as squares of speeds, let x = K.E. at 100 revolutions.

$$49400 : x :: (120)^2 : (100)^2,$$

$$\text{or } x = \frac{49400 \times 10000}{14400} = 34,300 \text{ ft.-lbs.}$$

$$\therefore \text{Work spent in punching} = 49400 - 34300 \\ = 15,100 \text{ ft.-lbs.}$$

Note.—All the above answers are approximate only, being obtained by means of a 10-inch slide-rule.

EXERCISES.

1. A girder weighing 10 tons is lifted on to supports 12 ft. above the ground. Calculate the amount of work done in placing it in position. *Ans.* 268,800 ft.-lbs.

2. A circular shaft 7 ft. in diameter and 120 ft. deep is full of water. How much work would be done in raising the water to the surface? *Ans.* 17,316,000 ft.-lbs.

3. A chain weighing 10 lbs. per foot of length is 240 ft. long and hangs vertically. What work is done in winding up the chain round a drum? *Ans.* 288,000 ft.-lbs.

4. What amount of work will be required to overcome the friction of a weight of 6 tons dragged along a level road for a length of 7 ft., when the coefficient of friction is 0.235? What would be the H.P. equivalent of this if the work is done in 2 seconds?
Ans. 22,100 ft.-lbs., 20.1 H.P.

5. The plunger of a force pump is 9 in. diameter, the length of stroke 2 ft. 6 in., and the pressure of the water 50 lbs. per sq. in. Find the number of units of work done in 1 stroke (neglecting friction).
Ans. 7950 ft.-lbs.

6. If the above pump makes 22 effective strokes per minute, calculate the H.P. to pump this water, and estimate the probable power of the pump if the efficiency is 80 per cent.
Ans. 5.3 and 6.63 H.P.

7. A body weighing 1610 lbs. was lifted vertically by a rope, there being a damped spring balance to indicate the pulling force *F* lbs. of the rope. The lifting forces for different distances (*D*) were as follows:—

D	0	11	20	34	45	55	66	76
F	4010	3915	3763	3532	3366	3208	3100	3007

Find approximately the work done on the body when it has risen 70 ft.
Ans. 246,570 ft.-lbs.

8. A locomotive exerts a steady pull of 2000 lbs. when drawing a train along a level track at the rate of 40 miles per hour. Find the rate of working in H.P.
Ans. 213.3.

9. Calculate the horse-power required to pump 5000 gallons of water per minute from a well 40 ft. deep, if the efficiency of the machinery employed is 60 per cent.
Ans. 101.

10. A double-acting steam engine has a cylinder 12 in. diameter and a stroke of 18 ins. If the mean effective pressure for both sides of the piston is 40 lbs. per sq. in., what is the H.P. of the engine when running at 120 revolutions per minute?
Ans. 49.4.

11. What will be the H.P. in the previous question if we take account of a piston rod 2 in. diameter at the front of the cylinder and a tail-rod $1\frac{1}{2}$ in. diameter at the back?
Ans. 48.3.

12. A flat bar of mild steel is 12 ft. long and has a section 3 in. $\times$ $\frac{3}{4}$ in. If a load of 9 tons is gradually applied, find the work

done in stretching the rod, taking E (the modulus of elasticity) = 13,500 tons. *Ans.* 35.8 ft.-lbs.

13. A centrifugal pump, whose efficiency is 45 per cent., is required to lift 6.2 cub. ft. of water per second to a height of 7 ft. What must be its H.P.? If it is to be driven by a continuous current electric motor whose efficiency is 85 per cent. at 200 volts, how many amperes of current must be supplied to the motor? *Ans.* 12.9 H.P. and 48 amps.

14. A weight of 110 lbs. is moving with a velocity of 3840 ft. per minute. How much work has been expended to produce this result?

15. An engine and train weighing 300 tons have a velocity of 30 miles per hour when steam is turned off. How far will they travel on a level rail if the mean coefficient of friction be 0.0035? *Ans.* 17,200 ft.

16. In an air compressor 10 cub. ft. of air at a gauge pressure of 5 lbs. per sq. in. is compressed adiabatically to a gauge pressure of 105 lbs. per sq. in. Calculate the work done during (a) an adiabatic compression, (b) an isothermal compression of the air. For (a) take $n = 1.408$. *Ans.* (a) 334, (b) 357 ft.-lbs.

17. In an air compressor 15 cub. ft. of air is drawn in at atmospheric pressure for each stroke, and the final pressure is 100 lbs. per square inch absolute. Find the work done during compression (a) if the air is compressed isothermally, (b) if the air is compressed adiabatically. *Ans.* (a) 427, (b) 404 ft.-lbs.

18. A double-crank engine indicates 250 H.P. at 80 revolutions per minute. Find the weight of fly-wheel necessary, with a radius of gyration of 7 ft., to control the speed variation within $\frac{1}{4}$ per cent. on either side of the mean. *Ans.* 6.9 tons.

19. A fly-wheel weighs 9 tons, its angular velocity is 6.25 radians per second, and its radius of gyration 6 ft. How many foot-tons of work are stored up in it? *Ans.* 196.5 ft.-tons.

20. In the last question, if it be required to store an additional work of 10 foot-tons, find what will be the increase of velocity. *Ans.* 1.41 radians per sec.

21. Two fly-wheels with rims of rectangular section are each made from the same drawing, but to two different scales such that all the dimensions of one wheel are twice the dimensions of the other. Find the ratio of their kinetic energies when rotating an equal number of times per minute. *Ans.* 32 : 1.

22. Taking the average piston speed of an engine as 400 ft. per minute, and the value of q as $\frac{1}{3}$, the fluctuation of speed as 1 per cent. on either side of the mean (*i.e.* $R=0.02$), the radius of the fly-wheel as twice the stroke; show that the weight of the rim may be taken as 50 lbs. per I.H.P. for a single-cylinder engine running at 100 revolutions per minute.

23. A single-cylinder engine of 45 I.H.P. runs at 120 revolutions per minute. What will be the weight of a fly-wheel 6 ft. diameter to keep the total fluctuation of energy within 2 per cent. ?

CHAPTER IX.

TRANSMISSION OF WORK THROUGH MACHINES.

A MACHINE, as we shall deal with it in the present chapter, is an agent for overcoming a resistance or load, usually larger than the effort applied to the machine. In the crab-winch or windlass or any other lifting appliance we have a machine through the agency of which a *load may be lifted at one part of the machine by the exertion of a force or effort applied to another part*—the load lifted being nearly always much larger than the effort applied. In a lathe or any other machine tool for shaping material into any desired form, the resistance is overcome at the point of the tool by an effort (or driving force) whose magnitude is equal to the difference of the tensions on the two sides of the belt.

Mechanical Advantage.—Hence we may say that any machine is *usefully* employed and an advantage gained when a resistance is overcome for some useful purpose by an effort or force usually less in magnitude than the resistance, and the **ratio of the magnitude of the resistance (W or R) to the magnitude of the effort (F) is called the mechanical advantage.**

Thus, in the case of a lifting appliance,

$$\text{Mechanical advantage} = \frac{\text{Load lifted}}{\text{Effort exerted}}.$$

Symbolically,
$$M_a = \frac{W}{F}.$$

In the case of a machine tool or other machine where a resistance not due to gravity is overcome,

$$\text{Mechanical advantage} = \frac{\text{Resistance overcome}}{\text{Effort applied}},$$

or
$$M_a = \frac{R}{F}.$$

It follows from the definition that

$$\text{Effort to overcome a given resistance} = \frac{\text{Resistance}}{\text{Mechanical advantage}},$$

$$\text{or} \quad F = \frac{R \text{ or } W}{M_a},$$

$$\text{also} \quad R \text{ or } W = M_a \times F.$$

It is scarcely necessary to point out that for any particular machine the magnitude of the effort or force will depend upon the magnitude of the load lifted or resistance overcome, and that the mechanical advantage will in a general way increase as the load increases. In Table XIX., p. 223, we have tabulated the results of a series of experiments on an ordinary Weston chain pulley block. In the second column the loads lifted are stated, and in the third column the corresponding values of the efforts exerted to lift these loads. These efforts were found by attaching weights to the hauling side of the chain sufficient to just lift the load. Thus, in experiment 5 a load of 21 lbs. was lifted by a force of 5.73 lbs. This gives a mechanical advantage of 3.66, or, in other words, the load lifted = 3.66 times the effort exerted. In experiment 10 an effort of 12.23 lbs. was sufficient to lift a load of 56 lbs., giving a mechanical advantage of 4.58. The last experiment, for a load of 140 lbs. and an effort of 29.05 lbs., gives $M_a = 4.82$. These results and an inspection of column 4 will show that *in this machine the mechanical advantage up to a certain point increases with the load.*

Velocity Ratio.—The student must have noticed, in watching any machine at work, that the load is lifted or resistance overcome through a distance usually small in comparison with the distance through which the effort is exerted in the same time. Hence we are led to define the **Velocity Ratio** as the ratio of the distance through which the effort is exerted to the distance through which the load is lifted or the resistance overcome in the same time.

$$\text{Velocity ratio} = \frac{F's \text{ motion}}{W's \text{ motion}} = V_r.$$

The velocity ratio for a given machine can be calculated from the significant dimensions of the machine. It often can be more easily determined, in the case of a machine already constructed, by **actually measuring** any distance through which the effort is exerted for the corresponding measured distance through which the load is lifted or resistance overcome.

A few worked examples on the determination of velocity ratio are given.

Example 1.—In a simple screw-jack the pitch of the screw is $\frac{1}{4}$ in. and the length of the toggle bar is such that the effort exerted by the hand can act at a distance of 15 in. from the axis of the screw. Let it be required to find the velocity ratio.

Suppose the screw to be turned round once; the effort would then be exerted through a distance equal to the circumference of a circle whose radius is 15 in., i.e. through 94.4 in. The load would be lifted in the same time through a distance equal to the pitch of the screw, and since $V_r = \frac{F^{\text{ns}} \text{ motion}}{W^{\text{ns}} \text{ motion}}$, we have for

the velocity ratio $\frac{94.4}{\frac{1}{4} \text{ in.}} = 377.6$, or, in other words, the effort is exerted through 377.6 times the distance through which the load is lifted; or the velocity at which the effort is being applied at any instant is 377.6 times the velocity at which the load is being lifted at the same instant.

Symbolically,

$F^{\text{ns}} \text{ motion} = 2\pi d$, where d = distance from axis to effort;

$W^{\text{ns}} \text{ motion} = p$, where p = pitch of screw;

hence

$$V_r = \frac{2\pi d}{p}.$$

Example 2.—In the Weston chain pulley blocks for which the results of the experiments are given in Table XIX., the number of link spaces in the two grooves of the top block were 8 and 7 respectively. What velocity ratio would this give?

Suppose the top block to be turned once round by the effort applied on the hauling side of the chain. Then the effort would be exerted through a distance equal to the length of 8 link spaces while one circumference of chain is unwound from the smaller pulley.

Owing to the use of the snatch-block, the weight would be raised $\frac{1}{2}(8-7)$ links, i.e. through a distance equal to the length of $\frac{1}{2}$ a link.

The velocity ratio therefore must be $\frac{8 \text{ link distances}}{\frac{1}{2} \text{ link distance}} = 16$.

More generally, let d_1 , d represent diameters of larger and smaller pulley in top block respectively. Then for one turn of top block

$$F^{\text{ns}} \text{ motion} = \pi d_1;$$

and since the two lifting sides of the chain are shortened by $\pi(d_1 - d)$, the weight must be lifted $\frac{1}{2}\pi(d_1 - d)$, which is $W^{\text{ns}} \text{ motion}$.

Hence
$$V_r = \frac{\pi d_1}{\frac{1}{2}\pi(d_1 - d)} = \frac{2\pi d_1}{\pi(d_1 - d)}, \text{ or simply } \frac{2d_1}{d_1 - d}.$$

Example 3.—In a crab-winch the dimensions are as follows:—Effective diameter of drum = 5 in.; number of teeth on spur-wheel fixed on drum = 150; number of teeth on pinion = 15; length of handle from handle-bar to axis of spindle = 15 ins. What is the velocity ratio of the winch?

Suppose the handle to be turned once round, then distance through which effort is exerted = $2 \times 15 \times 3.1416$. Now the drum is turned $\frac{15}{150} = \frac{1}{10}$ of a revolution in the same time.

$\therefore$ the weight is lifted $\frac{1}{10}$ of the circumference, i.e., $\frac{1}{10} \times 5 \times 3.1416$.

Hence
$$\text{Velocity ratio} = \frac{2 \times 3}{\frac{1}{10}} = \frac{6}{\frac{1}{10}} = 60.$$

If in this example the drum makes $\frac{1}{n}$ of a revolution for one turn of the handle,

then W 's motion = $\frac{1}{n}$ of πd (where d = diameter of drum);

F 's motion = $2\pi r$ (where r = length of handle);

hence
$$= \frac{2\pi r}{\frac{1}{n}\pi d} = \frac{2nr}{d}.$$

Having defined mechanical advantage and velocity ratio, we have next to consider a machine as an agent for the transmission of work.

If the machine worked without friction in the different parts, we should have

$$\left. \begin{array}{l} \text{* The useful work} \\ \text{given out by machine} \end{array} \right\} = \text{Work put into it in the same time.}$$

This statement is known as the *Principle of Work* applied to a *frictionless* machine.

With **actual machines**, taking into account friction, the principle of work would be stated thus:

$$\left. \begin{array}{l} \text{* Total work put} \\ \text{into machine} \end{array} \right\} = \left\{ \begin{array}{l} \text{Useful work obtained from it} \\ \text{+ work spent in overcoming friction.} \end{array} \right.$$

* These equations assume the machine to be working without *acceleration* of the moving parts.

It is obvious that the *useful work obtained from any machine must consequently be less than the total work applied*, and we are led to define efficiency.

Mechanical Efficiency of a machine is the ratio that the **useful work given out bears to the total work put in**. This ratio is often conveniently expressed as a *percentage*.

Thus, for example, suppose we say that the **mechanical efficiency of a steam engine**, regarded as a machine, is 80 per cent., we mean that the useful work (measured by brake horse-power) which can be taken from the engine shaft is 80 per cent. of the work (measured by indicated horse-power) developed in the engine cylinder—20 per cent. being lost in engine friction.

The mechanical efficiency of a machine for a given load or resistance may be calculated from the formula

$$M_e = \frac{W}{F \times V_r},$$

where F represents the magnitude of the force or effort required to overcome the load or resistance W (and friction), both quantities being expressed in terms of the same unit, and where V_r represents the velocity ratio of the machine. As all machines have a certain amount of friction, the mechanical efficiency must necessarily be less than unity, or less than 100 if expressed as a percentage. If we call the percentage of efficiency obtained η , we have

$$\eta = M_e \times 100 \quad \text{or} \quad M_e = \frac{\eta}{100}.$$

Referring to Table XIX., giving the results of the experiment on a differential chain pulley block, we find that 17·36 lbs. was sufficient to overcome a load of 84 lbs. with a velocity ratio of 16; so that, using the formula just given, we obtain as our mechanical efficiency at this load

$$M_e = \frac{84}{17 \cdot 36 \times 16} = 0 \cdot 3024,$$

or $\eta = 0 \cdot 3024 \times 100 = 30 \cdot 2$ per cent.

In order that the student may fully appreciate the significance of this statement of efficiency, let us estimate the amount of work put into the machine when the effort is exerted through a distance of (say) 1 ft. Then, as the force of 17·36 lbs. is exerted through a distance of 1 ft., the work put into the machine is 17·36 ft.-lbs. But for this expenditure of work, the load has been raised a distance of $\frac{1}{16}$ th ft., the number 16 being the velocity ratio of the machine. The useful work done, therefore, is that of

raising 84 lbs. against the force of gravity through a height of $\frac{1}{8}$ th of a foot, or 5.25 ft.-lbs. (practically). Then the ratio of useful work done to work put in, or mechanical efficiency, will be

$$\frac{5.25}{17.36} = 0.302 \text{ or } 30.2 \text{ per cent.}$$

To prove that, in any case, the mechanical efficiency is given by the expression

$$M_e = \frac{W}{F \times V_r} \quad \text{or} \quad M_e = \frac{M_a}{V_r},$$

let L represent the distance in feet through which W is overcome, and l the distance in feet through which the effort is exerted in the same time.

$$\text{Then} \quad M_e = \frac{\text{work obtained}}{\text{work put in}} = \frac{W \times L}{F \times l},$$

or, dividing both numerator and denominator by L , we have

$$M_e = \frac{W}{F \times \frac{l}{L}} = \frac{W}{F \times V_r},$$

since the ratio $\frac{l}{L}$ is equal to the velocity ratio of the machine.

Also, since mechanical advantage = $\frac{W}{F}$, the expression for efficiency may be written

$$M_e = \frac{M_a}{V_r}.$$

The proportion of the total work done that is absorbed in friction in any machine is obviously for a given load equal to 100 minus the percentage of efficiency obtained. Thus, in the test just quoted, for which an efficiency of 30.2 was obtained, the remainder of the work applied, equal to $(100 - 30.2)$, or 69.8 per cent. of the whole, is absorbed in overcoming the frictional resistances of the apparatus.

The friction can also be estimated as a part of the total effort applied, and is equal to the actual effort as found by experiment, required to overcome the resistance, less the effort which would have been required to overcome the same resistance had the machine worked without friction. This latter quantity we will term the "load effort," and it is found by dividing the load or useful resistance by the velocity ratio. The other portion of the effort we may term the "friction effort" or the amount of effort required to overcome the frictional resistance of the machine.

This can be found by subtracting the load effort (F_l) from the total effort (F).

Thus, if F = total effort required in lbs.,

F_l = load effort in lbs.,

F_f = friction effort in lbs.,

then $F = F_l + F_f$,

or $F_f = F - F_l$,

and $F_l = \frac{W}{V_r}$.

Reverting again to our experiment, we see from Table XIX., in Test No. 12, the load effort (F_l) is equal to

$$\frac{86.38}{16} = 5.4 \text{ lbs. (approx.),}^*$$

i.e. without friction in the pulley blocks, 5.4 lbs. effort would have been sufficient to lift or balance the load of 86.38 lbs.; but since the actual effort required is 17.36 lbs., it follows that (17.36 - 5.4) or 11.96 lbs. of the effort is used in overcoming friction. Hence F_f for this load is 11.96 lbs. The values of F , and F_f for all the tests are given in columns 5 and 6 of the table.

Having considered at some length the meaning of the terms mechanical advantage, velocity ratio, efficiency, and load effort, we will now examine briefly a few of the simpler machines and mechanical contrivances in actual use, and determine the method of calculating the value of the velocity ratio and load effort in each case.

In the *ordinary lathe*, for example, the work is done against the resistance to cutting at the point of the tool. Let this resistance be represented by R lbs., and the cutting speed in feet per minute by C , then the effective work done is measured as the product of R into C or $R \times C$. If, now, the belt has a velocity of S feet per minute, and the tensions are T and t lbs. in the tight and slack sides respectively, then the work put into the machine is equal to the product $(T - t) \times S$. If there were no frictional resistances, the work put in would be equal to the effective work obtained, or, symbolically,

$$S(T - t) = R \times C.$$

Or, if η equal percentage of efficiency actually obtained, taking friction into account, we have the relationship

$$S(T - t) \times \frac{\eta}{100} = R \times C.$$

* Weight of lower block must be taken into account in finding load effort.

Now, the effective tension of the belt is in reality the effort applied to the machine, and the resistance of the material is the load, so that we have the mechanical advantage

$$M_a = \frac{W}{F} = \frac{R}{T-t}.$$

Also the velocity ratio of the machine is the ratio of the speed of the belt B to the cutting speed of the tool,

or
$$V_r = \frac{B}{C}.$$

Example 4.—Take the following example:— $S=800$ ft. per minute, $C=20$ ft. per minute, $(T-t)=200$ lbs. To find R , the resistance at the point of the tool, assuming $\eta=50$ per cent.

Here
$$800 \times 200 \times \frac{50}{100} = R \times 20,$$

or
$$R = \frac{800 \times 200 \times 50}{100 \times 20} = 4000 \text{ ft.-lbs.}$$

TABLE XIX.—RESULTS OF EXPERIMENTS ON WESTON'S
DIFFERENTIAL CHAIN PULLEY BLOCKS.

Test No.	Total Load Lifted W lbs.	Total Effort Exerted F lbs.	Mechanical Advantage $M_a = \frac{W}{F}$	Load Effort, lbs. $F_l = \frac{W}{V_r}$	Friction Effort, lbs. $F_f = (F - F_l)$	Mechanical Efficiency per cent. = $\frac{W}{F \times V_r} \times 100$.
1	The bottom block	1.38	...	...	...	...
2	0 + block	1.84	...	...	...	13.59 ... per cent.
3	7	3.22	2.17	.59	2.63	19.02 "
4	14	4.60	3.04	1.02	3.58	22.90 "
5	21	5.73	3.66	1.46	4.27	24.34 "
6	28	7.19	3.89	1.90	5.29	26.04 "
7	35	8.40	4.17	2.34	6.07	26.76 "
8	42	9.81	4.28	2.77	7.04	27.76 "
9	49	11.03	4.44	3.21	7.82	28.62 "
10	56	12.23	4.58	3.65	8.59	28.91 "
11	70	15.13	4.63	4.52	10.61	30.24 "
12	84	17.36	4.84	5.40	11.96	30.36 "
13	98	20.17	4.86	6.27	13.90	30.21 "
14	112	23.17	4.83	7.15	16.02	30.29 "
15	126	26.00	4.85	8.02	17.98	30.12 "
16	140	29.05	4.82	8.90	20.15	...

Note.—1. The total effort includes the weight of the scale-pan.

2. The total load includes the weight of the bottom block.

Various experiments have been made to find the resistance offered to the cut, sometimes by measuring the upward pressure exerted at the point of the tool, or by measuring the actual pressure upon the tool in a direction parallel to its length; but no definite information on the subject seems available. It naturally depends upon the hardness of the material turned, the depth of cut,

Experiments on Weston's Differential Pulley Blocks.

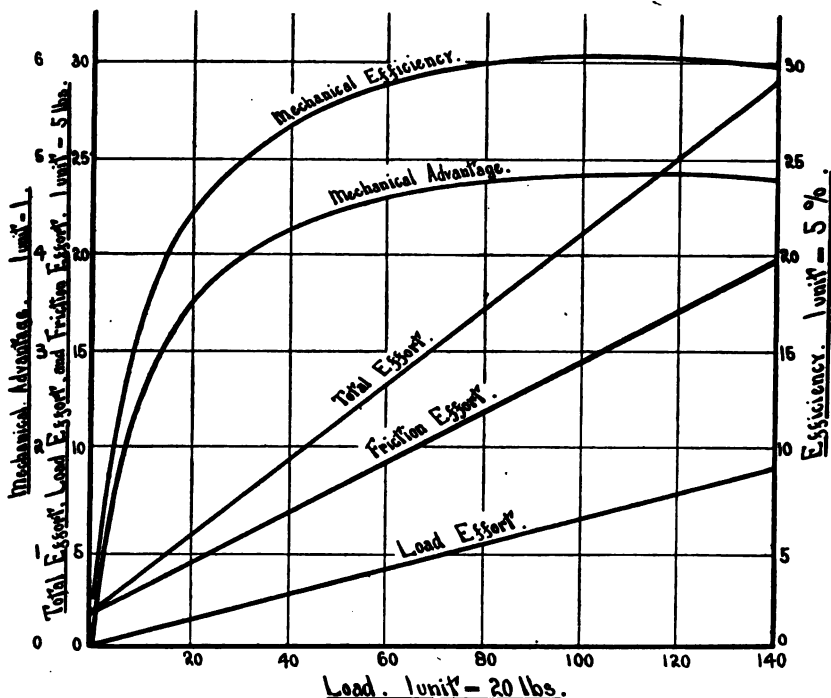


FIG. 127.

amount of feed, speed of cut, angle of tool, and the nature of the material cut.

We now give in Table XIX. the complete results for the experiments on the Weston blocks, and on p. 225 the additional information of the principal dimensions.

In fig. 127 these results are plotted on squared paper, and a close study of the curves obtained will amply repay the student for any time spent on them.

EXPERIMENT ON DIFFERENTIAL CHAIN PULLEY BLOCKS (WESTON'S TYPE).

Object of Experiment.—To find efficiency of tackle.

Data of Apparatus.

Top block—

Link spaces in larger and smaller sheaves = 8 and 7 respectively.

Approximate diameters of sheaves to centre of chain = 4.0 in. and 3.5 in.

Weight of top block = 4.2 lbs.

Diameter of pin = 0.658 in.

Length of sheaves on pin = 2.3 in.

Projected area of pin = 0.658 in. $\times$ 2.3 in. = 1.51 sq. in.

Bottom block—

Approximate diameter to centres of chain = 3.5 in.

Weight of block = 2.38 lbs.

Diameter of pin = 0.645 in.

Length of sheave on pin = 1.2 in.

Projected area of pin = 0.645 in. $\times$ 1.2 in. = 0.774 sq. in.

Chain—

Total length = 41.8 ft.

Total weight = 19.4 lbs.

Notes on test—

Pins lubricated.

Weight of effort—scale-pan ($\frac{1}{2}$ lb.) included in the total effort.

Velocity ratio—

Weight of bottom block included in total load lifted—

$$(1) \text{ Calculated} = \frac{8}{\frac{1}{2}(8-7)} = 16 \text{ to } 1.$$

$$(2) \text{ Measured, mean of 5 observations} = 15.967.$$

In the case of the *wheel and axle*, a weight or load W is raised by means of a rope or chain coiled round a drum mounted on the same axle as a larger wheel from which the effort is applied. Then, if F , represents the effort or force exerted from the wheel, R the distance from centre of axle to centre of the cord, and r the effective radius of the drum (to the centre of the rope or chain), without friction, we have

$$F \times R = W \times r,$$

and the velocity ratio is equal to $\frac{R}{r}$.

If we include friction, the load effort will be given by

$$F \times R = W \times r \text{ (including friction),}$$

and the mechanical efficiency will be $\frac{F_i}{F}$ or $\frac{W}{F \times V_r}$, as before.

To prove this by the principle of work, let the drum make one complete revolution, then the weight will be raised a distance equal to $2\pi r$, and the effective work done against gravity will be $W \times 2\pi r$.

But the work supplied to the machine during one revolution is $F \times 2\pi R$.

The velocity ratio is $\frac{R}{r}$, and the mechanical advantage $M_a = \frac{W}{F}$.

The mechanical efficiency is the ratio of useful work obtained to total work supplied, or

$$M_e = \frac{W \times 2\pi r}{F \times 2\pi R} = \frac{W \times r}{F \times R} = \frac{W}{F} \times \frac{1}{V_r} \quad \text{or} \quad \frac{M_a}{V_r}.$$

And we might write $V_r = \frac{W}{F_i}$; then

$$\begin{aligned} M_e &= \frac{W}{F} \times \frac{1}{V_r} = \frac{W}{F} \times \frac{F_i}{W} \\ &= \frac{F_i}{F}. \end{aligned}$$

Example 5.—If $F = 30$ lbs., $R = 12$ in., and $r = 2$ in., and efficiency = 60 per cent, find W or load raised.

$$\text{Here} \quad F_i = 30 \times \frac{60}{100} = 18 \text{ lbs.}$$

$$W = \frac{F_i \times R}{r} = \frac{18 \times 12}{2} = 108 \text{ lbs.} \quad \text{Ans.}$$

Example 6.—In a ship's capstan, three men, each exerting a force of 25 lbs. on bars 5 feet long from centre of capstan, can just raise a weight of 5 cwt. If the diameter of the barrel be 10 in., and that of the rope be 1 in., find the efficiency of the apparatus.

$$\text{Here } R = 5 \text{ ft.} = 60 \text{ in., and } r = \frac{10 \text{ in.} + 1 \text{ in.}}{2} = 5.5 \text{ in.}$$

$$\therefore V_r = \frac{R}{r} = \frac{60}{5.5} = 10\frac{10}{11}.$$

Weight raised = 5 cwt. = $5 \times 112 = 560$ lbs.

Total effort = $3 \times 25 = 75$ lbs.

$$\therefore M_a = \frac{560}{75} = 7\frac{7}{15}.$$

$$\text{Efficiency} = \frac{M_a}{V_r} = \frac{7\frac{7}{15}}{10\frac{11}{11}} = \frac{112}{15} \times \frac{11}{120} = 0.684.$$

Ans. Efficiency = 68.4 per cent.

Example 7.—The sum of the diameters of a wheel and axle is 36 in., and it is found that an effort of 40 lbs. will just support a load of 150 lbs. Find the sizes of the wheel and axle if the efficiency of the apparatus is 75 per cent.

$$\text{Here} \quad M_a = \frac{150}{40} = 3.75.$$

$$M_e = 0.75; \text{ and as } M_e = \frac{M_a}{V_r}$$

$$V_r = \frac{M_a}{M_e} = \frac{3.75}{0.75} = 5.$$

$$\therefore \frac{R}{r} \text{ or } \frac{D}{d} \text{ (taking diameters)} = 5 \text{ or } D = 5d.$$

But $D + d = 36$ in., or $5d + d = 36$ in.,

$$\therefore d = \frac{36 \text{ in.}}{6} = 6 \text{ in.,}$$

and

$$D = 5 \times 6 = 30 \text{ in.} = 2 \text{ ft. 6 in.}$$

Ans. Wheel = 2 ft. 6 in. diameter.

Axle = 6 in. diameter.

We shall next briefly consider the three systems of blocks and tackle, and find an expression determining the velocity ratio in each case.

System 1.—A simple combination of single-sheaved blocks. The effort (F) is applied at the free end of the rope leading from under the upper block, and the load W raised from the lower block.

If n = number of blocks employed, then

$$V_r = 2^n.$$

To illustrate this, let t represent the tension in the rope supporting the upper block, or (neglecting friction) $t = F$. Then

the second block will be supported by 2 ropes, each having a tension of $2t$ in it. The third block will be carried by a rope with $4t$ tension in each side, and so on. Thus, if the weight is suspended from the third block (neglecting friction), it will be equal to $8t$, or

$$\frac{W}{F} = \frac{8t}{t} = 8 = 2^3 = V_r.$$

The same result could be obtained by using the principle of work, and assuming the load to be raised 1 ft. Clearly the second block must be raised 2 ft., the upper block 4 ft., and the effort applied through a vertical distance of 8 or 2^3 ft. to accomplish this.

System 2.—Here the position of the pulleys is the reverse of above. The upper block is suspended from some fixed point, and the ends of the rope for same are attached to the weight and the next block respectively. If n = number of blocks, then

$$V.R. = 2^n - 1.$$

This can be proved by similar reasoning to that just given.

System 3.—Here a number of loose sheaves, all revolving independently of each other, are mounted in two blocks, one the upper block and a lower block. The former is suspended from some fixed point, and the latter supports the load.

As the rope is in one continuous length, one free end fixed to the lower block and the other end used for the application of the effort, it would have the same tension throughout in a frictionless machine. Let this tension = t lbs., then, if there are five ropes supporting the load, the latter will be $5t$, or without friction

$$F = t \quad \text{or} \quad \frac{W}{F} = \frac{5t}{t} = 5 \quad V_r = 5.$$

Generally, let n represent the number of ropes supporting the lower block, then

$$V_r = n,$$

and

$$\frac{W}{F} = n.$$

The number of sheaves used in each block can be varied to suit the requirements of load, but generally one block has one more sheave than the other. Sometimes the block with the larger number of sheaves is placed at the bottom, in which case the free end of the rope is attached to the upper block.

Winches or Cranes.—The calculations for the power or efficiency of ordinary winches or cranes are similar to those for the wheel and axle, except that we have to include the effect of the gearing. Referring to the illustration of the crane shown in fig. 128, and taking first the case of the single purchase, we see that, by applying the driving force to the handle or at the circumference of a pulley, we revolve the small pinion (n teeth) which gears into a

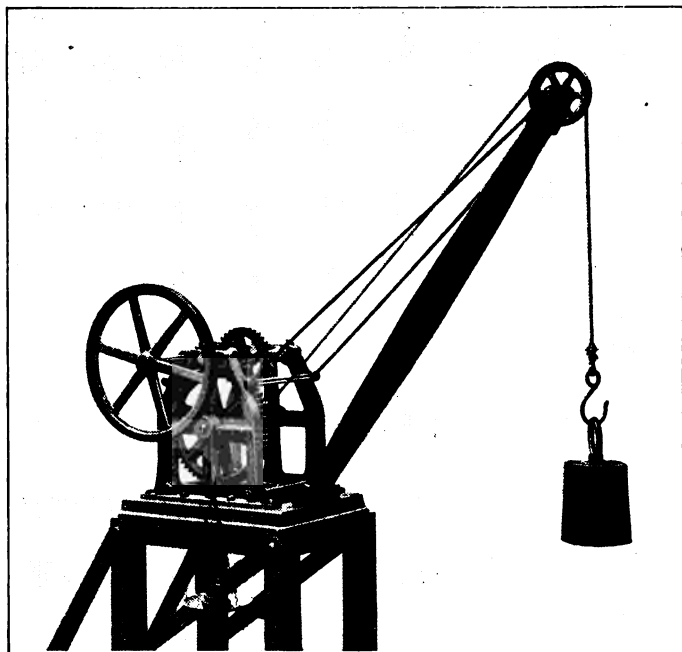


FIG. 128.—Crane, Single and Double Purchase.

larger wheel (N teeth) mounted on the same axle as the chain or rope drum D . For one revolution of the drum, the handle makes $\frac{N}{n}$ revolutions, and therefore the driving force F is exerted through a distance $2\pi R \times \frac{N}{n}$ (in inches) whilst the weight or load is lifted $2\pi r$ (in inches), R being the length of the handle in inches, and r the effective radius of the drum or rope barrel.

The work done by F_i , therefore, for one revolution of the drum is

$$F_i \text{ (lbs.)} \times \frac{2\pi R}{12} \times \frac{N}{n} \text{ (ft.) in ft.-lbs.,}$$

and the work obtained at the rope or load end of the machine is

$$W \text{ (lbs.)} \times \frac{2\pi r}{12} \text{ (ft.) in ft.-lbs.}$$

Neglecting friction, these two amounts must be equal, or

$$F_i \times \frac{2\pi R}{12} \times \frac{N}{n} = W \times \frac{2\pi r}{12}.$$

Dividing both sides by $\frac{2\pi}{12}$, we get

$$F_i \times R \times \frac{N}{n} = W \times r,$$

or
$$F_i = \frac{W \times r \times n}{R \times N},$$

or
$$W = \frac{F_i \times R \times N}{r \times n}.$$

The velocity ratio with this machine is therefore

$$\frac{R \times N}{r \times n}.$$

If the machine is geared in the double purchase (see fig. 128), that is, fitted with an extra wheel and pinion to increase its lifting power, let M and m respectively represent the numbers of teeth in the second wheel and pinion; then

$$V_r \text{ (double gear)} = \frac{R \times N \times M}{r \times n \times m},$$

$$F_i \text{ (neglecting friction)} = \frac{W \times r \times n \times m}{R \times N \times M},$$

and
$$W \text{ (neglecting friction)} = \frac{F_i \times R \times N \times M}{W \times r \times n \times m}.$$

If η represents the percentage of efficiency obtained, including friction, then the actual weight lifted will be reduced in the ratio of $\frac{\eta}{100}$ in each case, and the actual force required to lift a given

weight will be F lbs., where F is given by $F = \frac{100}{\eta} \times F$ in both cases.

Example 8.—In a single-purchase (or single-gear) crab, the handle-bars are 14 in. long and the rope drum 6 in. effective diameter. The gearing consists of a pinion with 15 teeth gearing into a wheel of 75 teeth. Find the velocity ratio of the machine, and calculate the weight which can be raised by two men, each exerting a force of 40 lbs., if the efficiency is 35 per cent.

$$\text{Here Velocity ratio} = \frac{R \times N}{r \times n} = \frac{14 \times 5}{3 \times 1} = \frac{70}{3} = 23\frac{1}{3}. \quad \text{Ans.}$$

$$\begin{aligned} \text{Then } W &= F \times V, \times \frac{\eta}{100} = 2 \times 40 \times \frac{70}{3} \times \frac{35}{100} \\ &= 653 \text{ lbs.} \quad \text{Ans.} \end{aligned}$$

(*Note.*—In such questions as the above, the assumption is made that the driving force exerted is constant, and always tangential to the circle described by the handles. In actual practice this is far from being realised.)

Example 9.—In a double-purchase winch, it is found that a driving force of 50 lbs. is just capable of raising $\frac{1}{2}$ ton. The sizes are as follows: radius of handle-bar = 16 inches, diameter of drum = 8 inches; first gear consists of a pinion of 12 teeth gearing into a wheel of 40 teeth, and in the second gear a pinion of 15 teeth gears into a wheel of 90 teeth on the drum axle. Find its efficiency.

$$\text{Here Velocity ratio} = \frac{R \times N \times M}{r \times n \times m} = \frac{16 \times 40 \times 90}{4 \times 12 \times 15} = 80.$$

$$\text{Actual mechanical advantage} = \frac{1120 \text{ lbs.}}{50 \text{ lbs.}} = 22.4.$$

$$\begin{aligned} \therefore \text{Percentage efficiency} &= \eta = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}} \times 100 \\ &= \frac{22.4}{80} \times 100 \\ &= 28 \text{ per cent.} \quad \text{Ans.} \end{aligned}$$

Screw and Worm Gears.—The simple screw or bottle-jack is a mechanical contrivance for raising heavy loads through a small distance. A screw is threaded through a nut fixed in the top of the bottle or case, and turned round by means of a bar or handle.

The claw at the top of the screw is provided with a spherical seat, so that it does not turn round with the screw.

Let F = force applied at end of bar, in lbs.;

R = length of bar from centre of screw to end, in ins.,

p = pitch of screw, in ins.,

P = pressure exerted by screw, or resistance overcome,

F_1 = load effort, or part of total effort required to lift load, exclusive of friction, in lbs.;

then, neglecting friction, by the principle of work,

$$F_1 \times 2\pi R = P \times p,$$

and

$$P = \frac{F_1 \times 2\pi R}{p}.$$

The velocity ratio is $\frac{2\pi R}{p}$, which can be made very high if the pitch is kept small; but practical considerations of the wear on the screw thread prevent the use of too small a pitch. The number of threads cut upon the screw, whether double or treble-threaded, does not affect the result if the pitch is measured correctly. Including friction, if η represents the percentage of efficiency obtained,

$$P = \frac{F \times 2\pi R}{p} \times \frac{\eta}{100}$$

or

$$F = \frac{P \times p}{2\pi R} \times \frac{100}{\eta}.$$

Example 10.—A bottle-jack has a screw $\frac{1}{2}$ -in. pitch, and it is found that a force of 20 lbs. at the end of a bar 3 ft. 6 in. long is just sufficient to raise 1 ton. Find its efficiency.

$$\text{Here Velocity ratio} = \frac{2\pi R}{p} = \frac{2 \times 22 \times 42}{7 \times \frac{1}{2}} = 528.$$

$$\text{Mechanical advantage} = \frac{2240 \text{ lbs.}}{20 \text{ lbs.}} = 112.$$

$$\text{Percentage efficiency, therefore} = \frac{112}{528} \times 100$$

$$= 21.2 \text{ per cent. } \textit{Ans.}$$

In larger sizes of screw-jack, the power is increased by making the nut form the boss of a worm wheel that can be turned slowly by an endless screw or worm driven by handle-bars, or the force can be applied at the circumference of a pulley if the apparatus

is required for experimental work, as shown in fig. 129. In this case, if

* N = number of teeth in worm wheel for a single-threaded worm,

F = total effort exerted at handle-bar or bars,

F_i = load effort,

R = radius of centre of handle-bar or bars,

p and P as before,

then $\text{Velocity ratio} = \frac{2\pi RN}{p},$

and $P \text{ (excluding friction)} = \frac{F_i \times 2\pi RN}{p},$

or $P \text{ (including friction)} = \frac{F \times 2\pi RN}{p} \times \frac{\eta}{100}.$

Example 11.—A geared screw-jack has a worm wheel of 16 teeth gearing into a single-threaded endless screw driven by winch handles each 14 in. long from the centre of the screw. If the total efficiency of the machine be 20 per cent., find the force required to lift 5 tons, the pitch of the main screw being 1 in.

$$\text{Velocity ratio} = \frac{2\pi RN}{p} = \frac{2 \times 22 \times 14 \times 16}{7 \times 1} = 1408.$$

Then

$$\frac{P}{F} = \frac{1408 \times 20}{100} \quad \text{and} \quad P = 5 \text{ tons} = 11,200 \text{ lbs.}$$

or

$$F = \frac{11200 \times 100}{1408 \times 20} = 40 \text{ lbs. (approx.) on both handle-bars. Ans.}$$

In the **Hydraulic Jack** a sliding cylinder rises on a fixed ram by means of water pressure produced by a small hand-pump at the top of the machine.

Let L = length of pump lever from fulcrum to end, in ins.,

l = length from fulcrum to centre of pump plunger,

d = diameter of pump plunger, in ins.,

D = diameter of ram, or water cylinder, in ins.,

F = force employed at end of pump handle, in lbs.,

P = pressure produced or resistance overcome,

F_i = load effort required (neglecting friction),

then $\text{Velocity ratio} = \frac{L}{l} \times \frac{D^2}{d^2},$

and (neglecting friction) $P = F_i \times \frac{L \times D^2}{l \times d^2};$

* N should be taken at one-half the number of teeth if a double-threaded worm be used, and one-third if a treble-threaded worm.

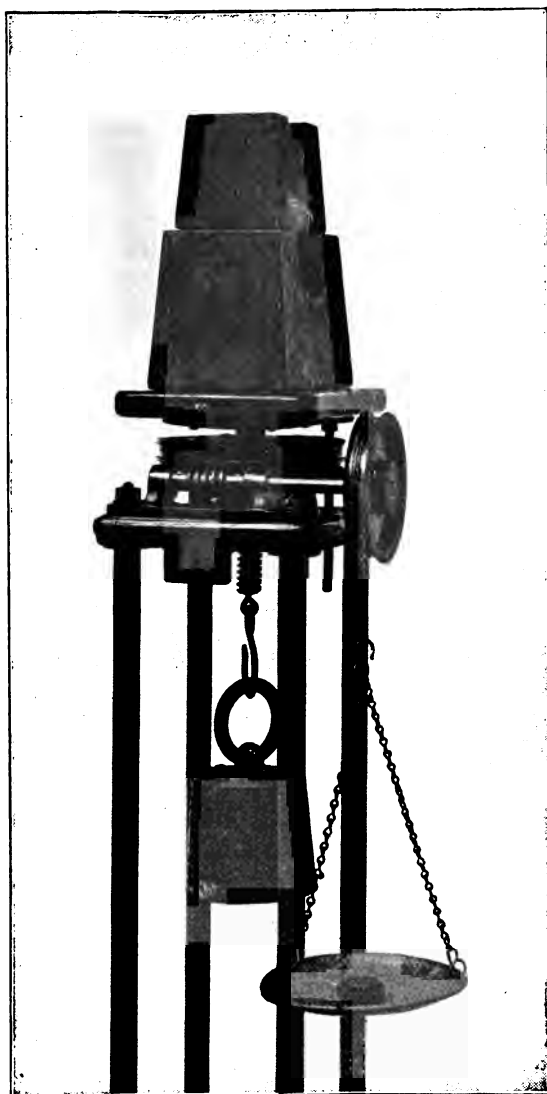


FIG. 129.—Worm, Worm-wheel, and Screw Lifting Jack.

or, if η = percentage efficiency of complete machine,

then (including friction), $P = F \times \frac{L \times D^2}{l \times d^2} \times \frac{\eta}{100}.$



FIG. 130.—Worm and Worm-wheel.

Example 12.—In a hydraulic lifting-jack, with a cylinder 3 in. diameter and pump $\frac{3}{4}$ in. diameter, the hand lever is 24 in. long over all, and the distance from fulcrum to lever is 1 in. Find

its lifting effect with a force of 60 lbs. at the end of the lever, and an efficiency of 60 per cent.

$$\text{Velocity ratio} = \frac{L}{l} \times \frac{D^2}{d^2} = \frac{24}{1} \times \frac{3^2}{(\frac{3}{4})^2} = 24 \times 16 = 384.$$

$$\therefore P = F \times 384 \times \frac{60}{100} = \frac{60 \times 384 \times 60}{100} = 13824 \text{ lbs.}$$

or 6.174 tons. *Ans.*

Calculations for other screw devices, such as coupling screws and copying presses, can all be worked out quite as easily if the principle of work is applied in each case.

EXERCISES.

1. Fig. 129 represents an experimental apparatus for determining the friction on a screw and worm and worm-wheel. The dimensions of the machine are:—pitch of screw, $\frac{1}{4}$ in.; diameter of pulley on screw, 7 in.; number of teeth in worm-wheel, 72; and diameter of effort pulley, 4.95 in. The worm can be thrown out of gear and the effort cord wound round the pulley on the screw so that the machine can be tested for friction of screw and for combined friction of screw, worm, and worm-wheel, etc. The following two sets of results were obtained—

(a) Worm in—

Load (lbs.)	100	150	200	250	300	350	400	450	500
Effort (lbs.)	0.7	0.9	1.1	1.4	1.6	1.8	2.1	2.5	2.8

(b) Worm out—

Load (lbs.)	0	50	100	150	200	250	300	350	400
Effort (lbs.)	0.5	3.1	5.0	8.1	11.1	14.5	16.5	21.0	24.0

Calculate as for Table XIX., p. 223, the results for each of these sets of results, and plot curves similar to those given in fig. 127, p. 224, and then obtain equations to any results which follow the linear law.

2. Fig. 130 represents a worm, worm-wheel, and drum-lifting apparatus whose dimensions are:—diameter of drum, 8 in.; teeth on worm-wheel, 55; and diameter of effort pulley, 4.95 in. The following results were obtained from experiment:—

Load (lbs.)	0	10	20	50	70	90	100
Effort (lbs.)	0.25	1.25	1.95	4.5	4.5	7.6	9.0

Work out these results as in Table XIX., and plot the results as in fig. 127, p. 224, obtaining equations to the straight lines.

3. A crab winch in double purchase gave the following results:—

Load (lbs.)	0	14	28	42	56	84	98	112	126	140
Effort (lbs.)	1.97	1.31	2.67	3.12	3.84	4.81	5.31	5.97	6.5	7.13

The circumference of the drum = 16.6 in.; circumference of effort pulley = 49.8 in.; gearing pinions, 10 and 10 teeth; wheels, 23 and 45 teeth; diameter of rope on drum, 0.3 in. diameter; and diameter of cord from pulley, 0.1 in. diameter. Calculate and plot the results as for Exercises 1 and 2.

4. An ordinary bottle-lifting jack, with a screw $\frac{1}{2}$ in. pitch, was fitted with a pulley $6\frac{1}{2}$ in. diameter and tested, with the following results:—

Load (lbs.)	14	21	28	42	56	84	112	140	168
Effort (lbs.)	1.2	1.48	1.8	2.48	3.04	4.23	5.29	6.48	7.61

Make the necessary calculations and plot the results.

5. In a single-purchase crab, the length of each of the handle-bars is 16 in., and the diameter of the rope barrel 4 in. The gearing consists of a pinion of 18 teeth gearing with a wheel of 90 teeth. Find the weight which can be raised by two men, each exerting a force of 45 lbs., if the efficiency is 56 per cent.

Ans. 2016 lbs.

6. A double-purchase crab has gearing of 4 to 1 and 6 to 1. The handles are each 12 in. long, and the drum barrel 8 in. diameter. If a force of 100 lbs. can just raise a load of 2 tons, find the efficiency of the crab.

Ans. 62.2 per cent.

7. What is the velocity ratio of a double-gearred crab, with pinions of 10 and 12 teeth, and wheels of 60 and 100 teeth respectively, if the handle-bars are twice the diameter of the chain barrel?

Ans. 200.

8. Find the force required to lift 1 ton on a screw jack, with pitch of screw $\frac{3}{8}$ in. and handle-bar 4 ft. long, assuming an efficiency of 20 per cent.

9. Find the weight which can be raised by a single-purchase crab, given the following particulars:—diameter of rope drum = 5 in., length of handles = 12 in., number of teeth in wheel = 80, ditto in pinion = 25, force exerted by two men = 70 lbs. each, total efficiency of machine = 60 per cent.

10. In a Weston's block and fall, with sheaves 8 in. and $7\frac{1}{2}$ in. diameter respectively, a man exerting a force of 30 lbs. is just able to raise a load of 300 lbs. Find the efficiency of the tackle.

Ans. 31.2 per cent.

11. What is the velocity ratio of a pair of differential blocks and fall in which the larger chain sheave has 15 teeth and the smaller 13 teeth? Also what force is required to lift 5 cwt. if the efficiency be 28 per cent.?

Ans. 15 and 133.3 lbs.

12. The following results were obtained in a test of a wheel and compound axle in which the velocity ratio is 25 to 1:—

Load raised (lbs.)	0	50	100	150	200	250	300
Force exerted (lbs.)	5	10	15	21	28	36	45

Calculate the mechanical advantage and the efficiency for each load, and plot the results on squared paper to a base line of load lifted, showing load, advantage, and efficiency curves.

13. In a pulley block tackle with a velocity ratio of 6 to 1, it is found by experiment that a force of 10 lbs. can just lift a load of 30 lbs., and a force of 50 lbs. a load of 170 lbs. Find the mechanical efficiency of the tackle at each of these loads, and ascertain the probable efficiency when lifting 1 cwt.

CHAPTER X.

TRANSMISSION OF POWER.

THE particular method adopted for the transmission of power from the prime mover to the machine, or from one shaft to another shaft, will depend very much upon the type of machine for which the power is required, the speed of rotation, and the room available. For general factory purposes, belts and ropes are perhaps most frequently employed as between the prime mover and the first driven shaft; but for some special purposes wheel gearing is more applicable. For many purposes direct transmission is effected through couplings and clutches.

Different methods of transmission are represented in fig. 131, and the rate per minute at which work is transmitted is

$$\frac{2\pi N \times \text{torque}}{33000} = \text{Horse-power.}$$

Power transmitted by Ropes.—Where the power to be transmitted is large, say, from 50 H.P. and upwards, a number of cotton or manilla ropes are frequently used, and the practical problem resolves itself into a calculation to determine the diameter and number of ropes to transmit the given horse-power at the required speed.

Let n = number of ropes,

T = maximum working tension in lbs. on the driving side of each rope,

t = tension on the slack side of the rope,

V = velocity of ropes in feet per minute,

H.P. = maximum safe horse-power to be transmitted;

then, as each rope gives a driving force of $(T - t)$ lbs., if there are n ropes, the total driving force exerted will be $n(T - t)$ lbs. This force is exerted through a distance of V feet per minute, so that the total work done per minute is

$$n(T - t)V \text{ ft.-lbs.}$$

Methods of Power Transmission.

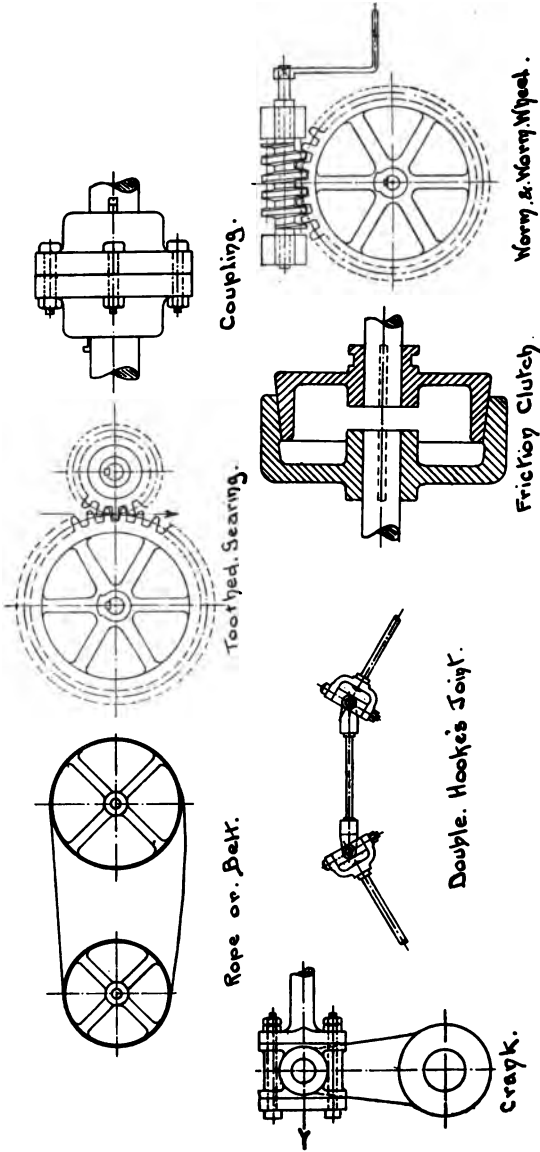


FIG. 181.

The power transmitted, therefore, will be

$$\text{H.P.} = \frac{n(T-t) \times V}{33000},$$

and

$$n = \frac{\text{H.P.} \times 33000}{V(T-t)}.$$

Let R denote the radius of one of the pulleys round which the ropes drive, and N the number of revolutions made by it per minute.

Then

$$V = 2\pi RN$$

and therefore

$$\text{H.P.} = \frac{n(T-t) \times 2\pi RN}{33000}.$$

But $R(T-t)n$ gives the torque or turning moment, so that

$$\text{H.P.} = \frac{2\pi N \times \text{torque}}{33000}, \text{ as stated above.}$$

Now, established practice indicates that for the best cotton driving ropes, and for speeds of from 3000 to 4500 ft. per minute (the usual range of speed), the value of $T-t$ can be expressed by $100 \times d^2$, when d denotes the diameter of the rope in inches. For higher speeds or ropes of the poorer quality, a lower value of $T-t$ should be taken, say $T-t = 50d^2$. Substituting $100d^2$ for $T-t$ in above formula, we obtain

$$\begin{aligned} n &= \frac{\text{H.P.} \times 33000}{V \times 100d^2} \\ &= 330 \frac{\text{H.P.}}{d^2 V}. \end{aligned}$$

For example, an engine of 200 B.H.P. is required to transmit power from a rope fly-wheel 14 ft. diameter running at 80 revolutions per minute. How many ropes $1\frac{1}{2}$ -in. diameter will be required?

$$\text{Here} \quad \text{H.P.} = \frac{n(100 \times d^2) \times V}{33000};$$

$$\text{or} \quad 200 = \frac{n(100 \times 1\frac{1}{2}^2) \times \pi \times 14 \times 80}{33000};$$

$$\begin{aligned} \text{and} \quad n &= \frac{200 \times 33000}{100 \times (1\frac{1}{2})^2 \times \pi \times 14 \times 80} \\ &= 8\frac{1}{2} \text{ ropes} \\ &= (\text{say}) \text{ 9 ropes.} \end{aligned}$$

Power Transmission by Belts.—For shorter drives, and for smaller powers, leather belts or bands are found more convenient for the transmission of power. These are termed single-ply or double-ply, according to their thickness, the former being about $\frac{3}{16}$ in. to $\frac{7}{16}$ in. thick, according to its width, and the latter being about $\frac{1}{2}$ in. thick.

The power transmitted by a belt obviously depends upon the frictional grip between the belt and the pulley, and the resulting difference of tensions between the tight side and the slack side of the belt.

Theory gives the expression

$$\log_e \frac{T}{t} = \mu \theta \quad \text{or} \quad \frac{T}{t} = e^{\mu \theta}$$

as representing the ratio of T to t (see p. 126), where μ is the coefficient of friction, and where θ is the angle of contact made by the belt on the pulley. μ has values of 0.2 to 0.4 for cotton or leather belts.

* If we take $\mu = 0.4$, $\theta = 180^\circ$, and $T = 80$ lbs. per inch width for single-ply belting, allowing for laced joints; then if w = width of belt, it can be shown that $T - t = 55w$. Substituting this value in the formula $\text{H.P.} = \frac{(T - t)V}{33000}$, we obtain the simple expression

$$\text{H.P.} = \frac{wv}{600}.$$

For double-ply belting take

$$\text{H.P.} = \frac{wv}{300}.$$

Solving these two equations, for w we get

$$w = 600 \frac{\text{H.P.}}{v} \text{ for single belt;}$$

$$w = 300 \frac{\text{H.P.}}{v} \text{ for double-ply.}$$

Example.—If 6 H.P. has to be transmitted from a pulley 3 ft. 6 in. diameter making 100 r.p.m., what width of single-ply belting would be required?

$$\text{Here} \quad V = 100 \times 3\frac{1}{2} \times 3\frac{1}{2};$$

$$\text{hence} \quad w = \frac{600 \times 6}{100 \times 3\frac{1}{2} \times 3\frac{1}{2}} = 3.27 \text{ in.,}$$

say $3\frac{1}{4}$ in. *Ans.*

* Professor Goodman's *Applied Mechanics of Engineering* (Longmans).

Power transmitted by Toothed Gearing.—Where the relative speeds of two shafts, the driver and the driven, must be a definite ratio, the use of belts or ropes is impossible, owing to the varying amount of slip which occurs between the pulleys and the belt or rope. Also in confined places it is sometimes impossible to obtain the requisite distance between the two shafts, and under these circumstances the use of toothed gearing of one kind or other becomes necessary. The power transmitted by toothed wheels will depend upon their size, speed, and material. It will also vary with the method of loading, that is, whether it may be assumed that the total driving force is bearing evenly upon two or more teeth at the same time, and whether the load per tooth is evenly distributed over the full breadth of the tooth, or only acting in places. It is safest to assume that the total driving force acts upon one tooth only at the time, for, though the force may theoretically be considered as acting upon two or even three teeth which may be in contact, yet, should there be any slight variation in the spacing out of the teeth, the load will be distributed unequally on them, and it may be that all the load will come upon one tooth. This is more necessary in the case of wheels cast from a pattern, and not machine cut, as the slight variations in contraction, rapping the pattern in the sand, or slovenly making up of the mould where broken, are quite sufficient to prevent the uniform distribution of the load upon the two or three teeth which would otherwise be in contact at one time. Also it is usual to assume the load to be equally distributed along the breadth of the teeth. Given these conditions, it is a comparatively simple matter to calculate the maximum safe pressure which should be allowed to act upon the ordinary tooth of a spur-wheel or rack.

Considering each tooth as a cantilever, fixed at one end and loaded at the other, where l represents the extreme length of the tooth from root to point, t its thickness at the root, and b its breadth or width on the face, we have

Bending moment = Moment of resistance

$$P \times l = f \times \frac{b \times t^2}{6},$$

$$\text{or} \quad P = f \times \frac{b \times t^2}{6l},$$

which gives the maximum safe load in terms of the safe stress allowable for the material, and b , t , and l the proportions of the tooth. As spur-gearing is usually of a somewhat fixed proportion, we can express b , t , and l in terms of the "pitch" p of the teeth, for b = about $3p$, t = $\frac{1}{2}p$, and l = $0.6p$.

Then the safe load $P = f \times \frac{p^2}{6} \frac{3 \times (\frac{1}{2})^2}{0.6},$

or $P = f \times \frac{p^2}{4.8}. \quad (\text{See pp. 134 and 135.})$

This gives the safe load in terms of the material and the pitch of the teeth. For cast-iron wheels, f , the safe stress allowable for a varying load, should not be taken at more than 1500 lbs.; for "shrouded" wheels it may be taken at about 1800 lbs. (due to the additional support given to the teeth by the external flanges or "shrouds"; and for cast-steel wheels f may be taken as high as 2500 lbs. per sq. in. For bevel wheels, as the pitch is measured at the large end of the wheel, f should be taken at about 0.8 of these values, to allow for the average pitch being smaller than the nominal pitch.

The horse-power transmitted will be represented by the formula

$$\text{H.P.} = \frac{P \times S}{33000}$$

where P = maximum safe load on teeth in lbs. (sometimes called the driving force or driving pressure),

S = speed of teeth of wheel in feet per minute.

Expressing P in terms of the material and pitch of teeth, we get:—

(1) *For cast-iron unshrouded wheels—*

$$\text{H.P.} = \frac{1500 \times \frac{p^2}{4.8} \times S}{33000} \quad \text{or} \quad \text{H.P.} = \frac{p^2 \times S}{105.6};$$

or (for all practical purposes) $\text{H.P.} = \frac{p^2 \times S}{110}.$

(2) *For cast-iron shrouded wheels—*

$$\text{H.P.} = \frac{1800 \times \frac{p^2}{4.8} \times S}{33000} \quad \text{or} \quad \text{H.P.} = \frac{p^2 \times S}{88}.$$

(3) *For cast-steel wheels (unshrouded)—*

$$\text{H.P.} = \frac{2500 \times \frac{p^2}{4.8} \times S}{33000} = \frac{p^2 \times S}{63.36},$$

or (practically) $\text{H.P.} = \frac{p^2 \times S}{60}.$

Power transmitted by Shafting.—When power is transmitted along or through a shaft, the latter is subjected to a torsional (or twisting) stress, and usually to a bending stress as well. If we consider the strength of a short piece subjected to a twisting stress, neglecting for the present the stress caused by the bending produced, the strength of the shaft is given by the fundamental rule—

Twisting moment = Moment of resistance of material,

$$\text{or} \quad P \text{ (lbs.)} \times r \text{ (ins.)} = f_s \times \frac{\pi}{16} \times d^3,$$

where P is the force in lbs. producing the twist,
 r is the radius at which P acts, in inches,
 f_s is the shearing stress produced, in lbs. per sq. in.,
 d is the diameter of the shaft, in inches.

A *safe value* of f_s for mild steel, subjected to a uniform twist, may be taken at 8000 lbs. per sq. in., and the value of $\frac{\pi}{16}$ may be taken at $\frac{1}{8}$ approximately (its exact value is 0.1963). These constants simplify the formulæ to

$$P \times r = 8000 \times \frac{1}{8} \times d^3,$$

$$\text{or} \quad P \cdot r = 1600d^3,$$

$$\text{or} \quad d = \sqrt[3]{\frac{P \cdot r}{1600}} = \frac{\sqrt{P \cdot r}}{11.7}.$$

The power transmitted by shafting, however, is usually expressed in horse-power transmitted at a given speed, so that, if

H.P. = horse-power transmitted,

N = number of revolutions per minute,

T = twisting moment produced = Pr ,

$$\text{then} \quad \text{Work transmitted in ft.-lbs.} = \frac{2\pi NT}{12}.$$

$$\text{H.P. transmitted} = \frac{2\pi NT}{12 \times 33000} = \frac{T \times N}{63025}.$$

But $T = 1600d^3$,

$$\therefore \text{H.P.} = \frac{1600d^3 \times N}{63025} \\ = \frac{d^3 N}{40} \text{ (approximately). (See p. 128.)}$$

From this simple rule, we learn the following two important relationships:—

(1) **The power which shafting is capable of transmitting varies as the cube of its diameter.** For example, a 2-in. diameter shaft will transmit 2^3 or 8 times as much power as a 1-in. shaft at the same speed; and a 3-in. diameter shaft will transmit 3^3 or 27 times as much power as a 1-in. shaft at the same speed.

(2) **The power which a shaft can transmit varies directly as its speed;** that is, for example, a shaft will transmit *twice* as much power if its speed is doubled, and only half as much power if its speed is halved.

The above rules apply to shafts which are subjected to a twisting stress only; in actual practice all shafts are subject to a bending stress as well as a torsional stress, and this makes the case a little more complicated. Consider the case of a length of line shafting in a factory, where the shaft has not only to withstand the twisting stress caused by the driving force, but also the bending caused by various belts or pulleys. For such a case the above formula can be modified to

$$\text{H.P.} = \frac{d^3 \times N}{100},$$

providing the number of pulleys and the various pulls of the belts are not excessive.

In the case of **crank-shafts for steam engines**, the conditions are again different. Here the torsional stress is constantly changing in magnitude during each revolution, owing to the varying torque, and the bending stress varies in character with each stroke, alternately tending to bend the shaft in opposite directions. For steam engines with a single crank, the rule

$$\text{H.P.} = \frac{d^3 \times N}{180}$$

has been found to answer well in practice. For steam engines with two cranks at right angles, the above rule can be modified to

$$\text{H.P.} = \frac{d^3 \times N}{160},$$

owing to the more even twisting moment on the shaft.

For **gas or oil engines**, or other internal combustion motors, where the power produced is in the form of an explosion, lasting only for a small portion of the stroke, and with a power stroke only once in every two revolutions, the crank-shafts have to be

made much larger to transmit the same power, and an approximate rule is

$$\text{H.P.} = \frac{d^3 \times N}{800}.$$

Examples.—(1) If a 1-in. diameter shaft will transmit 1 H.P. at 100 revolutions per minute, what horse-power will a 3-in. diameter shaft transmit at 80 revolutions per minute?

Here a 1-in. shaft at 100 revs. per min. transmits 1 H.P.

Then a 3-in. ,, 100 ,, ,, $\frac{1 \times 3^3}{1}$ H.P.

Or a 3-in. ,, 80 ,, ,, $\frac{1 \times 3^3 \times 80}{1^3 \times 100}$.

$$\therefore \text{H.P.} = \frac{1 \times 27 \times 80}{1 \times 100} = 21.6 \text{ H.P.} \quad \text{Ans.}$$

(2) If a 2-in. diameter shaft running at 100 revolutions per minute can safely transmit 8 H.P., what size of shaft is required to transmit 23.5 H.P. at 150 revolutions per minute?

Let d = diameter of shaft required.

Then, considering speed only, by proportion,

$$\text{as } 150 : 100 :: 2^3 : d^3;$$

and, considering power only, by proportion,

$$\text{as } 8 \text{ H.P.} : 23\frac{1}{2} \text{ H.P.} :: 2^3 : d^3;$$

then, combining the two proportions, we get—

$$\text{as } \left. \begin{array}{l} 150 : 100 \\ 8 : 23\frac{1}{2} \end{array} \right\} :: 2^3 : d^3,$$

$$\text{or} \quad d^3 = \frac{2^3 \times 100 \times 23\frac{1}{2}}{150 \times 8} = 15\frac{5}{8}.$$

$$d = \sqrt[3]{15\frac{5}{8}} = \sqrt[3]{\frac{125}{8}} = \frac{\sqrt[3]{125}}{\sqrt[3]{8}} = \frac{5}{2}$$

$$= 2\frac{1}{2} \text{ in. diam.} \quad \text{Ans.}$$

Additional exercises on power transmission are given in Chapter IV.

EXERCISES.

1. Calculate the number of cotton ropes required to transmit power from an engine giving 100 H.P. on the brake to a dynamo, having given the diameter of the fly-wheel pulley on the engine = 8 ft., and the diameter of the rope $1\frac{1}{4}$ in. revolutions per minute = 200.

Ans. $4.15 = (\text{say}) 5$ ropes.

2. Work is done by means of a belt driving a pulley 250 revolutions per minute at the rate of $12\frac{1}{2}$ H.P. If the pulley is 12 in. diameter, calculate the difference in tensions between the tight side and slack side, and assuming a ratio of tensions of $2\frac{1}{2} : 1$, calculate the width of belting required.

Ans. $4\frac{1}{2}$ in.

3. Find the width required for a single-ply belt which would transmit 5 H.P. from a pulley 6 in. diameter, rotating at the rate of 750 revolutions per minute.

Ans. 2.55 in.

4. A vessel is steaming at speed $15\frac{1}{2}$ knots an hour. If the resistance to its motion is 50,000 lbs., what is the twisting moment on the propeller shaft, rotating at 120 revolutions per minute?

Ans. 1,250,000 lb.-ins.

5. A mild-steel shaft is required to transmit 75 H.P. at 120 revolutions per minute. What would be a reasonable diameter (neglecting bending)?

Ans. 2.93 in. = (say) 3 in. diameter.

6. Calculate the H.P. that can be transmitted by a wrought-iron shaft $3\frac{1}{2}$ in. diameter, rotating 100 revolutions per minute, taking $f_s = 7500$ lbs. per sq. in.

Ans. 102.

7. If a 3-in. shaft will transmit 21 H.P. at 80 revolutions per minute, what size of shaft will transmit 1 H.P. at 150 revolutions per minute?

Ans. 0.883 = (say) 1 in. diameter.

8. A shaft transmits 35 H.P. at 75 revolutions per minute. What is the mean value of the twisting moment?

Ans. 29,410 lb.-ins

CHAPTER XI.

CENTRIFUGAL FORCE AND BALANCING.

Motion in a Circle.—When a body is compelled to move in a circle, its tendency at any position is to fly off at a tangent to the circle, the tangent being the direction or straight line in which the weight is moving at the instant considered. The compelling force acting on the weight or body acts radially towards the centre of the circle in order to compel the body to describe the circle. The radial reaction of the body away from the centre is called its “centrifugal force,” being a direct pull exerted on the axis of rotation. It can be proved that, if the body is rotating in a circle, with a uniform velocity v feet per second, it is constantly accelerated towards the centre by an amount equal to $\frac{v^2}{r}$, where r is the radius of the circle in feet per second. The force F required to produce this acceleration will be equal to the mass multiplied by the acceleration (see Chapter XII.)

$$F = \frac{W}{g}f = \frac{Wv^2}{gr},$$

where W is the weight of the rotating body in lbs., and g the acceleration due to gravity (32.2 feet per second per second). We thus arrive at an expression which gives us a means of solving many practical problems in engineering. A few practical applications of this formula are given.

Example 1.—The centre of gravity of a body weighing 100 lbs. is revolving at 15 in. from an axis at 250 revolutions per minute. What is its centrifugal force?

$$F = \frac{Wv^2}{gr},$$

$$v = \frac{250}{60} \times 2\pi \times 1\frac{1}{4} = 32.7 \text{ ft. per second.}$$

$$\therefore F = \frac{100 \times (32.7)^2}{32.2 \times 1.25} = 2660 \text{ lbs. (approx.).}$$

Effect of Unbalanced Weights.—The above example serves to show the importance of balancing rotating masses running at a high speed of rotation, as otherwise even a very small unbalanced mass moving at considerable speed will be sufficient to set up vibration, produce straining actions, and shake fastenings loose.

Example 2.—A pulley making 420 revolutions per minute has an unbalanced set screw weighing $\frac{1}{2}$ lb. fixed at a distance of 3 ft. from the axis of the shaft. What pull will this bring to bear on the bearings of the shaft?

$$v = \frac{420}{60} \times 2 \times \pi \times 3 = 132 \text{ ft. per second,}$$

$$F = \frac{0.5 \times (132)^2}{32.2 \times 3} = 90 \text{ lbs.}$$

Example 3.—In connection with a petrol engine flywheel, it was found that an unbalanced weight of 1 lb., acting at 12 in., and running at 2000 revolutions per minute, was productive of serious trouble. What pull would this bring to bear upon the shaft?

$$v = \frac{2 \times \frac{22}{7} \times 1 \times 2000}{60} = 209.5.$$

$$F = \frac{1 \times (209.5)^2}{32.2 \times 1} = 136 \text{ lbs.}$$

This will be quite sufficient to illustrate the great importance of correct balancing.

To Balance a Single Rotating Weight.—The problem of balancing complex rotating and reciprocating masses (even approximately) is not altogether a simple one, and to deal with this problem efficiently would take us beyond the scope of the present work. It is sufficient for our present purpose that the importance of balancing should be made clear. Let W represent a weight rotating at a distance R from the axis of rotation. It is required to balance the pull on the axis by means of a counter-balance weight acting on the other side of the radius produced at a distance r from the axis of rotation.

Let these two weights revolve N revolutions per minute. Then for a perfect balance the outward pull or centrifugal force of each mass must be the same.

Let V denote the circumferential velocity of the large weight W , and v the circumferential velocity of the small one; then

$$V = \frac{2\pi RN}{60} = \frac{\pi RN}{30},$$

$$v = \frac{2\pi rN}{60} = \frac{\pi rN}{30},$$

$$\frac{WV^2}{gr} = \frac{W\left(\frac{\pi RN}{30}\right)^2}{gR},$$

$$\frac{wv^2}{gr} = \frac{w\left(\frac{\pi rN}{30}\right)^2}{gr},$$

$$\frac{W\left(\frac{\pi RN}{30}\right)^2}{gR} = \frac{w\left(\frac{\pi rN}{30}\right)^2}{gr};$$

i.e., simplifying,

$$W \times R = w \times r,$$

$$w = \frac{WR}{r},$$

$$r = \frac{W}{w}R.$$

Example 4.—An engine crank disc has an unbalanced crank pin weighing 10 lbs., fixed at 2 ft. from the axis of rotation. If the engine makes 200 revolutions per minute, what pull would this exert on the shaft, and what weight fixed at 18 inches would balance this pull?

$$V = \frac{2 \times 22 \times 2 \times 200}{7 \times 60} = 41.9 \text{ ft. per second.}$$

$$\text{Pull} = \frac{10 \times (41.9)^2}{32.2 \times 2} = 271 \text{ lbs.}$$

If w denotes the balance weight

$$w \times 18 = 10 \times 24,$$

or
$$w = \frac{10 \times 24}{18} = 13\frac{1}{3} \text{ lbs.}$$

Tensile Stress in Flywheel Rims caused by Centrifugal Forces.

—We have seen that the centrifugal force set up in a rotating body

varies as the square of the speed, the weight of the body, and inversely as the radius of gyration. To find the tensile stress caused by this centrifugal force in the rims of wheels and pulleys rotating at high speed, let p (fig. 132) represent the pull caused in the rim per 1 inch length of rim, r the radius to centre of

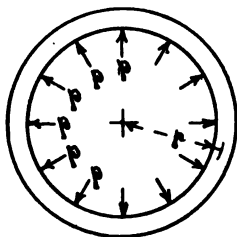


FIG. 132.

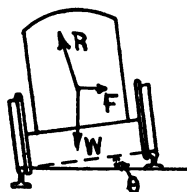


FIG. 133.

gyration in feet, and w the weight of a portion of the rim of the wheel 1 inch long. Then

$$P = \frac{wv^2}{gr}.$$

Let b = breadth of wheel in inches, and d = depth of rim in inches (assuming a rectangular section of rim); then $w = bd \times 1 \times \rho$ where ρ denotes the weight of a cubic inch of the material. Substituting in the above equation we obtain

$$p = \frac{bd\rho v^2}{gr}.$$

But since $12rp = bdf_t$ (where f_t denotes the intensity of the stress per sq. in. in the material), therefore

$$12r \frac{db\rho v^2}{gr} = bdf_t,$$

or

$$\frac{12\rho v^2}{g} = f_t.$$

If the material is of cast iron, $\rho = 0.26$, and

$$\therefore f_t = \frac{v^2}{10} \text{ (practically).}$$

We thus see that the tensile stress is entirely independent of the section of the rim, width of face, or thickness, but varies only as the square of the velocity of its centre of gyration. For all practical purposes, this latter may be taken as the centre of area of cross section of rim for ordinary flywheels or pulleys.

Having fixed f_s , the safe tensile stress permissible in the material, we can calculate the maximum velocity thus. For cast-iron f (safe stress) = $1\frac{1}{4}$ tons = 2800 lbs.; therefore, to find v , we have

$$2800 = \frac{v^2}{10} \quad \text{or} \quad v^2 = 28000,$$

$$\text{or} \quad v = 167 \text{ ft. per second.}$$

To allow for any initial stress that may remain in the wheel owing to unequal contraction in cooling, the usual velocity allowed is practically one-half of this, or 80 to 100 ft. per second.

Example 5.—Find the tensile stress due to centrifugal force in the rim of a flywheel, 10 ft. mean diameter, running at 200 revolutions per minute.

$$\text{Here} \quad v = \frac{200}{60} \times \pi \times 10 = 105 \text{ ft. per second.}$$

$$f_t = \frac{(105)^2}{10} \text{ (approx.)} = 1100 \text{ lbs. per sq. in.}$$

Example 6.—Find the limiting speed in revolutions per minute of a cast-iron wheel with a radius of gyration of 3 ft. 6 in., so that the tension in the rim due to centrifugal force shall not exceed 1 ton per square inch.

Let v = limiting velocity in feet per second ;

$$\text{Then} \quad 2240 = \frac{v^2}{10} \quad \text{or} \quad v = \sqrt{22400} = 150 \text{ ft. per second.}$$

Let N = revolutions per minute corresponding with this speed ;

$$\text{then} \quad \frac{N}{60} \times 2\pi \times 3.5 = 150 ;$$

$$\text{or} \quad N = \frac{60 \times 150}{2 \times \pi \times 3.5} = 410 \text{ revolutions per minute.}$$

Effect of Trains rounding Curves.—The centrifugal force set up by a train moving in a curve is counteracted by raising the outer rail, as shown in fig. 133. To find the angle θ for any given

speed, if F is the centrifugal force per ton of weight, and r the radius of the track in feet, then

$$F = \frac{Wv^2}{gr};$$

but $\frac{F}{W} = \tan \theta$ (see triangle of forces, fig. 133),

$$\text{or } \frac{\frac{Wv^2}{gr}}{W} = \tan \theta = \frac{v^2}{gr}.$$

If the train is going at 30 miles per hour ($v = 44$ ft. per second) and the radius is, say, 1000 ft.,

$$\tan \theta = \frac{(44)^2}{32.2 \times 1000} = 0.06 \text{ or } \theta = 3.26^\circ.$$

The amount, therefore, that the outer rail is laid above the inner rail, for standard gauge (4 ft. 8½ in.),

$$0.06 \times 56.5 \text{ (in.)} = 3.39 \text{ in.}$$

Example 7.—A train describes a horizontal circle of 250 yards radius at a speed of 40 miles per hour: what is the ratio of its centrifugal force to its weight? If the gauge be 1 metre, find the amount the outer rail should be laid above the inner rail.

$$v = \frac{40 \times 5280}{60 \times 60} = 58.7 \text{ ft. per second.}$$

$$\frac{F}{W} = \frac{v^2}{gr} = \frac{58.7^2}{32.2 \times 250 \times 3} = 0.142.$$

$$0.142 \times 39.3 \text{ (in.)} = 5.6 \text{ in.}$$

EXERCISES.

1. A segment of a fly-wheel, with the arm to which it is attached, weighs 3500 lbs., and its radius of gyration is 8 ft. The wheel makes 40 revolutions per minute. Find the force tending to pull the segment and arm from the boss of the wheel.

Ans. 15,240 lbs.

2. A fly-wheel, 30 ft. in diameter, runs at 80 revolutions per minute. Find the tension in its rim due to centrifugal force.

Ans. 1580 lbs. per sq. in.

3. A disc, rotating on a shaft at 200 revolutions per minute, carries an unbalanced pin weighing 5 lbs. at a distance of 1 ft. from the centre of the shaft. Find the pull on the bearings due to this unbalanced mass, and the volume (in cubic inches) of a cast-iron balance weight placed at 8-in. radius on the other side of the disc to balance the pin. *Ans.* 68 lbs. and 29 cub. in.

4. A wheel weighing 20 tons has its centre of gravity at a distance $\frac{3}{4}$ in. from the axis of the shaft on which it is mounted. If it runs at 80 revolutions per minute, find the pull on the bearings of the shaft. *Ans.* 3050 lbs.

5. Find the size of a counter-weight of lead, to be fixed at a distance of 10 ft. from the centre of the shaft, required to balance the wheel in Question 4, and make a sketch showing the exact position of the weight. *Ans.* 560 lbs.

6. Find the limiting speed (in revolutions per minute) of a motor fly-wheel 15 in. diameter, so that the tension in the rim of the wheel shall not exceed 1800 lbs. per sq. in. *Ans.* 2050.

7. A leather belt weighing $1\frac{1}{2}$ lbs. per foot-run transmits power by driving on to a pulley 7 ft. diameter, making 120 revolutions per minute. Find the tension in it by reason of its centrifugal force.

8. A train of 250 tons weight is rounding a curve of 400 yards radius at a speed of 25 miles per hour. What is its centrifugal force? If the rails are specially laid for this speed, what should be their difference in height, assuming a gauge of 4 ft. $8\frac{1}{2}$ in.?

CHAPTER XII.

ACCELERATION, MOMENTUM, FORCE-ACTION.

MANY engineering problems require for their solution a more particular knowledge of the action of force upon matter than has hitherto been dealt with in this work. We shall include the present chapter for two reasons—(a) to give the practical engineer the information which will enable him to deal with such problems as may present themselves to him in connection with his practical work; (b) because it is highly important and necessary that an engineering student should at the commencement of his studies form a clear idea of what is implied in the use of the terms *mass*, *force*, *acceleration*, *momentum*, and of the fundamental units in terms of which these are expressed.

Momentum.—We must first notice that, if a force of definite magnitude act upon a body free to move, it produces in the body a definite change in the *quantity of motion* it possesses, and the change of motion takes place in the direction in which the force acts. If the body is initially at rest with respect to surrounding objects, then the quantity of motion with respect to its environment is *nil*, and any motion then imparted to it by the action of a force would be the change in the quantity of motion. The quantity of motion possessed by a moving body is called its *momentum*, and its magnitude at a given instant is found by multiplying the mass of the body into the velocity at which the body is moving at the instant considered. Thus, if Q denotes quantity of momentum, m mass, and v velocity, then

$$Q = mv.$$

If, however, the body is moving at a given instant with a given initial velocity v_1 , and a force act upon it so that its velocity is increased to v_2 , then the change of momentum (Q_c) may be expressed symbolically by

$$Q_c = m(v_2 - v_1).$$

The term *mass* used above is an abbreviation denoting **quantity**

of matter, which, for ordinary purposes in this country, is estimated in terms of the standard pound avoirdupois, or multiples and sub-multiples of this. This standard pound, therefore, constitutes the **unit of mass**. The engineer, however, often uses the **larger unit of 32.2 lbs.**, and a body of this weight is said to have unit mass. Thus, if W represents the mass (weight) in lbs., and M the mass in terms of the engineer's unit, then

$$M = \frac{W}{32.2}.$$

*what a useless
circumlocution*

Velocity is usually estimated in feet per second, though other units may sometimes be adopted in calculations on momentum.

The expression for quantity of motion or momentum may now be written

$$Q = MV = \frac{WV}{32.2}.$$

Or, again, if V_1 be the initial velocity, and V_2 the velocity after some force-action on the body, then change in momentum,

$$Q_c = \frac{W(V_2 - V_1)}{32.2}.$$

Example 1.—If the table of a planing machine weighing 3500 lbs. moves at a speed of 25 ft. per minute on the cutting stroke, what will be its momentum?

Substituting the values of mass and velocity in the expression

$$Q = \frac{WV}{32.2},$$

we obtain $\frac{3500 \times 25}{32.2 \times 60}$ or **453 units of momentum.**

why not state them.

Example 2.—Suppose the table referred to in the previous example attains a velocity of 30 ft. per minute after the tool runs off at the end of the cut. What would then be the change of momentum?

Momentum of table when moving at 25 ft. per minute = 453 units.

$$\begin{array}{rclclcl} \text{,,} & \text{,,} & \text{,,} & 30 \text{ ft.} & \text{,,} & = \frac{3500 \times 30}{32.2 \times 60} \\ & & & & & = 543. \end{array}$$

The increase in momentum is then given by

$$543 - 453 = 90 \text{ units.}$$

The change in momentum could have been obtained directly by substitution in the formulæ already given ; thus

$$Q_c = \frac{W(v_2 - v_1)}{32.2} = \frac{3500(30 - 25)}{32.2 \times 60} = 90 \text{ units.}$$

The momentum or change of momentum can also be expressed numerically in terms of the magnitude of the force producing the change and the time during which it acts. Later on we shall show reason for the statement that momentum or change of momentum equals

Force $\times$ time,

and also that **Force $\times$ time = Mass $\times$ velocity.**

This may be written symbolically

$$\mathbf{F \times T = m \times v.}$$

For practical calculations these four quantities would have to be expressed in suitable units, and the formula

$$\mathbf{F \times T = \frac{W \times V}{32.2} ,}$$

where F is the magnitude of the force in lbs. producing the change in momentum, T the time in seconds during which the force acts, W the weight of the body in lbs., and V the velocity in ft. per second. The product on the first side of the equation ($F \times T$) is called by the engineer the *impulse of the force*, and the principle embodied in the equation may be expressed by the statement that *the change of momentum is numerically equal to the impulse which produces it.*

Example 3.—A sudden shove whose average value is 2000 lbs. is given by an engine to a truck, the buffers remaining in pushing contact for half a second. What is the impulsive action of the force with respect to the momentum it is capable of imparting to the truck ?

Impulsive action measured by $F \times T = 2000 \times \frac{1}{2} = 1000 \text{ units.}$

Example 4.—If the truck referred to in the previous example weighs 10 tons, what momentum would be given to it, and what would be its velocity just after leaving the buffers ?

Neglect resistance offered to motion during impact. Momentum given to truck = impulse of the force = 1000 units.

To find velocity, let V denote velocity just after impact; then momentum possessed by truck

$$= \frac{10 \times 2240 \times V}{32.2} = 1000.$$

$$\therefore V = \frac{1000 \times 32.2}{10 \times 2240} = 1\frac{1}{16} \text{ ft. per second.}$$

Having given any three of the four quantities F , T , W , V , the fourth can be determined.

Example 5.—Suppose a train weighing 300 tons to be running at a given instant at 30 miles an hour, when the steam is suddenly shut off and the brakes applied, so that the train is brought to rest in 6 seconds. Express the total mean resistance to motion offered by brakes, friction, etc., in the direction of motion as a fraction of the weight of the train.

pretty rapid work!

Let F denote the resistance in tons, then, impulsive action opposing motion $= F \times 6 \times 2240$, and this must equal the momentum of the train.

$$\text{Hence} \quad 2240 \times F \times 6 = \frac{300 \times 2240 \times 44}{32.2},$$

44 being the velocity, reduced to feet and seconds, at which the train is moving.

In this equation 2240 occurs on both sides, so that it is obvious that the formula

$$F \times T = \frac{W \times V}{32.2}$$

is true whether F and W are taken in lbs. or tons or in any other units, so long as both are expressed in the same terms. Thus

$$F \times 6 = \frac{300 \times 44}{32.2},$$

and, consequently,

$$F = \frac{300 \times 44}{32.2 \times 6} = 68.3 \text{ tons};$$

that is to say, brakes and other resistances would be equivalent to a direct opposing force of 68.3 tons acting for 6 seconds, and this would be sufficient to bring the train to rest. Notice that

$$\frac{68.3}{300} = 0.228 \text{ of the weight of the train.}$$

Example 6.—Water is scooped up into the tank of a locomotive tender while the engine is running. If the head or height of delivery is 8 ft., and the frictional resistance in the pipe equal to one-third of the head, what will be the lowest speed at which the engine can run to make the feeding of the tanks possible?

The total pressure per square inch to be overcome during the scooping = weight of column of water 8 feet high + one-third weight of column of water 8 ft. high

$$= \frac{8 + \frac{1}{3} \text{ of } 8}{2.3} = 4.64 \text{ lbs. per sq. in.}$$

since a head of water of 2.3 ft. gives 1 lb. pressure per sq. in.

Now, if V denotes minimum speed of the engine in feet per second, then the momentum given to the water delivered through 1 sq. in. section of pipe

$$= \text{mass and velocity} = \frac{V}{2.3 \times 32.2} \times V$$

and this must equal

$$F \times T = \frac{4 \times 8}{3 \times 2.3} \times 1.$$

$$\text{Hence} \quad \frac{V^2}{2.3 \times 32.2} = \frac{4 \times 8}{3 \times 2.3}$$

$$\text{and} \quad V = \sqrt{\frac{4 \times 8 \times 32.2}{3}} = 18.54 \text{ ft. per second,}$$

or 12.65 miles per hour.

Acceleration.—If a constant force act upon a body free to move, it will produce a constant change in the velocity of the body during each second of time that the force acts. If the force acts so as to increase the velocity, then the increase or added velocity per second is called the **acceleration**. If, however, the force acts to oppose the motion of the body, then the decrease in each second is called **retardation** or **negative acceleration**. In elementary mechanics we use the formula $v = fT$ as giving the velocity imparted to a body at the end of T seconds by the action of a constant force— f denoting the acceleration. In this formula v may be said to be the total acceleration of velocity at the end of T seconds, and it is used in this connection in the equation $F \times T = m \times v$. If T be taken as 1 second, the equation becomes

$$F \times 1 = \frac{mv}{32.2};$$

or, since f is usually taken to denote the acceleration or added velocity per second, this result may be written

$$F = \frac{wf}{32.2};$$

that is to say, we obtain the value of the force in lbs., which, acting for 1 second on a mass of w lbs. produces an acceleration represented by f . From this result we get

$$f = \frac{F}{m} = \frac{32.2 \times F}{w}.$$

Example 7.—Two weights, A and B (fig. 134), whose masses are 200 and 180 lbs. respectively, are connected together by means of a cord passing over a pulley, so that as the heavier descends with a uniform acceleration the lighter one ascends with the same acceleration. Assuming that 10 lbs. at the circumference is sufficient to overcome the friction, find the acceleration of each weight.

The mass moved

$$= \frac{200 + 180}{32.2} = \frac{380}{32.2} \text{ units.}$$

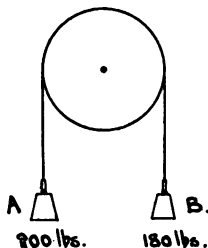


FIG. 134.

The force producing motion = $200 - 180 - 10 = 10$ lbs.; and since $f = \frac{F}{m}$, we get, by substitution,

$$\frac{10}{\frac{380}{32.2}} = \frac{10 \times 32.2}{380} = 0.92 \text{ ft. per second.}$$

Thus, A in descending or B in ascending will have its velocity increased by 0.92 ft. per second every second.

If the fundamental equation $F \times T = m \times v$ be divided throughout by T , we obtain,

$$\text{Force} = \frac{\text{Momentum}}{\text{Time}} = \frac{mv}{T} = \frac{WV}{32.2T},$$

that is to say, the numerical value of the force is equal to the rate of change of the momentum with the time. In other words,

$$\text{Force} = \text{Change of momentum per second.}$$

Example 8.—Taking the case of a planing machine where the table together with the work bolted to it weighs 5 tons, find the

time which must elapse in reducing its velocity of 20 ft. per minute to zero and reversing it to twice the speed in the opposite direction, assuming an average effective force between the rack and pinion of 200 lbs.

Momentum destroyed in reducing table to rest

$$= \frac{2240 \times 5}{32 \cdot 2} \times \frac{20}{60}.$$

Momentum restored in opposite direction on reversal

$$= \frac{2240 \times 5}{32 \cdot 2} \times \frac{20}{60} \times 2;$$

the total change of momentum

$$= \frac{2240 \times 5 \times 20 \times 3}{32 \cdot 2 \times 60};$$

and this equals force $\times$ time $= 200 \times T$.

$$\therefore T = \frac{2240 \times 5 \times 20 \times 3}{32 \cdot 2 \times 60 \times 200} = 1 \cdot 74 \text{ seconds.}$$

Example 9.—If a service pipe 60 ft. long is delivering water at a velocity of 50 ft. per second, what mean pressure would be produced if the valve is closed so that the water is brought to rest uniformly in one-fifth of a second?

Mass of water brought to rest per square inch section per second

$$= \frac{60}{2 \cdot 3 \times 32 \cdot 2}.$$

Momentum destroyed

$$= \frac{60}{2 \cdot 3 \times 32 \cdot 2} \times 50;$$

and this equals the pressure $P \times \frac{1}{2}$.

$$\therefore P = \frac{60}{2 \cdot 3 \times 32 \cdot 2} \times 50 \times 2 = 108 \text{ lbs.}$$

Example 10.—A cylindrical jet of water, $1\frac{1}{2}$ in. diameter, impinges on a fixed plane at right angles to its motion with a velocity of 25 ft. per second. What will be the pressure on the plane, taking a cubic foot of water to weigh $62\frac{1}{2}$ lbs.?

Since the force can be measured by the rate of change of momentum with the time, the pressure P on the plane is equivalent to the momentum destroyed per second, or

$$P = \frac{W \times V}{g},$$

where W = the weight of water brought to rest per second, and V = the velocity in feet per second, and g = the number 32.2.

Substituting, we get

$$\frac{\frac{3}{4} \times \frac{3}{4} \times \frac{22}{7} \times 25 \times 12 \times 62\frac{1}{2} \times 25}{1728 \times 32.2} = 14.9 \text{ lbs.}$$

Example 11.—Find what the pressure would be if the jet in the previous question impinging on a fixed plane at a given angle.

If v denote the velocity of the jet impinging on a fixed plane at an angle of θ , then the pressure on the plane is found by calculating the momentum destroyed per second in the direction at right angles to the plane.

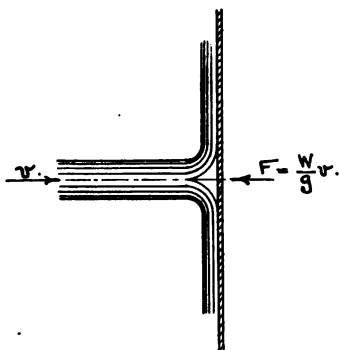


FIG. 135.

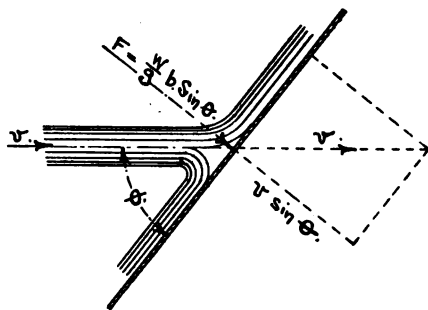


FIG. 136.

Resolving v along the normal, we have

$$v_n = v \sin \theta,$$

and therefore the pressure

$$P = \frac{W}{g} v \sin \theta.$$

Force or Pressure exerted when the Momentum possessed by a Moving Body is destroyed or reduced.—Suppose a body of mass m moving at a given instant with a velocity of v ft. per second to be opposed by a force F such that it is reduced to rest in the time T . The magnitude of the force we know to be given by the equation

$$F = \frac{mv}{T},$$

and an inspection of this formula shows us that the magnitude of the force to destroy the motion of the body increases as the duration of the time during which it acts is diminished. An example will make this very important principle clear.

Example 12.—An iron tup weighing 100 lbs. is suspended from a chain fixed above its centre of gravity, and is used for driving keys into the bosses of large wheels. It swings like a pendulum, and is pulled out by means of a rope and then allowed to swing back and strike the head of the key. On first striking the key the impulse of the blow is prolonged owing to the key sliding into its position. As the key is driven further and further in by repeated blows from the tup, less time is taken up in destroying the momentum of the tup. Assuming the striking velocity to be 8 ft. per second, find in each case the mean force of the blow when the time of impact is $\frac{1}{2}$ of a second, $\frac{1}{5}$ of a second, $\frac{1}{10}$ of a second, $\frac{1}{20}$ of a second, and $\frac{1}{100}$ of a second.

The mean force when the impact lasts for $\frac{1}{2}$ of a second is given by the equation

$$F \times \frac{1}{2} = \frac{100 \times 8}{32 \cdot 2}, \text{ whence } F = 50 \text{ lbs. (approx.).}$$

For $\frac{1}{5}$ of a second

$$F \times \frac{1}{5} = \frac{100 \times 8}{32 \cdot 2}, \text{ whence } F = 125 \text{ lbs.}$$

When $T = \frac{1}{10}$ of a second

$$F \times \frac{1}{10} = \frac{100 \times 8}{32 \cdot 2}, \text{ therefore } F = 250 \text{ lbs.}$$

Similarly, for $\frac{1}{50}$ of a second F works out to 1250 lbs., and for $\frac{1}{100}$ of a second 2500 lbs.

For $\frac{1}{200}$ of a second the force will be 5000 lbs. An inspection of the graphical representation of these results is most instructive.

In fig. 137 the different values of the forces and the times are plotted out to scale along the ordinates and abscissæ respectively. The scales used are shown.

A careful study of the example given and the graphical representation of the results should be sufficient to make clear to the student a very important principle with many applications in practical engineering. The curve obtained showing graphically the relationship between force and time is a rectangular hyperbola having an equation $F \times T = \text{a constant quantity}$, the constant quantity depending upon the magnitude of the impulse for any particular problem. In the example given this impulse or constant quantity

is in each case measured by the number 25, being the momentum destroyed, or $\frac{100 \times 8}{32.2}$. This is equivalent to $\frac{1}{2} \times 50 = \frac{1}{2} \times 125 = \frac{1}{10} \times 250 = \frac{1}{100} \times 2500$, and so on. The student will notice that the curve never meets either of the two axes, showing that (a), however great the force, the momentum of a moving body cannot be destroyed instantaneously, a definite interval of time being required; (b) as the time or duration of the

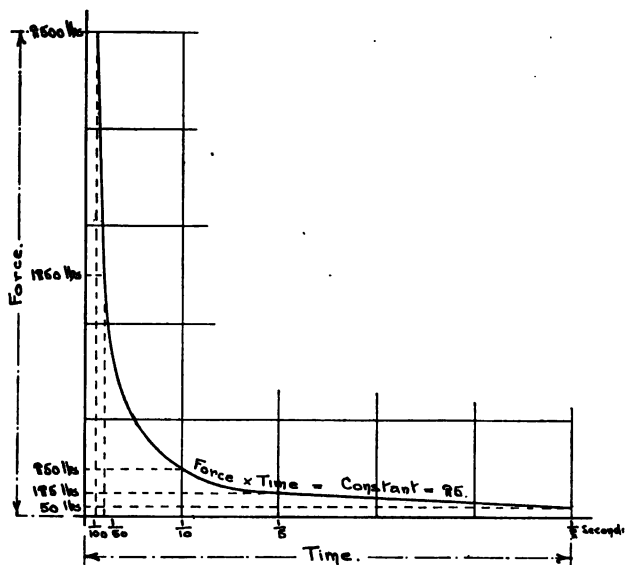


FIG. 137.

impact increases, the average value of the force to destroy the momentum diminishes.

Distinction between Mass and Weight.—We have already defined mass as quantity of matter. The unit of mass in terms of which a quantity of matter is measured was defined by Act of Parliament to be the imperial standard pound avoirdupois, and is the quantity of matter contained in the platinum weight deposited in the office of the Board of Trade in London. Copies of this and a number of multiples and sub-multiples of this, called **weights**, are made and distributed about the country; and quantity of matter, commonly called weight, is determined by balancing in an ordinary beam-scale against these

copies. The student should, however, clearly **distinguish between weight and mass**. Weight is essentially a force, and really measures the mutual attraction between the earth and a body whose weight is expressed. Thus, for example, if we speak of a body as weighing 5 lbs., we should understand that it is attracted towards the earth with a force five times as great as that exerted by the earth on the standard pound.

The earth's pull on the standard pound thus gives us a unit of force in terms of which to express other forces. This unit is the **British gravitation unit of force**. It is important for scientific accuracy that the place at which the earth's pull on the standard pound should be specified, for, as we shall see presently, this pull varies with the geographical position or latitude, and for exact calculation we require an invariable unit of force.

If, however, we defined the gravitation unit of force as the force equivalent to the earth's pull on the standard pound in London, we should obtain an absolute or invariable unit.

The process of weighing a body in an ordinary balance, the body being placed in the scale-pan at one end of the beam and the standard weight in the scale-pan at the other end, requires for a balance that the earth's pull on the body weighed or mass placed in the scale-pan shall be equal to the earth's pull on the standard weight in the other pan. Now, experiment shows that the earth's pull on a body, or the weight of a body, is independent of the kind of matter, bodies of equal masses placed in the same position of altitude or latitude relative to the earth being attracted equally; but that *at the same place the earth's pull or weight is proportional to the mass*. Hence the masses of bodies may be compared at any particular place by comparing their weights, or the earth's attractive force on them. It is obvious, therefore, that, when weighing with an ordinary scale balance, we

(1) Express the earth's pull at the particular place in which the weighing is done in terms of the earth's pull on the standard pound at the same place.

(2) Express the mass or quantity of matter in the body in terms of the mass or quantity of matter in the standard pound.

We have already pointed out that the earth's pull on a standard weight, although always the same for a given position with respect to the earth, varies with the latitude, and with the altitude above the surface of the earth; but this variation would not affect the accuracy of any determination of mass by weighing in an ordinary lever balance, because, as the earth's pull on the standard weight varies with its geographical position, so also the earth's pull on the body weighed would vary at the same rate. Thus, for example, a pound of steel at the poles would have the same

quantity of matter or mass as a pound of steel weighed in London ; but the earth's attractive force on the pound mass of steel would be different in the two places. The earth's pull on the body at the poles is about 0.2 per cent. greater than the earth's pull at the equator. This might be shown by means of a delicately made spring balance having a scale-reading decimally divided. The earth's pull on a weight would then be balanced against the extension of the spring.

Suppose such a spring balance be made and calibrated in London by suspended weights which are multiples or sub-multiples of the pound weight. Then if the balance and the standard pound weights were taken to different latitudes, it would be found that the balance would show increasing readings for the same weight in passing from the equator to the poles.

The usual method of determining the actual pull exerted by the earth on the unit of mass, and consequently on any mass, is to find the value of g , or the acceleration of a body falling freely under the influence of gravity, by means of a swinging pendulum. We shall not, however, trouble the student with the details of this method.

It can be shown that *the ratio of the earth's pull on a given body to the value of g is invariable for all places on the earth's surface*, and that this ratio is equal to the mass as already defined. Thus if w_1, w_2, w_3 , etc., represent the earth's attractive force on a given body at three different places, and g_1, g_2, g_3 the corresponding values of g at those places, then we get the relationship—

$$\frac{w_1}{g_1} = \frac{w_2}{g_2} = \frac{w_3}{g_3} = \text{mass} = \frac{\text{* weight by ordinary balance}}{32.2}.$$

Consequently, if we estimate the mass at a place by balancing the body at one end of a lever against standard weights at the other, and we call this weight w , then the mass in engineering units would be $\frac{w}{32.2}$, and this is true for any place on the earth's surface. If,

however, we estimate the earth's actual pull on the body, which we have shown to be slightly different from the result we should get by weighing in the ordinary way, and divide this by the acceleration due to gravity at the particular place, we should still obtain the same numerical value for the mass given by weight per balance $\div 32.2$.

Laws of Motion, Force Measurement.—We shall now state Newton's laws of motion and what is implied by them,

* 32.2 is the value of g for London.

though the student will find, if he has worked through the exercises already given, that he has mastered the principles formally stated sufficiently well to enable him to deal with many interesting problems. We shall also have to consider force-action from a strictly scientific point of view.

First Law.—*Every body continues in its state of rest or of uniform motion in a straight line, except in so far as it may be compelled to change that state by an impressed force.* This law is merely a statement of the **inertia of matter**, or the inability of a body to change its own state of rest or of motion unless acted upon by some force. The first part of the law, viz. that bodies at rest remain at rest unless acted upon by some external or impressed force, the student will find no difficulty in realising, as it agrees with his general experience. The second part of the statement, to the effect that a body moving with a uniform velocity in a straight line will continue with the same linear uniform velocity *ad infinitum*, unless acted upon by a force, although not quite so obvious on the face of it, will yet appear as obvious after a little reflection. A truck shunted along a line with a definite velocity comes to rest owing to the opposition of the forces and friction or other resistances, and it is fairly obvious that, the less the friction and resistance, the further the truck will go along the rails before coming to rest; and if the friction and resistance were reduced to *nil*, the motion could be continued indefinitely if the rails were continued indefinitely.

This first law may be extended to include rotating bodies, for it is equally true that a rotating body, such as a fly-wheel, could only be brought to rest by the action of some force, when rotation is once established.

Second Law.—*Change of quantity of motion or momentum is proportional to the force, and the time the force acts, and takes place in the direction in which the force acts.* This law has been illustrated by a number of examples, but we shall deal a little more fully with its full signification.

Taking the statement as it stands, we are enabled by it to compare forces in terms of the change of momentum they produce. Thus, suppose two forces whose magnitudes are F and F_1 act for the same interval of time on two bodies A and B, whose masses are M and M_1 respectively. Let the change in A's velocity be denoted by V and the change in B's velocity by V_1 , then by the law—

$$F : F_1 :: MV : M_1V_1 \quad \text{or} \quad \frac{F}{F_1} = \frac{MV}{M_1V_1}.$$

If $M = M_1$, that is to say, if the two masses are equal, then

$$\frac{F}{F_1} = \frac{V}{V_1}.$$

In other words, the total accelerations produced, V and V_1 , are directly proportional to the force. This statement is true for any interval of time, as for 1 second; so that, using f and f_1 for the change in velocities per second, we get

$$\frac{F}{F_1} = \frac{f}{f_1}.$$

These results become of practical value when suitable units are chosen in which to measure force, mass, and acceleration, or velocity.

First, with respect to the unit of force. The **British absolute unit of force** is called the **poundal**, and is a force of such magnitude that, acting for 1 second on a mass of 1 lb., produces in it a velocity or acceleration of 1 ft. per second; so that, if a force of F poundals imparts to a mass of M lbs. in 1 second a change of velocity which is represented by f , then, comparing this force with the unit force of 1 poundal, we get

$$F : 1 :: Mf : 1 \times 1.$$

From this proportion we get $F = Mf$.

Example 13.—A mass of 5 lbs. is acted upon by a constant force, the acceleration produced being 10 ft. per second. What is the magnitude of the force (*a*) in poundals; (*b*) in lbs.? Compare the magnitude of the force of 1 lb. with the force of 1 poundal.

Substituting in the above formula,

$$F = 5 \times 10 = 50 \text{ poundals.}$$

Substituting in formula previously given,

$$F = \frac{5 \times 10}{32 \cdot 2} = 1 \cdot 537 \text{ lbs.}$$

Since we are dealing with the same force, which is 50 poundal absolute units of 1·537 lbs., it follows that 1 poundal is equivalent

to
$$\frac{1 \cdot 537}{50} = 0 \cdot 03075 \text{ of a lb.,}$$

or $0 \cdot 03075 \times 16 = 0 \cdot 492$ of an ounce—say, roughly, $\frac{1}{2}$ oz.

Example 14.—If a force acting for 1 minute on a mass of 5 lbs. gives to it a velocity of 10 ft. per second, what velocity would be given to it at the end of half an hour?

By the second law the change of momentum is proportional to the time the force acts, and therefore, since the mass is constant, the velocity will vary directly as the time.

Hence the required velocity is 10×30 , or **300 ft. per second.**

Example 15.—Equal forces, of magnitude 10 and 10 lbs., act for the same time, 5 seconds, on two bodies A and B, whose masses are 10 and 50 lbs. respectively. What will be the ratio of their velocities at the end of this time?

Since the impulses of the two forces are the same (10×5), the momenta given to the bodies A and B will be equal, and it follows, therefore, that the velocities will be in the inverse ratios of the masses. Hence

$$\text{Velocity of A : velocity of B} :: 50 : 10.$$

So that A at the end of 5 seconds will be moving at 5 times the velocity of B.

The student should verify this by substitution in the formula connecting impulse with momentum.

Example 16.—A mass of 250 lbs. being free to move, is acted upon for 10 seconds by a force of 20 lbs. A second mass of 50 lbs., also free to move, is acted upon for 2 seconds by a force of 50 lbs. Compare the momenta given to the masses, and find their final velocities.

The impulse in each case being equal to the momenta imparted, the ratio of the momenta is therefore

$$\frac{20 \times 10}{50 \times 2} = 2 : 1.$$

The velocity of the mass of 250 lbs., found from the equation

$$20 \times 10 = \frac{250 \times v}{32 \cdot 2},$$

is **25·76 ft. per second.**

The velocity imparted to the second mass is just twice this, or **51·52 feet per second.**

Third Law.—*Action and reaction are equal and opposite.*

This law has been illustrated in almost every exercise in this chapter, and we add additional exercises which should be carefully worked through.

EXERCISES.

1. If forces of equal magnitude act for the same time on two bodies from a state of rest whose masses are in the ratio of 7 : 5, what would be the ratio of the velocities generated? *Ans.* 5 : 7.

2. A moving body A, whose mass is 5, is acted upon for 4 seconds by a force of 10 units. A moving second body B, whose mass is 10, is acted upon for 8 seconds by a force of 15 units. Compare the added velocities of the two bodies. *Ans.* 2 : 3.

3. A train is travelling at a uniform speed on the level. The weight of the brake van at the rear of the train is 10 tons, and the weight of the remaining part of the train is 90 tons. If the brakes are applied to the brake van, what will be the force on the brake-van couplings, the coefficient of friction of the wheels on the rails being 0.1? (I.C.E., Feb. 1906.)

4. Explain how forces are measured, and distinguish between the mass of 1 lb. and the weight of 1 lb. Determine what force will be necessary to change the velocity of a mass of 400 lbs. from 15 to 25 ft. per second in 8 seconds. *Ans.* 15.5 lbs. (I.C.E., Feb. 1906.)

5. A shot is projected from a gun. Explain why—

(1) The momentum of the shot is equal (under certain conditions) to the momentum of the gun.

(2) The energy of the shot when it leaves the gun is greater by far than the energy communicated to the gun.

Example.—Shot, 100 lbs.; gun, 1 ton; velocity of shot, 1200 ft. per second. Find velocity of recoil of the gun and the energy of the gun. *Ans.* 53.5 ft. per second, 44.7 ft.-tons. (I.C.E., Feb. 1905.)

6. In the car of a balloon a piece of iron is hung from a spring balance. The balance registers 5 lbs. when the car is at rest. What will it register when the car is rising with an acceleration of 2 ft. per second per second? *Ans.* 5.31 lbs. (I.C.E., Feb. 1905.)

7. A gun fires 500 bullets per minute, each weighing $\frac{3}{4}$ oz. Assuming a velocity of discharge = 2000 ft. per second, calculate the average force exerted on the gun.

8. If the planing machine table in Example 8, p. 261, is to be reversed to four times the speed of the cutting stroke, find the time of reversal. *Ans.* 2.9 seconds.

9. A jet of water $1\frac{3}{4}$ in. diameter (circular section), moving at a velocity of 30 ft. per second, strikes a rigid plate fixed with its plane at right angles to the direction of the jet. What is the force exerted by the jet on the plane?

10. If the plane in the previous exercise were fixed at an angle of 120° with the jet, what would be the normal intensity of the pressure?

11. A locomotive going at 40 miles an hour scoops up water from a trough. The tank is 8 ft. above the mouth of the scoop, and the delivery has an area of 50 sq. in. If half the available head is wasted at entrance, find the velocity at which the water is delivered into the tank, and the number of tons lifted in a trench 500 yards long. What, under these conditions, is the increased resistance? and what is the minimum speed of train at which the tank can be filled?

(Univ. of Lond., B.Sc. Engineering, 1906.)

12. A locomotive draws a train of 100 tons with a uniform acceleration such that a speed of 60 miles per hour is attained in 4 minutes on the level. If the frictional resistances are 10 lbs. per ton, and the resistance of the air, which varies as the square of the speed, is 120 lbs. at 20 miles per hour, find the pull exerted by the locomotive at 30 miles and at 60 miles per hour.

(I.C.E., Feb. 1906.)

13. Water is scooped into a locomotive tender to a height of $6\frac{1}{2}$ ft. above the trough. If the engine is running at 35 miles an hour, what is the velocity of delivery, assuming that the frictional resistance is equal to a head of water $6\frac{1}{2}$ ft. high? Also calculate the lowest speed of the engine for which scooping is possible.

14. The speed of a motor car is determined by observing the times of passing a number of marks placed 500 ft. apart. The time of traversing the distance between the first and second posts was 20 seconds, and between the second and third 19 seconds. If the acceleration of the car is constant, find its acceleration in feet per second per second, and also the velocity in miles per hour at the instant it passes the first post.

(I.C.E., Feb. 1905.)

15. If a piston, whose weight is 60 lbs., flies off the rod at the commencement of the stroke, calculate the velocity with which it would strike the cover, having given: steam pressure = 100 lbs. per sq. in. (assumed constant); diameter of piston, 3 ft. 6 in.; stroke, 4 ft. 2 in.; frictional resistance of rings, 80 lbs.

16. A thin circular disc 12 in. radius has a projecting axle $\frac{1}{2}$ in. diameter on either side. The ends of this axle rest on two parallel inclined straight edges inclined at a slope of 1 in 40, the lower part of the disc hanging between the two. The disc rolls from rest through 1 ft. in $53\frac{1}{2}$ seconds. Neglecting the weight of the axle and frictional resistances, find the value of g .
(I.C.E., Feb. 1905.)

17. A fly-wheel weighing 10 tons, whose radius of gyration is 5 ft., rests on bearings 10 in. diameter. If the coefficient of friction of the axle and bearings is 0.006, find the constant moment which must act upon the wheel to get up a speed of 20 r.p.m.
(I.C.E., Feb. 1906.)

18. A gyrostat is made to spin in the usual way by a string wrapped round the spindle. If the string be 2 ft. long and be pulled with a uniform force of 10 lbs., find, in revolutions per second, the speed of the gyrostat, assuming that this is equivalent to a circular disc 4 in. in radius and weighing 2 lbs.
(Univ. of Lond., B.Sc. Eng.)

Note.—The radius of gyration of disc = radius $\times \frac{\sqrt{2}}{2}$.

SECTION III.

Boiler and Heat-Engine Calculations.—The most important of the practical calculations required by manufacturers of heat-engine plant, and by the users of power, are not really of an advanced character, though often considered as such; and we propose, therefore, in this section to deal with some of these calculations.

Let us first ask ourselves what are the principal problems required to be solved by those who are connected with the development and the use of power, and in answering this question let us have in view a steam-engine plant consisting of boiler, engine, and accessories. We summarise a few of the principal problems as follows :—

First, with respect to the boiler.

1. *In designing.*—(a) The determination of the quantity of heating surface to be given to the boiler so as to evaporate a given weight of water per hour; and (b) the area of grate surface to burn economically the fuel necessary to give this evaporation; (c) the volumes of water and steam space; (d) the diameter of the boiler, and thickness of plates to carry safely a given working pressure.

These are the principal items determining the dimensions of the boiler.

2. *Concerning the steam generation and the fuel consumption.*—(a) The average evaporation or steam supply for each pound of coal burnt; (b) the efficiency of the boiler, or the extent to which the heat of the coal burnt is utilised by the boiler in the formation of the steam; (c) the heat losses; (d) the quality of the steam supplied by the boiler.

Secondly, with respect to the engine.

1. *In designing.*—(a) The dimensions of the cylinders (diameters and stroke) to develop a given horse-power for a given piston speed ; (b) the most economical cut-off, and the probable mean effective pressure to allow for in calculating cylinder dimensions.
2. *Concerning the performance and efficiency of the engine.*—(a) The cylinder or indicated horse-power developed by the engine ; (b) the useful or brake horse-power available at the engine shaft, and the mechanical efficiency of the engine, considered as a machine ; (c) the steam consumption per I.H.P., per B.H.P., or, if a dynamo be driven, per electrical horse-power, or per kilowatt hour ; (d) the heat efficiency of the engine and the heat losses, or the extent to which the heat of the steam supply is converted into mechanical work.

Finally, we have to deal with the **combined efficiency of the plant**, as between (a) the boiler furnace and the indicated power, (b) the boiler furnace and the brake horse-power, and (c) boiler furnace and the dynamo terminals (where an electric generator is driven by the engines).

These technical problems ultimately become commercial ones, and the user and the designer alike are concerned with the question, "How much power can I get for so much money?"

CHAPTER XIII.

PRELIMINARY CALCULATIONS.

Fahrenheit and Centigrade Scales of Temperature.—On the Fahrenheit (F.) scale the temperature of boiling water at atmospheric pressure is marked 212° , while the temperature of freezing water is 32° ; there is thus an interval of $(212 - 32) = 180^{\circ}$. On the Centigrade (C.) scale these two limits are marked 0° and 100° , giving an interval of 100° .

Hence 180° F. correspond to 100° C., so that

$$\frac{\text{degrees C.}}{\text{degrees F.}} = \frac{100}{180} = \frac{5}{9},$$

and conversion from one scale to the other becomes quite a simple calculation.

(1) *To convert degrees C. to degrees F.*—Multiply by $\frac{9}{5}$ and add 32° .

(2) *To convert degrees F. to degrees C.*—Subtract 32° and multiply by $\frac{5}{9}$.

Example 1.—Express 230° C. on the Fahrenheit scale.

$$\text{Ans. } 230^{\circ} \text{ C.} = 230 \times \frac{9}{5} + 32 = 446^{\circ} \text{ F.}$$

Example 2.—A block of iron is chilled to 10° C. below the freezing point of water. What is the corresponding temperature on the Fahrenheit scale?

$$\text{Ans. } -10^{\circ} \text{ C.} = -10 \times \frac{9}{5} + 32 = 14^{\circ} \text{ F.}$$

Example 3.—A boiler is working at a steam pressure of 150 lbs. □" absolute. Find the temperature of the steam on the Centigrade scale.

Referring to the steam table, p. 290, we find the temperature on the F. scale to be 358° . By rule (2) above,

$$358^{\circ} \text{ F.} = \frac{5}{9}(358 - 32) = 181.1^{\circ} \text{ C.} \quad \text{Ans.}$$

Example 4.—Convert 25° F. to degrees C.

$$\text{Ans. } 25^{\circ} \text{ F.} = \frac{5}{9}(25 - 32) = -3.88^{\circ} \text{ C.,}$$

or 3.88° C. below the freezing point of water.

Example 5.—The number B.Th.U. above 32° F. in 1 lb. of steam at a temperature of 316° F. is 1178.3 (see tables, p. 290). What is this in pound-degree-centigrade units above 0° C.?

(First see Exercises 1 and 2, p. 276.)

Since temperatures F. are to temperatures C. as 9 is to 5, therefore B.Th.U.'s F. are to lb.-degree-C. units in the same ratio.

Hence

$$1178.3 \text{ B.Th.U.} = \frac{5}{9} \times 1178.3 = 654.6 \text{ lb.-degree-C. units.}$$

Units of Heat.—We may define the unit of heat* in general terms as equivalent to the amount of heat required to raise unit mass of water through 1° rise in temperature.

The **British Thermal Unit** (B.Th.U.) is the heat required to raise 1 lb. of water 1° F.

The **Pound-Degree-Centigrade Unit** is the heat required to raise 1 lb. of water through 1° C.

The **Calorie** (or Gramme-Degree-Centigrade Unit) is the heat required to raise 1 gramme of water through 1° C. (from 0° C. to 1° C.). When the unit of mass taken is 1000 grammes or 1 kilo-gramme, then the heat required to raise this quantity through 1° C. is frequently called a **major calorie** or a **kilo-calorie**.

Example 1.—Calculate the heat required to raise 2500 lbs. of water through 50° F. rise in temperature.

$$\text{Ans. } 2500 \times 50 = 125,000 \text{ British thermal units (B.Th.U.)};$$

$$\text{or } 2500 \times 50 \times \frac{5}{9} = 69,444.44 \text{ degree-pound-C. units};$$

$$\text{or } 2500 \times 453.6 \times \frac{5}{9} \times 50 = 31,500,000 \text{ calories} = 315 \times 10^5 \text{ calories};$$

$$\text{or } 2500 \times 0.4536 \times \frac{5}{9} \times 50 = 31,500 \text{ major calories.}$$

Example 2.—A boiler containing 5 tons of water at a temperature of 200° F. is supplied with water at 60° F. from a feed-pump delivering 20 gallons per minute. If the pump be kept running

* Since the specific heat and density of water are not quite constant, but vary slightly with the temperature, it is more accurate to define the unit of heat as the quantity of heat required to raise the temperature of unit mass of water through 1° at some particular temperature, such as its point of maximum density (4° C. or 39.2° F.).

for 10 minutes, what will be the resulting temperature of the boiler?

Weight of water in boiler before pumping = $2240 \times 5 = 11,200$ lbs.
 „ „ pumped into boiler = $20 \times 10 \times 10 = 2000$ lbs.

There are two ways of working this calculation—

(a) To equate the heat lost by the original water in the boiler to the heat gained by the feed-water added.

(b) To consider the problem as a mixture of two quantities of heat.

By (a), if t be the resultant temperature,

$$11200(200 - t) = 2000(t - 60).$$

By (b), $11200 \times 200 + 2000 \times 60 = \text{total heat in mixture}$
 $= (11200 + 2000)t.$

Hence, by either (a) or (b) we obtain

$$t = \frac{11200 \times 200 + 2000 \times 60}{11200 + 2000} = 178.78^\circ \text{ F.}$$

Specific Heat Calculations.—Equal masses of different substances or different kinds of matter require different quantities of heat to raise them through an equal rise of temperature. Consequently we are led to define the **specific heat** (sp. ht.) of a substance, in general terms, as the ratio that the quantity of heat required to raise unit mass of the substance 1° is to the heat required to raise unit mass of water 1° .

The following are the approximate specific heats of a few substances of common use in engineering:—

TABLE XX.—SPECIFIC HEATS.

Aluminium . . .	0.214	Iron (cast) . . .	0.120
Brass (average) . . .	0.090	Iron (wrought) . . .	0.110
Copper	0.093	Lead	0.031
Carbon	0.250	Mercury	0.033
Glass	0.198	Petroleum	0.460
Glass (crown) . . .	0.160	Platinum	0.082
Glass (flint) . . .	0.117	Steel	0.117
Ice	0.500	Tin	0.056
India-rubber . . .	0.200	Zinc	0.094

Example 1.—A number of brass condenser tubes weighing 540 lbs. were at a temperature of 58° F. before the condenser was at

work ; afterwards, when in use, the mean temperature was 110° F. How many B.Th.U. did they take up ?

$$\text{Rise of temperature} = 110 - 58 = 52^{\circ} \text{ F.}$$

The heat required to raise 540 lbs. of water 52° F. would be 540×52 B.Th.U. But since the thermal capacity of brass as indicated by the sp. ht. is 0.09 of that of an equal weight of water,

$$\left. \begin{array}{l} \text{The heat absorbed} \\ \text{or taken up} \end{array} \right\} = 0.09 \times 540 \times 52 = 2527.20 \text{ B.Th.U.}$$

Example 2.—What weight of water is the metal in the above exercise equivalent to with respect to its capacity for heat ?

$$\text{Ans. } 540 \times 0.09 = 48.6 \text{ lbs. ;}$$

i.e. 48.6 lbs. of water raised through any temperature would absorb as much heat as 540 lbs. of brass raised through the same temperature.

Example 3.—A cast-iron plate weighing 120 lbs. is immersed in 240 lbs. of water. If the temperature of the plate is 150° F., and that of the water 50° F., what would be the final temperature (t) of both ?

$$\begin{aligned} \text{Heat gained by water} &= 240(t - 50). \\ \text{,, lost by plate} &= 120(150 - t) \times 0.12 \\ &= (18000 - 120t) \times .12 \\ &= 2160 - 14.4t \text{ B.Th.U.} \end{aligned}$$

And since these two quantities must be equal,

$$\begin{aligned} 240(t - 50) &= 2160 - 14.4t. \\ 240t - 12000 &= 2160 - 14.4t. \\ 254.4t &= 14160. \end{aligned}$$

$$\begin{aligned} t &= \frac{14160}{254.4} \\ &= 55.66^{\circ} \text{ F.} \end{aligned}$$

Example 4.—A calorimeter similar to the one described on p. 283 is made up of material as under. Calculate the total water equivalent of the apparatus as shown in the last two columns.

Note.—Take the product of the weight into the sp. ht. (as in Example 2) for each material, and add the results.

TABLE XXI.

Material.	Weight in Grammes.	Specific Heat.	Equivalent to Grammes of Water.
Glass beaker . . .	231.24 gms.	0.198	45.80
„ bell . . .	45.23 „	0.198	8.96
Brass-work . . .	102.61 „	0.090	9.23
Iron-work . . .	13.92 „	0.110	1.53
Platinum crucible .	6.96 „	0.082	0.22
Copper gauge . . .	28.77 „	0.093	2.68
Thermometer glass .	8.64 „	0.198	1.71
Mercury in thermometer	12.62 „	0.033	0.42
Water . . .	2000 „	1.000	2000.00
Total water equivalent of apparatus . .			2070.55

Heat carried away by Flue Gases.—Calculations are made on the results obtained from boiler trials to approximate the amount of heat carried away by the products of combustion. We give several practical exercises below, but first make the following statements :—

Heat carried away by products of combustion measured above air inlet temperature

$$= \left\{ \begin{array}{l} \text{Total mass of} \\ \text{products} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Elevation of tem-} \\ \text{perature at exit} \\ \text{above the air inlet} \\ \text{temperature} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Mean specific} \\ \text{heat of pro-} \\ \text{ducts.} \end{array} \right.$$

The mean specific heat of a mixture of gases is found by multiplying the mass of each constituent gas by its specific heat, adding the products thus obtained, and dividing the result by the total mass of the mixture.

Specific Heat of Gases (at constant pressure).

Steam superheated (H_2O)	0.560	Oxygen (O)	0.218
Carbon dioxide (CO_2)	0.216	Hydrogen (H)	3.4
Carbon monoxide (CO)	0.245	Marsh gas (CH_4)	0.593
Nitrogen (N)	0.244	Olefiant gas (C_2H_4)	0.404
Air			0.238

Example.—The results given below were obtained from trials on a steam boiler; calculate—

- (1) The mean specific heat of the products of combustion.
- (2) The heat carried away by the products of combustion for each pound of dried fuel.
- (3) The heat carried away by the air admitted per lb. of dried fuel in excess of that required for complete combustion.
- (4) The heat used in evaporating the moisture in 1 lb. of coal as fired, and in superheating the steam produced and carried away with flue gases.
- (5) The total heat leaving boiler with flue gases.

Heat value of 1 lb. of dried fuel = 14,240 B.Th.U.

Temperature of air before admission to boiler furnace } = 62° F. = t .

Temperature of flue gases at exit from boiler } = 452° F. = t_f

Weight of products of combustion per 1 lb. of dry fuel } = 12.6 lbs.

Weight of air in excess of that required for complete combustion per lb. of dried fuel } = 13.35 lbs.

Moisture per lb. of coal burnt = 0.019 lb.

Proportions by weight of products of combustion for each 1 lb. of dried fuel } = $\begin{cases} \text{CO}_2 = 0.244 \\ \text{CO} = 0.000 \\ \text{H}_2\text{O} = 0.039 \\ \text{N} = 0.717. \end{cases}$

- (1) Mean specific heat of products

$$= \frac{0.244 \times 0.216 + 0.48 * \times 0.039 + 0.244 \times 0.717}{0.244 + 0.039 + 0.717} = 0.246.$$

- (2) Heat carried away by products of combustion per 1 lb. of dried fuel

$$= \left\{ \begin{array}{l} \text{Weight of products per lb.} \\ \text{of dried fuel} \end{array} \right\} \times t_f - t \times \left\{ \begin{array}{l} \text{Specific heat of} \\ \text{products} \end{array} \right.$$

$$= 12.6(452 - 62) \times 0.246 = 1209 \text{ B.Th.U.}$$

$$1209 \text{ B.Th.U. is } \frac{1209}{14240} \times 100 = 8.5 \text{ per cent. of heat value of fuel.}$$

* 0.56 is more in accordance with the latest researches on superheated steam.

$$(3) \left. \begin{array}{l} \text{Heat carried} \\ \text{away by the} \\ \text{excess air} \end{array} \right\} = 13.35(452 - 62) \times 0.238 = 1239 \text{ B.Th.U.}$$

$$1239 \text{ is } \frac{1239}{14240} \times 100 = 8.7 \text{ per cent. of heat value of fuel.}$$

$$(4) \left. \begin{array}{l} \text{Heat lost by eva-} \\ \text{porating and super-} \\ \text{heating moisture in} \\ \text{1 lb. of fuel} \end{array} \right\} = 0.019\{(212 - 62 + 966) + 0.48(452 - 212)\}$$

$$= 24 \text{ B.Th.U.}$$

$$= 0.168 \text{ per cent. of heat value of fuel.}$$

$$(5) \text{ The sum of above losses} = 17.368 \text{ per cent. of heat value of fuel.}$$

Relation between Heat and Work.—The student will no doubt have thought a good deal about the conversion of heat into mechanical work. He will have fully realised that the mechanical and useful work obtained from the engine shaft was originally in the form of heat energy in the boiler furnace. The internal heat or molecular energy developed during combustion has been converted into mass or molar energy from the engine shaft.

Now, experiment proves (and the Law of Conservation of Energy deduced therefrom states) that, for each unit of heat which disappears as heat, 778 ft.-lbs. of mechanical work are done, and that conversely, when work is converted into heat, 778 ft.-lbs. are required to produce 1 B.Th.U. of heat. This number 778 is the **Mechanical Equivalent of Heat**, more often known as **Joule's Equivalent**, and is denoted by **J**.

In the **C.G.S. system**, $J = 4.2 \times 10^7$ ergs to the calorie.

Example 1.—An engine has a cylinder 10 in. in diameter, and a stroke of 12 in. If the M.E.P. is 40 lbs. per sq. in. and the engine makes 200 r.p.m., calculate (a) the work done per stroke; (b) work per minute; (c) the heat equivalent of the work done.

$$\left. \begin{array}{l} \text{Total average pressure} \\ \text{on piston} \end{array} \right\} = 10 \times 10 \times 0.7854 \times 40 = 78.54 \times 40.$$

$$(a) \left. \begin{array}{l} \text{Work per stroke} \\ \text{in ft.-lbs.} \end{array} \right\} = 10 \times 10 \times 0.7854 \times 40 \times 1$$

$$= 78.54 \times 40 \text{ ft.-lbs.}$$

$$(b) \text{ Work per minute} = 78.54 \times 40 \times 200 \times 2 = 1,256,640$$

$$\text{ft.-lbs.}$$

$$(c) \left. \begin{array}{l} \text{Heat equivalent of} \\ \text{work done} \end{array} \right\} = \frac{1256640}{778} = 1615.2 \text{ B.Th.U.}$$

Example 2.—Find the heat equivalent of 1 H.P. minute.

1 H.P. minute = 33,000 ft.-lbs. of work per minute, and, since each B.Th.U. can do 778 ft.-lbs. of work, $\frac{33000}{778}$, or 42.41, gives the heat value or expenditure to produce 1 H.P. minute.

Heat Value of a Fuel.—The calorific or heat value of the fuel is required in calculating the thermal efficiency of a steam boiler. For each pound of coal burnt in the boiler furnace a definite quantity of heat is developed, this heat being, for the most part, transferred to the water in the boiler for the formation of steam; and, as the student will see presently, the thermal efficiency of the boiler is the ratio of the heat transmitted to the water for each pound of coal burnt, to the heat of combustion of 1 lb. of coal. The heat value of a fuel is determined by means of a calorimeter, of which there are many forms on the market. One of the most useful of these is the Thomson calorimeter represented in fig. 138, p. 284, the principle of which is very simple. A definite small quantity of finely powdered fuel is burnt in a stream of oxygen, and the heat of combustion is imparted to a comparatively large weight of water, with a resulting rise in its temperature. The heat value of the fuel burnt is then given by the rise in temperature into the water equivalent of the apparatus. Fig. 138B shows the apparatus complete when the fuel is being tested, and fig. 138A the several details of the apparatus separately.

The process of testing a fuel is somewhat as follows:—First, sample the coal-heap, breaking up the lumps, thoroughly mixing, and dividing and subdividing until a sufficiently small representative quantity is obtained. Take a small portion of this sample, and grind by means of a mortar and pestle into a very fine powder. This finely-divided sample should then be thoroughly dried to expel the moisture it contains. The fuel will then be ready for testing in the calorimeter. A small quantity of this fuel, such as a gramme weight of it, is placed in a small platinum crucible A, the crucible then being removed to the stand or support B. A small piece of glowing touch-paper is then placed on the surface of the coal in the crucible, and the hollow bell-jar C, and its attachment, is placed over the stand B between the clips shown. The end G of the long tube passing down the middle of the bell-jar is connected up to a supply of oxygen, which is carefully admitted to the interior of the bell just above the surface of the fuel. The touch-paper glows intensely in the atmosphere of oxygen, and the fuel then takes fire, the products of combustion forcing their way out from the bottom of the bell through the holes shown in the base of the stand. The apparatus is then lifted bodily,

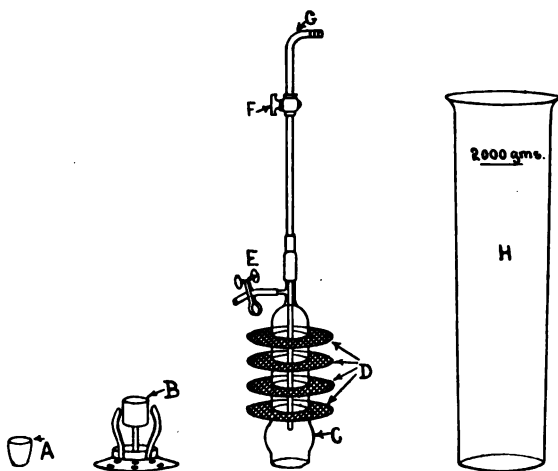


FIG 138A.

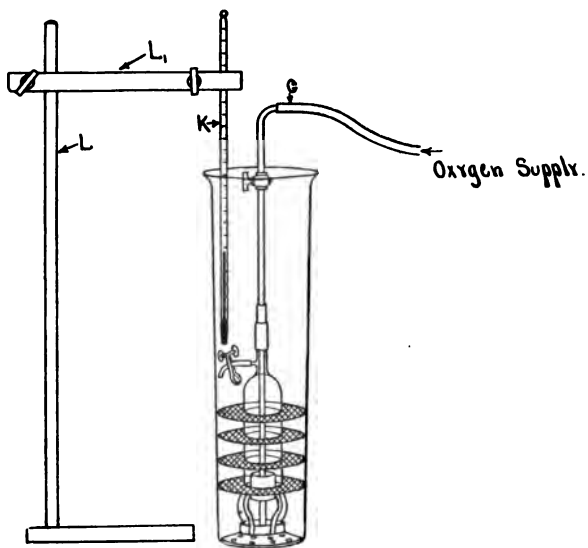


FIG. 138B.

and placed in the glass beaker H, which has previously been filled with water to the level of the 2000 grammes mark, and the temperature of which was noted. As the combustion proceeds the bubbles of gas are forced up through the water in the beaker, giving up the heat of combustion to the water and apparatus. Round the bell-jar a number of rings of wire gauze D are fixed, to break up and impede the bubbles of gas on their way to the surface of the water. The apparatus is taken by the stem G and given a slow vertical up-and-down motion, thus intimately mixing the gas bubbles with the water, so as to make sure that the whole of the heat of combustion has been imparted to the water. The combustion of the 1 gramme may take anything from five to ten minutes, or even longer, and as soon as it is complete, which is indicated by an absence of glow in the crucible, the apparatus is drawn up in the beaker, the clip E taken off the india-rubber tube, and the apparatus replaced in the water. The water then rushes down into the interior of the bell, cooling the crucible, which has become very hot, and the heat of which should have been given to the water. To make sure that a uniform temperature has been given to the apparatus, the bell-jar is moved slowly up and down through the water in the beaker, the oxygen supply having been shut off by means of the tap F. The final temperature of the water in the apparatus is then taken by means of the thermometer K, and, knowing the total water equivalent of the apparatus and the rise of temperature, the heat value of the fuel can then be determined. Any inaccuracy in the result due to the loss of heat from the apparatus by radiation may be avoided by commencing the experiment with the temperature of the water about as many degrees below the temperature of the room as the final temperature is likely to be above.

Example.—One gramme of coal was tested in a calorimeter, and gave a rise of temperature of 6.3° F. Calculate the heat value of the fuel, (a) in calories, (b) in B.Th.U. per pound of coal, having given that the water equivalent of the apparatus was equal to 153.67 grammes of water.

$$(a) \ 6.3^{\circ} \text{ F.} = 6.3 \times \frac{5}{9} = 3.496^{\circ} \text{ C.}$$

$$\begin{aligned} \text{Heat value per gramme weight of fuel} &= (2000 + 153.67) \times 3.496. \\ &= 7529 \text{ calories.} \end{aligned}$$

(b) Since heat value of 1 gramme = $2153.67 \times 6.3 = 13,568$ gramme-degree-F. units, it follows that, if 1 lb. weight be taken as the unit, the heat value in lb.-degree-F. units would be 13,568 B.Th.U.

The Condensation of Steam.*—In a condensing engine the steam, after doing its work in the cylinder, is exhausted into some form of condenser, which may be jet, surface, or evaporative. In the jet condenser the steam is mixed directly with the water employed to condense it. In the surface condenser the condensing or injection water is usually passed through a series of tubes, while the steam is exhausted round the tubes. In designing a condensing plant it is necessary to find first of all the amount of circulating water required to condense the steam from the engine, in order that the pump capacity, where a pump is used, may be calculated to supply the water. This may be done by first finding the amount of water to condense 1 lb. of steam.

The Jet Condenser.—Let W represent the weight of water to condense 1 lb. of steam, T_s the initial temperature of the exhaust steam, T_i the initial temperature of the injection water, and T_h the temperature of the hot well or mixture of water and steam.

Heat added to water = heat taken from exhaust steam.

$$W(T_h - T_i) = L + T_s - T_h = 1114 - 0.7T_s + (T_s - T_h) \\ = 1114 + 0.3T_s - T_h.$$

$$\therefore W = \frac{1114 + 0.3T_s - T_h}{T_h - T_i}.$$

Example 1.—An engine, with a jet condenser, indicates 100 H.P. when making 150 revolutions per minute, and uses 20 lbs. of steam per I.H.P. per hour. If the temperatures of the exhaust steam, the injection water, and hot well are 110° F., 50° F., and 100° F. respectively, calculate (a) the weight of circulating water required per hour, (b) the weight of circulating water per double stroke of air pump.

$$\left. \begin{array}{l} \text{Lbs. water to condense} \\ \text{1 lb. of steam} \end{array} \right\} = \frac{1114 + 0.3 \times 110 - 100}{100 - 50} = 20.94.$$

$$\text{Weight per hour} = 20.94 \times 20 \times 100 = 41880.$$

$$\text{Weight per double stroke} = \frac{41880}{60 \times 150} = 4.654 \text{ lbs.}$$

The Surface Condenser.—The hot-well temperature T_h is the temperature of the condensed exhaust, and T_i we shall take as

* The student should read the first part of the next chapter, dealing with the Properties of Steam, if unable to understand the next two paragraphs.

representing the final temperature of the injection water after leaving the condenser. Our equation becomes

$$W(T_f - T_i) = L + T_s - T_h.$$

$$\therefore W = \frac{1114 + 0.3T_s - T_h}{T_f - T_i}.$$

Example 2.—Having given the temperatures of injection, discharge, hot well, and exhaust steam as 50°, 100°, 120°, and 150° F. respectively, calculate the weight of water required to condense 1 lb. of steam. If the temperature of the injection is 70° F., how much water is required?

$$W = \frac{1114 + 0.3 \times 150 - 120}{100 - 50} = 20.8 \text{ lbs.}$$

If the injection is at 70° F.

$$W = \frac{1114 + 0.3 \times 150 - 120}{100 - 70} = 34.6 \text{ lbs.}$$

EXERCISES.

1. Convert the following temperatures on the F. scale to temperatures on the C. scale:—

–15°, 0°, 18°, 50°, and 320°.

2. Convert the following temperatures on the C. scale to temperatures on the F. scale:—

200°, 20°, 0°, –4°, and –20°.

3. If 3500 lbs. of water at 200° F. are mixed with 2000 lbs. at 50° F., what is the final temperature? *Ans.* 145½° F.

4. Two lbs. of aluminium at 50° F., 4 lbs. of copper at 100° F., and 10 lbs. of cast iron at 200° F., are all simultaneously plunged into 3 cub. ft. of water at 40° F., what is the final temperature of the mixture? *Ans.* 41° F.

5. One gramme of coal was tested in a Thomson calorimeter whose total water equivalent was 2120 grammes. The initial and final temperatures of the apparatus were 54° F. and 60.85° F. What was the calorific value of the coal?

Ans. 14520 B.Th.U. per lb., or 8066 calories per gramme.

6. An engine indicated 73.87 H.P., and the heat entering the engine per minute to produce this power was 24,223 B.Th.U.

Calculate (a) the heat equivalent of the I.H.P., (b) the heat admitted to the engine per H.P., (c) the ratio of the heat equivalent of the I.H.P. to the total heat admitted to the engine. How do you account for the difference between the heat admitted and the heat equivalent of the I.H.P.?

7. A steam-generating plant uses $2\frac{1}{2}$ lbs. of coal per I.H.P. per hour. If the heat value of each pound of coal is 13,550 B.Th.U., what is the ratio of the heat utilised as work on the pistons to the heat energy of the fuel producing that work? Why are the two quantities not equal?

Ans. 7.2 per cent.

8. If for each pound of coal burnt in the boiler furnace 10 lbs. of water at 70° F. are converted into steam at 350° F., what is the ratio of the heat given to the steam to the heat value of 1 lb. of coal, the latter being, say, 14,500 B.Th.U.?

Ans. 72.3 per cent.

9. 350 lbs. of coal are burnt on a boiler grate per hour and 3000 lbs. of water are passed through the economiser, the temperature of the water being raised 150° F. and the temperature of the waste gases being lowered 300° F. If the specific heat of the flue gases is 0.24, what weight of furnace gases are produced per pound of coal burnt?

Ans. 17.85 lbs.

(I.C.E., Feb. 1906.)

10. An engine indicates 250 H.P. when making 200 r.p.m. with a steam consumption of 18 lbs. per I.H.P. per hour. Having given the following temperatures, calculate the weight of injection water delivered for each double stroke of the circulating pump:—

Temperatures of injection, discharge, hot well, and exhaust steam—

55°, 110°, 125°, and 138° F. respectively.

Ans. 7 lbs.

11. If 25 lbs. of circulating water are required for each pound of exhaust steam condensed, and the temperature of the circulating water in passing through the tubes is increased from 56° to 98° F., calculate the temperature of the exhaust steam when the temperature of the hot well is 108° F.

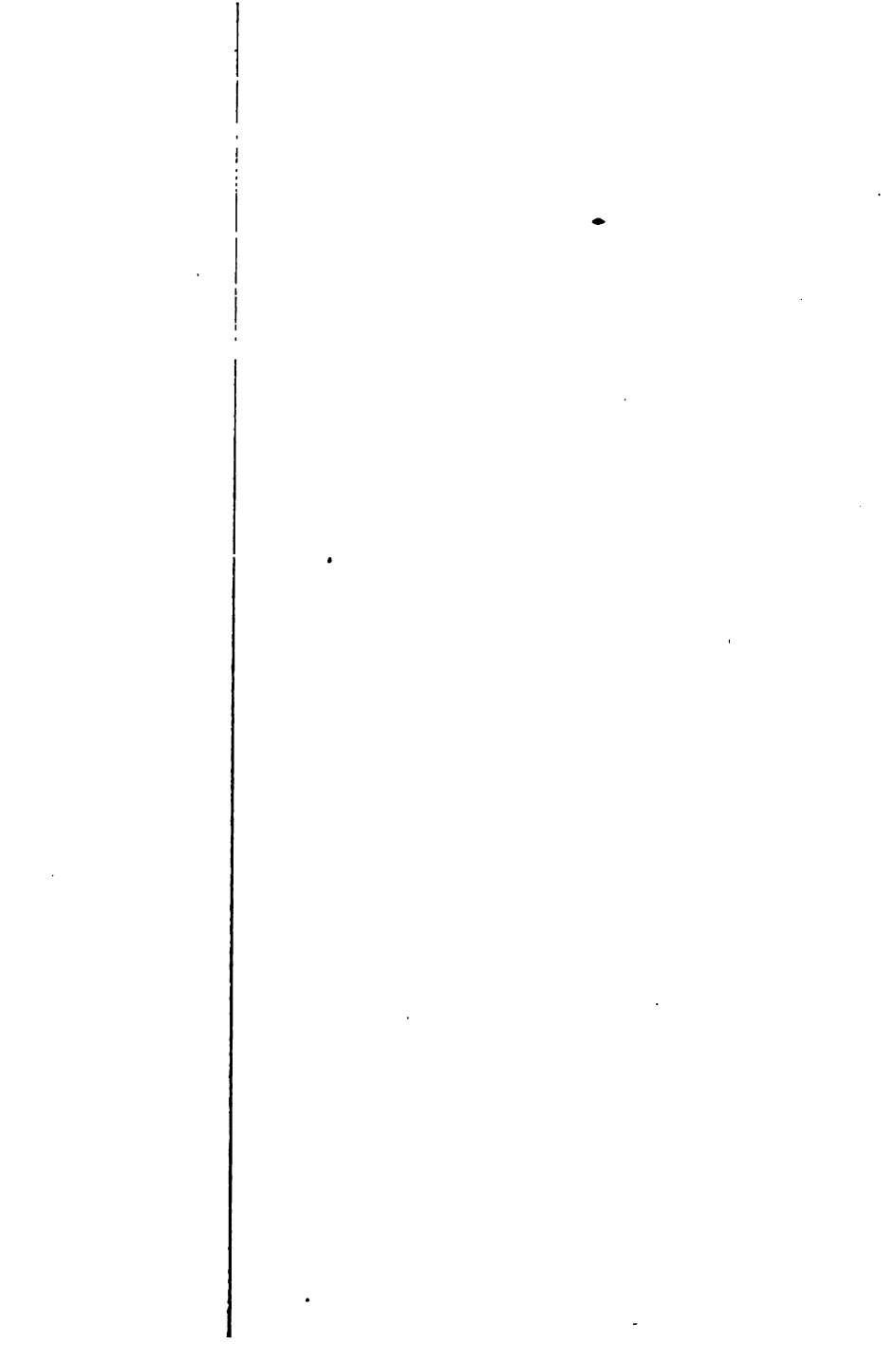
Ans. 146.6° F.

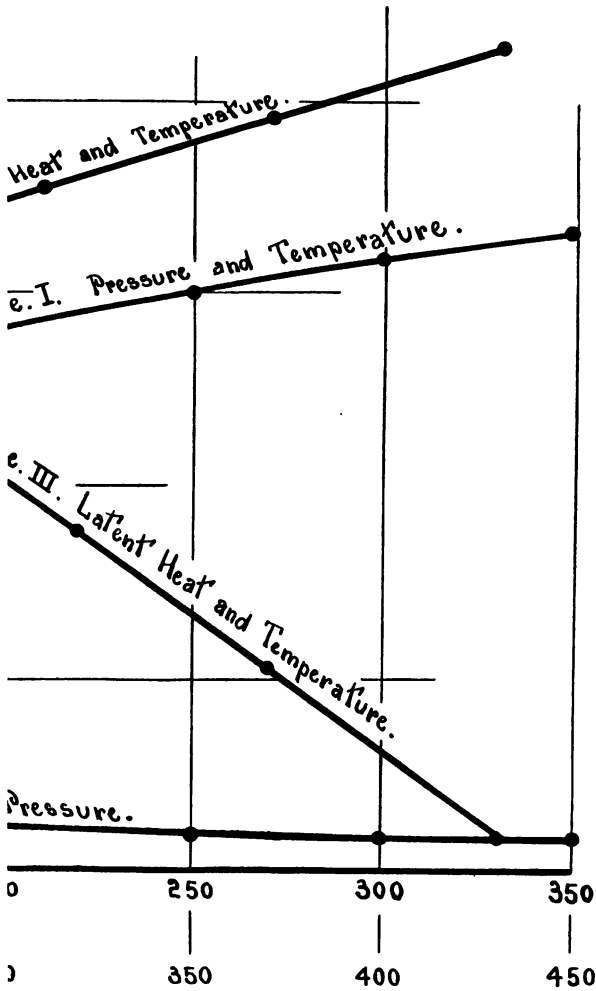
12. The temperature of the injection is 50° and of the discharge 102° F. Calculate the temperature of the discharge if the temperature of the injection is increased to 60° F., and if the weight of injection is increased by 15 per cent.

Note.—Weight of injection $\times$ rise in temperature = constant.

Ans. 105° F.

13. State clearly the reason why such a large quantity of water is required to condense 1 lb. of exhaust steam.





CHAPTER XIV.

THE STEAM BOILER AND ITS CALCULATIONS.

Steam Tables.—Our knowledge of the properties of steam is derived chiefly from the experiments of Regnault made many years ago under the authority of the French Government.

The chief results required by the practical engineer are embodied in Table XXII., p. 290, and they are represented graphically in fig. 139.

Column I. gives the **absolute pressure of the steam** in pounds per square inch (lbs. per $\square''$). This is found by adding the atmospheric pressure (take at 14.7 lbs. per sq. in.) to the pressure as recorded by the gauge. Thus

$$\text{Absolute pressure} = \text{Gauge pressure} + 14.7.$$

Column II. gives the **temperatures** on the Fahrenheit scale corresponding to the various absolute pressures tabulated in the first column. The temperatures corresponding to the various pressures, as tabulated, were determined experimentally, and Curve I., fig. 139, shows this relationship graphically.

Column III. gives the **sensible heat (h)** (see p. 290) for 1 lb. of water at the temperature given in column II. This is very nearly $= t - 32$.

Columns IV. and V. give the **total and latent heats of evaporation** for 1 lb. of saturated steam. These results are derived from experiment. Curves II. and III., fig. 139, show graphically the relationship between the temperature and the total and latent heats. For definitions, see pp. 291 and 292.

Column VI.—The numbers in this column, giving the **volume** in cubic feet of the steam formed from each pound of water evaporated at the given temperatures and pressures, were calculated from theoretical considerations, and the other relationships, as tabulated, from the experimental results. Curve IV., fig. 139, shows v plotted on a pressure base.

TABLE XXII.—PROPERTIES OF STEAM.

I. Pressure in lbs. per $\square$ " Absolute.	II. Temperature. ° F.	III. Sensible Heat above 32°. B.Th.U.	IV. Latent Heat. B.Th.U.	V. Total Heat above 32° F. B.Th.U.	VI. Volume of 1 lb. Steam in Cubic Feet.
<i>p</i>	<i>t</i>	<i>h</i>	<i>L</i>	<i>H=L+h</i>	<i>v</i>
1	102.0	70.0	1043.0	1113.0	332.0
2	126.3	94.4	1026.1	1120.5	173.0
3	141.6	109.8	1015.3	1125.1	118.0
4	153.1	121.4	1007.1	1128.5	90.1
5	162.3	130.7	1000.8	1131.5	72.8
10	193.3	161.8	979.2	1141.0	37.9
14.7	212.0	180.7	966.0	1146.7	26.4
15	213.1	181.8	965.2	1147.0	25.9
20	228.0	196.8	954.7	1151.5	19.8
25	240.1	209.1	946.0	1155.1	16.0
30	250.3	219.4	939.0	1158.4	13.5
35	259.2	228.4	932.6	1161.0	11.7
40	267.2	236.4	927.0	1163.4	10.3
45	274.4	243.7	921.8	1165.5	9.22
50	280.9	250.4	917.2	1167.6	8.35
55	287.0	256.5	912.9	1169.4	7.64
60	292.6	262.2	908.9	1171.1	7.04
65	297.8	267.6	905.2	1172.8	6.53
70	302.8	272.7	901.6	1174.3	6.10
75	307.4	277.4	898.3	1175.7	5.70
80	311.9	282.0	895.1	1177.1	5.38
85	316.1	286.3	892.1	1178.4	5.08
90	320.1	290.4	889.2	1179.6	4.81
95	323.9	294.3	886.4	1180.7	4.57
100	327.6	298.0	883.8	1181.8	4.36
105	331.2	301.7	881.2	1182.9	4.16
110	334.6	305.2	878.7	1183.9	3.99
115	337.9	308.6	876.4	1185.0	3.82
120	341.1	311.9	874.0	1185.9	3.67
125	344.1	315.0	871.9	1186.9	3.54
130	347.1	318.1	869.7	1187.8	3.39
135	350.0	321.0	867.6	1188.7	3.29
140	352.8	324.0	865.5	1189.5	3.18
145	355.6	326.8	863.6	1190.4	3.07
150	358.2	329.6	861.6	1191.2	2.98
155	360.7	332.2	859.7	1191.9	2.89
160	363.3	334.8	857.9	1192.7	2.79
165	365.7	337.4	856.1	1193.5	2.72
170	368.2	339.9	854.4	1194.3	2.65
175	370.6	342.3	852.7	1195.0	2.58
180	372.9	344.7	851.0	1195.7	2.51

TABLE XXII.—PROPERTIES OF STEAM—*continued*.

I. Pressure in lbs. per □" Absolute.	II. Temperature, ° F.	III. Sensible Heat above 32° B.Th.U.	IV. Latent Heat. B.Th.U.	V. Total Heat above 32° F. B.Th.U.	VI. Volume of 1 lb. Steam in Cubic Feet.
p	t	h	L	$H = L + h$	v
185	375.2	347.0	849.3	1196.3	2.45
190	377.4	349.3	847.7	1197.0	2.38
195	379.4	351.6	846.1	1197.7	2.32
200	381.6	353.8	844.6	1198.4	2.27
205	383.7	355.9	843.1	1199.0	2.22
210	385.8	358.1	841.6	1199.6	2.17
215	387.8	360.1	840.1	1200.2	2.12
220	389.7	362.2	838.7	1200.9	2.08
225	391.7	364.2	837.2	1201.4	2.03
230	393.6	366.2	835.8	1201.9	1.99
240	397.3	370.0	833.1	1203.1	1.91
250	400.9	373.8	830.5	1204.3	1.84
260	404.4	377.4	827.9	1205.3	1.77
270	407.8	380.9	825.4	1206.3	1.71
280	411.1	384.3	823.0	1207.3	1.65
290	414.3	387.7	820.6	1208.3	1.59
300	417.4	391.9	818.3	1209.3	1.54
350	431.3	406.0	807.6	1213.6	1.37

Formation of Steam at Constant Pressure.—This is substantially what occurs in the working of a steam boiler when steam is delivered from it at a given rate per hour with constant gauge pressure. We have first of all to consider the **heat supply** which goes to the formation of this steam from the feed-water pumped into the boiler.

Case 1.—*When the steam leaving the boiler is in the dry saturated condition*, for which we assume the steam tables to hold good. Consider each pound of feed-water, whose temperature denote by t_p , pumped into a boiler where the surface water and the steam have a higher and common temperature, which denote by t . Then the heat transmitted through the heating surface to the 1 lb. of water is made up of—

(a) The heat necessary to raise the temperature from t_p to t . This quantity, which for each pound is given very nearly by $(t - t_p)$, may be called the **sensible, liquid, or water heat** above the temperature t_p . We shall denote this by h_p . If we imagine the heating of 1 lb. of water to commence at 32° F., then heat supply

or $(t - 32)$ is usually known as the sensible heat, and is generally denoted by h . If the temperature t is on the Centigrade scale, then $h = t$ in pound-degree-Centigrade units.

(b) The heat required to convert into steam each pound of water thus brought to t degrees, the temperature of the steam. This quantity is called the **latent heat of evaporation**. It is called latent (hidden) because, during evaporation, heat disappears from direct and immediate thermometric measurement—the temperature of the steam formed being the same as the temperature of the water from which it is generated. It is usually denoted by L .

The **total heat of evaporation** (H) is the sum of the sensible heat and latent heat. Symbolically,

$$H = h + L.$$

If we consider steam at ordinary atmospheric pressure (14·7 lbs.), and the corresponding temperature * 212° F., the sensible heat (reckoning above 32° F.) is obviously $212^\circ - 32^\circ = 180$ B.Th.U. (assuming the specific heat of water to be unity). The latent heat experiment shows to be 966 B.Th.U. Hence the total heat is $180 + 966 = 1146$ B.Th.U. Now, Regnault's experiments and the results of these, as tabulated, show that the total heat increases by 0·3 of a B.Th.U. for each degree rise in the temperature at which the steam is formed. Hence, denoting the temperature of the steam by t , we have at once the relationship

$$H = 1146 + 0\cdot3(t - 212).$$

This formula gives the heat of formation in B.Th.U. of 1 lb. of water raised from 32° F. to the temperature t and converted into steam at the same temperature.

Clearing the brackets and simplifying, we obtain

$$H = 1082 + 0\cdot3t.$$

If the Centigrade scale is used, the formula becomes

$$H = 606\cdot5 + 0\cdot3t,$$

where t is temperature in degrees Centigrade, and H = heat in pound-degree-Centigrade units.

The latent heat of steam formed at atmospheric pressure (14·7 lbs.), and temperature 212° F., is 966 B.Th.U. (see column IV., Table XXII.,) and this is the quantity of heat necessary to convert 1 lb. of water already at the boiling point (212°) into steam. For any other boiling point or temperature t , Regnault's experiments, as

* The temperature of the steam formed is in each case (column II.) the temperature of the boiling point, i.e. of the water from which the steam is formed.

tabulated, show that, for each degree rise above 212° , the latent heat decreases by 0.7 of a B.Th.U. Hence we get for temperature t

$$L = 966 - 0.7(t - 212),$$

or

$$L = 1114 - 0.7t.$$

If t is in degrees Centigrade,

$$L = 606.5 - 0.7t,$$

and L is then given in pound-degree-Centigrade units.

Example 1.—Dry saturated steam is formed at a pressure of 200 lbs. per sq. in., and corresponding temperature 381.6° F. Calculate

- (a) The sensible heat ;
- (b) The latent heat of vaporisation ;
- (c) The total heat of vaporisation ;

for each pound of steam formed.

$$(a) \text{ Sensible heat per lb.} = 381.6 - 32 = 349.6 \text{ B.Th.U.} \\ \text{(above } 32^{\circ} \text{ F.)}$$

$$(b) \text{ Latent heat} = 1114 - 0.7 \times 381.6 = 846.88 \text{ B.Th.U.}$$

$$(c) \text{ Total heat} = 1082 + 0.3 \times 381.6 = 1196.48 \text{ B.Th.U.}$$

Example 2.—Feed-water at a temperature of 60° is pumped into a boiler whose working pressure = 150 lbs. per sq. in. by gauge at the rate of 3000 lbs. per hour. Assuming that the steam leaves the boiler at the same rate, and in the dry saturated condition, and that the water level and pressure are kept constant, find the total heat required for the formation of—

- (a) Each pound of steam leaving the boiler ;
- (b) The total output per hour.

First Solution.—Steam tables available.

$$\text{Absolute pressure} = 150 + 15 = 165.$$

$$\text{Corresponding temperature from tables} = 365.7^{\circ}.$$

$$\text{Total heat of formation of 1 lb. above } 32^{\circ} \text{ F., from table} \\ = 1193.5.$$

But the feed-water is delivered at 60° F., so that $60 - 32$ or 28 B.Th.U. less heat will be required than for 1 lb. initially at 32° F.

$\therefore$ Total heat required to convert 1 lb. of water at 60° into steam at $365.7^{\circ} = 1193.5 - 28$, or 1165.5 B.Th.U.

And the total amount per hour is then found by taking the product of the heat per lb. into the output per hour, i.e.

$$\text{Heat supply per hour} = 3000 \times 1165.5 \\ = 3,496,500 \text{ B.Th.U.}$$

Symbolically, W = weight of steam per hour.

$$\text{Total heat formation} = W(H - h_f).$$

Second Solution.—Steam tables not available.

The temperature is given as 365.7° F.

$$\begin{aligned} (a) \text{ Total heat per lb.} &= 1082 + 0.3 \times 365.7 = 1193.5 \text{ B.Th.U.,} \\ &\text{less } 60 - 32 \text{ or } 28 = 1193.5 - 28 \text{ or } 1165.5 \text{ B.Th.U., as} \\ &\text{by the first method; and heat per hour is, as before,} \\ &1165.5 \times 3000. \end{aligned}$$

Case 2.—When the steam leaving the boiler is wet, and therefore carries with it a certain amount of entrained water.

Let q = weight of steam present in each pound of wet steam leaving the boiler.

Then the total heat of formation for 1 lb. is—

$$\left. \begin{array}{l} \text{Latent heat of actual steam} \\ \text{present} \end{array} \right\} + \left\{ \begin{array}{l} \text{Water heat of 1 lb. at the} \\ \text{boiler temperature;} \end{array} \right.$$

i.e.

$$qL + h.$$

Heat of formation may also be obtained thus :—

$$\left. \begin{array}{l} \text{Total heat of 1 lb. dry} \\ \text{saturated steam} \end{array} \right\} - \left\{ \begin{array}{l} \text{Latent heat equivalent of} \\ \text{entrained moisture;} \end{array} \right.$$

i.e.

$$H - (1 - q)L.$$

Example 3.—A boiler supplies 3000 lbs. of steam containing 5 per cent. of suspended moisture per hour. If the working pressure is 150 lbs. per sq. in. by gauge, with corresponding temperature = 365.7° F. , and the temperature of the feed = 60° F. , calculate the heat required per hour by the steam for its formation.

Total heat of formation for 1 lb. of dry saturated steam from water at 60° F. to steam at 365.7° F. = 1165.5 B.Th.U. (see Example 2). But since each pound of wet steam contains 0.05 lb. suspended moisture, there is 0.05 lb. of actual steam less for each pound leaving the boiler, and so the latent heat value of this quantity must be subtracted from the total heat.

$$\begin{aligned} \therefore \text{Heat of formation} \left\{ \begin{array}{l} \text{per lb.} \end{array} \right. &= 1165.5 - 0.05 \times (1114 - 0.7 \times 365.7) \\ &= 1117.6 \text{ B.Th.U.} \end{aligned}$$

$$\begin{aligned} \text{Heat of formation of 3000 lbs. from} \left\{ \begin{array}{l} \text{water at } 60^\circ \text{ F.} \end{array} \right. &= 1117.6 \times 3000 \\ &= 3,352,800 \text{ B.Th.U.} \end{aligned}$$

Case 3.—When the steam leaving a superheater fixed in connection with the boiler and flues is superheated.

In the process of superheating the steam, any moisture present in the steam entering the superheater is first evaporated, and the temperature of the steam is then raised above the temperature corresponding to the boiling point or temperature of saturation—the rise in temperature being called the degree of superheat.

The heat of formation of 1 lb. is then obviously

The total heat for 1 lb. of dry } + { The heat to superheat this
saturated steam } steam.

The heat of superheat for } = { Mass $\times$ rise of temperature $\times$
any mass of steam is } specific heat of superheated
steam.

Engineers, for the purpose of approximation, assume the specific heat of superheated steam as constant, and in the present work we shall take this constant as 0.56; hence

Heat of formation for 1 lb. superheated }
steam, reckoning from 32° F. } = $H + 0.56(t_s - t)$,

where t_s = temperature of superheated steam,

and t = „ corresponding to the pressure of saturated
steam first formed in the boiler.

If the feed temperature t_f is taken as the datum from which to reckon the heat supply, then

$$\begin{aligned}\text{Heat of formation} &= H - h_f + 0.56(t_s - t) \\ &= (1082 + 0.3t) - (t_f - 32) + 0.56(t_s - t).\end{aligned}$$

Example 4.—If the steam supply given in the above examples be superheated to 600° F., what is the heat of formation for each pound, reckoning above the feed temperature?

$$\begin{aligned}\text{Heat of formation } \left\{ \begin{array}{l} \text{(by calculation)} \end{array} \right\} &= \left\{ \begin{array}{l} (1082 + 0.3 \times 365.7) - (60 - 32) + 0.56 \\ (600 - 365.7) \end{array} \right\} \\ &= 1193.5 - 28 + 131.2 \\ &= 1296.7 \text{ B.Th.U.}\end{aligned}$$

$$\begin{aligned}\text{Heat of formation } \left\{ \begin{array}{l} \text{(using tables)} \end{array} \right\} &= 1193.5 - 28 + 131.2 \\ &= 1296.7.\end{aligned}$$

Boiler Efficiency, Evaporation, and Fuel Calculations.—The performance of a boiler is measured by

- (a) Its thermal efficiency;
- (b) The evaporation produced per lb. of coal supplied to the furnace.

To calculate the thermal efficiency, first find the weight of steam (wet or otherwise) leaving the boiler per lb. of actual fuel supplied to the furnace; then calculate the heat of formation of this steam, and express the result as a ratio of the heat value of 1 lb. of dry fuel. This ratio, which is less than unity, gives the thermal efficiency of the boiler. It is convenient to express this as a percentage by multiplying by 100. Thus, if H_s denote the heat of formation of the steam leaving the boiler per lb. of actual fuel, and H_r the heat value of 1 lb. of dry fuel, $\frac{H_s}{H_r} \times 100$ gives the percentage thermal efficiency.

In boiler practice the efficiencies range from 50 to as much as 85 per cent., the lower percentage representing a bad performance and the higher percentage a very good performance. 70 per cent. may be considered as a moderately good result to obtain for Lancashire and water-tube boilers. By a 70 per cent. efficiency we should understand that 70 per cent. of the heat which the fuel is capable of giving out in complete combustion goes to the formation of the steam leaving the boiler. If an economiser be used, the efficiencies may be increased by 5 to 15 per cent., though in a combined boiler and economiser plant the efficiency is seldom greater than 85 per cent.

In making an efficiency trial of a boiler it is necessary to determine—*

(1) The weight of steam and suspended moisture leaving the boiler in a given time. This may be found by measuring the water taken from the feed-tanks, or the condense from the hot well of the engine;

(2) The weight of coal supplied to the furnace over the time the measured feed is supplied;

(3) The percentage of suspended moisture present in the steam leaving the boiler, which can be approximated by means of a steam calorimeter;

(4) The heat value of the dried fuel, by means of a fuel calorimeter;

(5) The percentage moisture present in the fuel.

We shall first find an expression for the thermal efficiency of a boiler (1) when the steam is supplied in the ordinary wet condition; (2) when the steam is superheated before leaving the boiler plant.

* Unless the water level, boiler pressure, and output are kept fairly constant and the stoking uniformly carried out during these determinations, it will be necessary to make certain corrections on the results obtained.

The following notation will be used in the formulæ obtained :—

H_s = heat of formation of 1 lb. of wet steam.

H = heat of formation (from tables) for 1 lb. of ordinary saturated steam at the boiler pressure.

h_b = heat (from tables) in 1 lb. of water at the boiler temperature.

h_f = heat (from tables) in 1 lb. of water at the feed temperature.

W = weight of steam with its moisture leaving the boiler per lb. of dry coal, and its value is found by dividing the total feed per hour from feed-tanks by the weight of fuel per hour, less the moisture it contains.

Thus

$$W = \frac{\text{total feed per hour}}{\text{fuel per hour} - \text{fuel per hour} \times \text{moisture in 1 lb.}}$$

t = temperature of the steam at the boiler pressure.

t_f = " " feed-water.

w = weight of moisture in the W lbs. of steam leaving the boiler = $(1 - q)W$.

q = the dryness fraction of the steam.

First consider the boiler supplying steam in the ordinary wet condition.

$$\left. \begin{array}{l} \text{Heat of formation} \\ \text{of the } W \text{ lbs. of} \\ \text{wet steam leaving} \\ \text{the boiler per lb.} \\ \text{of fuel} \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat of formation} \\ \text{above the feed} \\ \text{temperature for} \\ W \text{ lbs. of dry} \\ \text{steam} \end{array} \right\} - \left\{ \begin{array}{l} \text{The latent} \\ \text{heat of} \\ w \text{ lbs.} \end{array} \right.$$

Using the symbols given above,

$$H_s = W(H - h_f) - w(H - h_b);$$

but since $H - h_b = L$, this may be written,

$$H_s = W(H - h_f) - wL.$$

A close approximation to this result may be arrived at from Regnault's formula and the temperatures, thus :—

$$\begin{aligned} H_s &= W[1082 + 0.3t - (t_f - 32)] - w(1114 - 0.7t) \\ &= W[1114 + 0.3t - t_f] - w(1114 - 0.7t). \end{aligned}$$

In the case of a boiler supplying superheated steam it is necessary to take the temperature of the steam after being superheated. Call this temperature t_s , and assume the specific heat of steam to be constant within the ranges of temperature

used for superheating, and equal to 0.56. There will be no moisture present in the steam, since it is superheated, and consequently

$$\left. \begin{array}{l} \text{The heat of formation} \\ \text{of steam leaving the} \\ \text{boiler per lb. of fuel} \\ \text{used in furnace} \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat above feed tem-} \\ \text{perature in W lbs.} \\ \text{of steam leaving the} \\ \text{boiler} \end{array} \right\} + \left\{ \begin{array}{l} \text{Heat of} \\ \text{super-} \\ \text{heat.} \end{array} \right.$$

Symbolically, this may be written,—

$$H_s = W[(H - h_f) + 0.56(t_s - t)].$$

This result may be arrived at by calculation in terms of temperatures only, thus:—

$$H_s = W[1114 + 0.3t - t_f + 0.56(t_s - t)].$$

Water evaporated per lb. of Coal.—Another way of expressing the performance of a boiler is to find the number of lbs. of water evaporated per lb. of coal used in the furnace.

(1) To determine the weight of water evaporated for each lb. of fuel as fired in the furnace.

W_1 = the feed-water pumped into the boiler per hour.

w_1 = the fuel per hour fired in the boiler furnace (this equals the genuine fuel plus the moisture present).

q_1 = the ratio of the actual fuel to the total weight of the fuel together with its moisture. This we shall call the dryness fraction of the fuel.

q = the dryness fraction of the steam.

H = the total heat of 1 lb. of saturated steam (above 32° F.).

h_f = the heat in 1 lb. of feed-water (above 32° F.).

Steam evaporated per hour = feed per hour less the moisture present in the same weight of steam

$$= W_1 - W_1(1 - q) = qW_1.$$

Hence water evaporated per lb. of fuel as used = $\frac{qW_1}{w_1}$.

(2) To find the equivalent evaporation from and at 212° F. per lb. of coal. In order to compare the results obtained from different boiler trials, it is usual to express the actual evaporation in terms of the water which could be evaporated at atmospheric pressure by the same utilisation of heat.

$$\text{Equivalent evaporation from and at } 212^{\circ} \text{ F.} \left\} = \left\{ \begin{array}{l} \text{The heat given to the water actually} \\ \text{evaporated per lb. of fuel (measured} \\ \text{above the feed temperature) divided} \\ \text{by 966, the latent heat of evapora-} \\ \text{tion of steam at } 212^{\circ} \text{ F.} \end{array} \right.$$

Equivalent evaporation from and at 212° F.

$$= \frac{\text{Weight of water evaporated} \times (H - h_f)}{966} = \frac{qW_1}{w_1} \times \frac{H - h_f}{966}.$$

The expression $\frac{H - h_f}{966}$ is called the **evaporation factor**, and its value may be calculated by taking values of H and h_f from the steam tables, or its value may be taken directly from any good mechanical engineers' pocket-book for the given temperatures of boiler steam and feed. This equivalent evaporation for actual boiler working varies from 5 to $12\frac{1}{2}$ lbs. per lb. of coal, the amount depending on the efficiency of the boiler and stoking.

(3) To find the **weight of water evaporated per lb. of dried fuel**. That is to say, we require to determine the evaporation per lb. of the actual coal used in the furnace, after deducting for the water present in the coal as fired.

$$\begin{aligned} \text{Evaporation per lb. of dried fuel} &= \frac{\text{Actual evaporation per hour}}{\text{Actual fuel used}} \\ &= \frac{qW_1}{w_1 - w(1 - q_1)} = \frac{qW_1}{q_1w_1}. \end{aligned}$$

(4) To find the **equivalent evaporation from and at 212° F. per lb. of dried fuel.**

$$\begin{aligned} \text{Equivalent evaporation} \left\} &= \left\{ \begin{array}{l} \text{Water evaporated per} \\ \text{lb. of dried fuel} \end{array} \right\} \times \text{Evaporation factor} \\ &= \frac{qW_1}{q_1w_1} \times \frac{H - h_f}{966}. \end{aligned}$$

Boiler and Engine Steam; Determination of the Dryness Fraction.—Wet Steam.—The steam taken from the boiler plant is seldom or never in the dry saturated condition, but contains a varying percentage of suspended or entrained moisture. If the amount is appreciably large, it is called **priming water**. The presence of water in the steam supplied to an engine is uneconomical, and may be injurious, if too much priming takes place. Its amount depends upon many factors, such as the quality of the water in the boiler and its liability to foam during ebullition;

the activity of ebullition and the rate of steam supply ; the area of the water surface and the volume of the steam space ; the density of the steam and the position of the boiler stop-valve. The quantity of moisture present in the steam entering the engine stop-valve depends upon the radiation from the steam pipe, fluctuations of pressure due to intermittent demand, in addition to the foregoing. In the engine cylinder the moisture present in the steam is a varying quantity during expansion. Determinations of the moisture present in the steam leaving the boiler or entering the engine are attempted by means of steam calorimeters, of which there are various forms in use.

Dryness Fraction.—The actual amount of steam present in 1 lb. of wet steam is known as the dryness fraction of the steam (q).

It is also equal to $\frac{W - w}{W}$, where W represents a definite weight

of wet steam, and w the moisture present in it. We shall explain briefly the principles and necessary calculations for the calorimeters commonly used for the determination of this moisture, but would first point out that the dryness fractions should be determined by sampling the steam just outside the boiler and engine stop-valves—the first, to enable the boiler efficiency to be calculated ; and the latter, for the engine calculations.

(1) *The Separating Method.*—It is the common practice in high-class steam engines to use a separator to abstract almost the whole of the water from the steam just before it enters the engine. A special separating calorimeter on a small scale can also be applied to the steam main anywhere between the boiler and engine, for the purpose of sampling the steam. The method consists in passing steam through the calorimeter, mechanically separating the moisture, and condensing the approximately dry steam passed on in a small condenser provided for the purpose.

Let w = weight of steam condensed,

w_1 = weight of moisture separated ;

then the dryness fraction (q) = $\frac{w}{w + w_1}$.

Example.—A test with a separating calorimeter gave 2.24 lbs. of steam condensed, and 0.25 lb. of water in the separator ; what was the dryness of the original steam ?

$$q = \frac{2.24}{2.24 + 0.25} = 0.90, \text{ or } 90 \text{ per cent.}$$

(2) *The Condensing Method.*—Sample steam from the main is

allowed to flow for a short time into a condenser containing a definite weight of water, the weight of the condenser being taken both before and after the steam is admitted. The gain in weight represents the wet steam condensed.

Let W_c = weight of water in condenser together with the water equivalent of the condenser.

W = weight of the steam together with the moisture run into the condenser.

t_s = temperature of steam entering the condenser.

t_i = initial temperature of condenser.

t_f = final temperature of condenser.

$$\text{Heat gained by condenser} = W_c(t_f - t_i).$$

$$\left. \begin{array}{l} \text{Heat lost by the} \\ \text{wet steam added} \\ \text{to condenser} \end{array} \right\} = \left\{ \begin{array}{l} \text{Latent heat of} \\ \text{the } qW \text{ lbs. of} \\ \text{actual steam} \end{array} \right\} + \left\{ \begin{array}{l} \text{Water heat of } W \\ \text{lbs. above } t_f \end{array} \right\}$$

$$= qW(1114 - 0.7t_s) + W(t_s - t_f).$$

But Heat gained by condenser = Heat lost by steam.

$$\therefore W_c(t_f - t_i) = qW(1114 - 0.7t_s) + W(t_s - t_f);$$

and hence

$$q = \frac{W_c(t_f - t_i) - W(t_s - t_f)}{W(1114 - 0.7t_s)},$$

or

$$= \frac{W_c(t_f - t_i) - W(t_s - t_f)}{WL},$$

where L represents the latent heat of 1 lb. of steam corresponding to the temperature t_s as taken from the tables.

(3) *Wire-drawing or Throttling Calorimeter*.—If ordinary wet steam expands without doing work, or without receiving or rejecting heat, it becomes drier during expansion; and consequently, if the initial moisture present is sufficiently small, the steam after expansion may be in a superheated condition. This principle is taken advantage of in the construction and use of a wire-drawing calorimeter. Fig. 140 represents a throttling calorimeter as applied to a steam-pipe. A represents a cross section of the steam-pipe; B , the collecting tube for the calorimeter; C , the receptacle for thermometer giving initial temperature of steam; D , a plate bolted between two flanges, with a fine orifice about $\frac{1}{16}$ in. diameter through the centre, through which the steam is wire-drawn; E , a receptacle for thermometer to give temperature of steam after being wire-drawn and superheated.

Let L = latent heat and t = temperature of steam to be tested.
 L_p = latent heat of steam corresponding to reduced pressure.
 t_a = temperature of steam after being wire-drawn.
 h = the water heat of steam before wire-drawing.
 h_p = " " " after " corresponding to reduced pressure.
 t_p = temperature of saturated steam corresponding to the reduced pressure after wire-drawing.

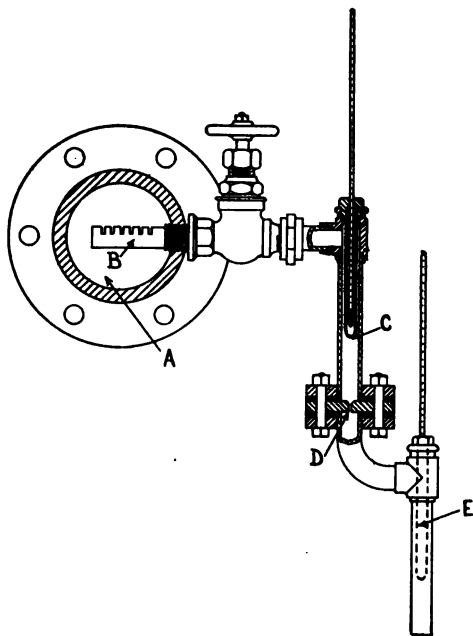


FIG. 140.

Dealing with 1 lb. of steam,

Total heat of steam } = Total heat of steam after throttling.
 before throttling

But the total heat of 1 lb. of the steam before throttling = $qL + h$,
 and

The total heat after } = \left\{ \begin{array}{l} \text{Total heat of 1 lb. of} \\ \text{saturated steam at} \\ \text{the reduced pressure} \\ \text{corresponding to the} \\ \text{temperature } t_p \end{array} \right\} + \left\{ \begin{array}{l} \text{The heat} \\ \text{of super-} \\ \text{heat} \end{array} \right\}

$$\begin{aligned}
 &= L_p + h_p + 0.56(t_a - t_p); \\
 \text{hence} \quad qL + h &= L_p + h_p + 0.56(t_a - t_p), \\
 \text{and} \quad \therefore q &= \frac{L_p + h_p - h + 0.56(t_a - t_p)}{L}.
 \end{aligned}$$

This expression is in a convenient form for calculation when the steam tables are used. It may, however, be expressed in terms of the temperature and Regnault's formulæ, thus :—

$$\begin{aligned}
 q &= \frac{(1114 - 0.7t_p) + (t_p - 32) - (t - 32) + 0.56(t_a - t_p)}{1114 - 0.7t} \\
 &= \frac{1114 + 0.3t_p - t + 0.56(t_a - t_p)}{1114 - 0.7t} \\
 &= \frac{1114 + 0.56t_a - t - 0.26t_p}{1114 - 0.7t}.
 \end{aligned}$$

Example.—In a test with a throttling calorimeter the steam in the steam-pipe is at a pressure of 102 lbs. per square inch absolute, and at a temperature of 329° F. The temperature and pressure after throttling are 220° F. and 14.7 lbs. per square inch absolute. What was the dryness fraction of the steam leaving the boiler (the temperature of saturated steam at 14.7 lbs. per square inch is 212° F.). [Lond. Univ. B.Sc. Engineering.]

$$\begin{aligned}
 \text{Ans.} \quad q &= \frac{1114 + 0.56 \times 220 - 329 - 0.26 \times 212}{1114 - 0.7 \times 329} \\
 &= 0.96.
 \end{aligned}$$

When the steam exhausts into the atmosphere after throttling, the reduced pressure may be taken at 16 lbs. per square inch, with a corresponding final temperature after throttling of 216° F.

If the steam is very wet, the final temperature falls to 212° F., the wire-drawing being insufficient to dry the steam.

It is important that the chamber into which the steam exhausts after being wire-drawn should be sufficiently large to allow the eddies to die out before the steam escapes, and also that the radiation losses from the apparatus are reduced as much as possible by a covering.

Professor Unwin states* that the throttling calorimeter cannot be used if the wetness exceed the following values :—

Initial Temperature.	Initial Wetness.
250° F.	0.8
300° F.	2.44
350° F.	4.21
400° F.	6.13.

* *Proceedings of the Institution of Mechanical Engineers*, Parts 1 and 2, 1895.

(4) *Combined Separating and Wire-drawing Calorimeter.*—This method is a combination of (1) and (3). Fig. 141 represents a combined apparatus. By referring to this figure, it will be seen that a separating chamber F is introduced between the steam main and the throttling calorimeter. The bulk of the moisture is separated and collected in F, the level being indicated by a gauge-glass G. The quantity of moisture thus separated in a given time can be drawn off from a tap, and weighed, or the

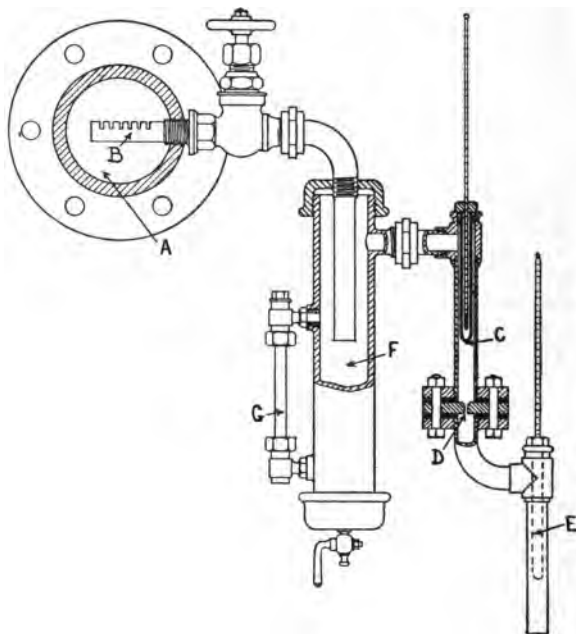


FIG. 141.

gauge glass can be calibrated. It is necessary when using this form of apparatus to conduct the steam after wire-drawing into a condenser, in order to determine the total weight of steam passing out.

We shall show that the dryness fraction of the steam may be found by first expressing the ratio of the weight (w lbs.) of steam condensed to the total weight ($w + w_1$) of steam and water passing through the apparatus, and multiplying this result by the dryness fraction of the steam entering the throttling part of the calorimeter— w_1 representing the water collected in the separator.

$$\left. \begin{array}{l} \text{Dryness fraction of steam} \\ \text{after leaving separator,} \\ \text{just before being wire-} \\ \text{drawn} \end{array} \right\} = q_1 = \frac{L_p + h_p - h + 0.56(t_a - t_p)}{L}.$$

Water in 1 lb. of steam entering the throttling calorimeter = $1 - q_1$.

$$\therefore \text{Water in } w \text{ lbs.} = w(1 - q_1).$$

Total water in steam taken from steam main = $w_1 + w(1 - q_1)$.

Dryness fraction of this steam,

$$q = \frac{w + w_1 - \{w_1 + w(1 - q_1)\}}{w + w_1} = \frac{wq_1}{w + w_1}.$$

Fuel and Flue Gases.—Other very important, though somewhat more advanced, calculations are given below, and illustrated throughout by means of a practical exercise, the data for which is taken from the report of the Committee on Steam Engine and Boiler Trials, under the authority of the Institution of Civil Engineers, and in working through this exercise, the authors have followed closely the recommendations of this committee.

The analyses of the fuel and flue gases are given, and the calculations scheduled in (a), (b) . . . to (j) are obtained from them.

Analyses of Fuel by Weight.		Analyses of Flue Gases by Volume.	
Carbon	0.840	Carbonic acid . .	0.082
Hydrogen	0.056	Carbonic oxide . .	0.000
Ash	0.030	Oxygen	0.112
Oxygen, etc. . . .	0.074	Nitrogen	0.806
	1.000		1.000

(a) In making calculations from the above analyses, we must first convert the volumetric analysis of the flue gases into the composition by weight. The results are shown in Table XXIII.

The numbers in the third column are obtained by taking half the sum of the combining proportions by weight of the elements. Thus for CO_2 we have $\frac{1}{2}(12 + 2 \times 16) = 22$. The results in the third column are found by taking the products of the relative densities (column 3) into the proportions by volume (column 2), and the composition by weight in unit mass of the flue gases is given by dividing the results in column 4 by their sum (14.868).

(b) We next find the weight of carbon present in 1 lb. of dry flue gases. It is usually assumed that the whole of the carbon in

the flue gases is derived from the coal burnt, and if there are no unburnt hydrocarbon gases, this carbon is contained in the CO_2 and the CO (if any of the latter is present). Now CO_2 contains $\frac{3}{11}$ th part by weight carbon, and CO contains $\frac{3}{7}$ th, therefore,

Carbon in 1 lb. of dry flue gases = $\frac{3}{11} \times 0.121 + \frac{3}{7} \times 0.00 = 0.033$ lb.

(c) We can now obtain the *weight of dry flue gases per lb. of carbon* by taking the reciprocal of the above result, i.e. $\frac{1}{0.033} = 30.3$ lbs.

(d) From this result and the weight of carbon (0.84) in the solid and dry fuel we calculate the *weight of dry flue gases per lb. of dry coal*, thus,

Dry flue gases per lb. of dry fuel = $0.84 \times 30.3 = 25.45$ lbs.

TABLE XXIII.

	Volumetric Analysis.	Relative Density.	Weight Ratios.	Composition by Weight.
CO_2	0.082	22	1.804	0.121
CO	0.000	14	0.000	0.000
O_2	0.112	16	1.792	0.120
N_2	0.806	14	11.284	0.759
	1.000		14.868	1.000

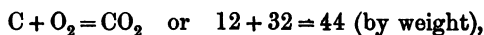
(e) We have, however, to deal with flue gases containing *steam* as one of the products of combustion (not shown in the analysis of the gases), and also the steam derived from the evaporation of the water in the coal. The weight of the first quantity is obviously nine times the weight of the hydrogen present in the fuel, and in this example this is $0.056 \times 9 = 0.50$ lb.

(f) Consequently the *total weight of the flue gases per lb. of dried fuel* is (in this example) $25.45 + 0.50 = 25.95$.

(g) The *weight of air drawn in per lb. of dried coal burnt* is obviously found by subtracting the weight of the combustible and gaseous matter in the coal from this last result, and = $25.95 - 0.970 = 24.98$ lbs., say, practically 25 lbs.

But since this result = air theoretically required for combustion + the excess air, we must next calculate the air theoretically required from the combustible matter in the coal. The student

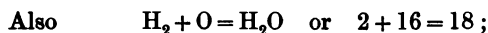
will, no doubt, remember that air contains practically 23·2 parts by weight oxygen and 76·8 parts nitrogen. The calculation may be set out as follows:—



i.e. 12 lbs. of C requires 32 lbs. O ;

$\therefore$ 1 lb. „ „ $\frac{32}{12} = \frac{8}{3}$ lbs. O,

and 1 lb. „ „ $\frac{32}{12} \div \frac{23\cdot2}{100} = 11\cdot49$ lbs. of air.



i.e. 1 lb. of H requires 8 lbs. of O,

or 1 lb. „ „ $8 \div \frac{23\cdot2}{100} = 34\cdot48$ lbs. of air.

(*h*) From these results and the analysis of the coal we obtain *air theoretically required per lb. of coal*

$$= 0\cdot84 \times 11\cdot49 + 0\cdot056 \times 34\cdot48 = 11\cdot58 \text{ lbs.}$$

(*i*) Subtracting this from total air drawn in, we have $25 - 11\cdot58 = 13\cdot42$ lbs. as the *excess air per lb. of dried fuel*.

(*j*) One other important calculation, in addition to those already given, is in the determination of the heat lost in evaporating and superheating the moisture from the fuel. This is given by

$$\left. \begin{array}{l} \text{Weight of moisture} \\ \text{per lb. of coal} \end{array} \right\} \times (212 - t) + 966 + 0\cdot56(t_f - 212),$$

where t denotes the temperature of inlet air and t_f the temperature at gas exit from boiler.

Boiler for a given Steam Output.—To enter scientifically into the numerous details of boiler design would take us quite beyond the scope of this volume ; but as it is of practical importance to be able to form rapidly some idea of the size of boiler to furnish a given steam output, we shall conclude this chapter by giving a rough method of determining the diameter and length of a Lancashire boiler. For this purpose we take a simple exercise, and give in conclusion Table XXIV.

Let it be required to find the dimensions of the boiler to furnish steam for a good compound condensing engine giving 100 H.P. on the brake. As the brake or effective horse-power is required to be 100, the indicated horse-power developed in the cylinders

might have to be some 15 per cent. more than this to allow for the internal friction of the engine and the power required for the necessary pumps. The steam consumption for a high-class condensing engine of this size should not be more than about 18 lbs. per I.H.P. per hour, which gives the quantity of steam required as $115 \times 18 = 2070$ lbs. of steam per hour, or 207 gallons of water per hour to be evaporated by the boiler. An approximate rule for the steaming capacity of an ordinary Lancashire or two-flued boiler is, that the number of gallons of water evaporated per hour is equal to one and a half times the product of the diameter of the boiler (in feet) multiplied by its length (in feet). This rule may be symbolically represented thus:—

$$W = D \times L \times 1\frac{1}{2},$$

where

W = gallons of water per hour,

D = diameter of shell of boiler in feet,

L = length of boiler in feet.

Thus if the boiler has to evaporate 207 gallons of water per hour, the product

$$D \times L \times 1\frac{1}{2} = 207,$$

and therefore

$$D \times L = \frac{207}{1\frac{1}{2}} = 138.$$

Now if D be taken as 6 ft., L works out to 23 ft.; or, if D be taken at 6 ft. 6 in., then L must be 21 ft. Either of these two sizes of boiler therefore should, with average conditions of draught and fuel, furnish sufficient steam for the engine specified under the assumed conditions. Table XXIV., p. 309, gives the approximate heating surfaces and steaming capacities of ordinary Cornish and Lancashire boilers, not allowing for Galloway tubes, assuming an evaporation of 4 lbs. of water per hour per square foot of heating surface.

An approximate rule for finding the *heating surface of an ordinary cylindrical boiler*, is to add $\frac{1}{2}$ of the circumference of the shell to the circumference of the flue or flues, and multiply the sum by the length of the boiler, all dimensions being expressed in feet; and the result gives the heating surface in square feet.

Thus for the 6 ft. 6 in. $\times$ 21 ft. 0 in. Lancashire boiler just quoted, the approximate heating surface will be:—

$$\left\{ \frac{3\frac{1}{2} (6 \text{ ft. } 6 \text{ in.})}{2} + 2 \times 3\frac{1}{2} (2 \text{ ft. } 6 \text{ in.}) \right\} \times 21,$$

i.e.

$$3\frac{1}{2} (3\frac{1}{2} + 5) \times 21 \\ = 545 \text{ sq. ft.}$$

This rule is approximate only, as it assumes a height of side flue such that the exposed portion of the shell is one half of the whole, and it neglects the area of the back plate of the boiler, which is, however, comparatively small. This size of boiler supplying 545 square feet of heating surface to evaporate 207 gallons of water per hour, gives an evaporation of about 3·8 lbs. of water per square foot of heating surface per hour.

It is useful to remember that the most economical results for Lancashire boilers are given with evaporations varying from 3 to 4½ lbs. per square foot of heating surface.

TABLE XXIV.—APPROXIMATE HEATING SURFACE AND STEAM CAPACITIES OF STEAM BOILERS.

CORNISH BOILERS.						LANCASHIRE BOILERS.									
Diam. of Boiler.		Diam. of Flue.		Length of Boiler.	Approx. Heating Surface. sq. ft.	Gallons Water evapo- rated per Hour.	Diam. of Boiler.		Diam. of Flues.		Length of Boiler.	Approx. Heating Surface. sq. ft.	Gallons Water evapo- rated per Hour.		
ft.	in.	ft.	in.	ft.	in.		ft.	in.	ft.	in.	ft.	in.			
5	0	2	6	15	0	235	94	6	0	2	3	15	0	355	142
5	0	2	6	20	0	315	126	6	0	2	3	20	0	470	188
5	0	2	6	25	0	390	156	6	0	2	3	25	0	590	236
5	6	2	9	15	0	260	104	6	0	2	3	30	0	705	282
5	6	2	9	20	0	345	138	6	6	2	6	20	0	520	208
5	6	2	9	25	0	430	172	6	6	2	6	25	0	650	260
6	0	3	0	15	0	280	112	6	6	2	6	30	0	775	310
6	0	3	0	20	0	375	150	7	0	2	9	20	0	566	226
6	0	3	0	25	0	470	188	7	0	2	9	25	0	705	282
6	0	3	0	30	0	565	226	7	0	2	9	30	0	850	340
6	6	3	3	20	0	410	164	7	6	3	0	20	0	610	244
6	6	3	3	25	0	510	204	7	6	3	0	25	0	765	306
6	6	3	3	30	0	610	244	7	6	3	0	30	0	920	368
7	0	3	6	20	0	440	176	8	0	3	3	25	0	825	330
7	0	3	6	25	0	550	220	8	0	3	3	30	0	990	396
7	0	3	6	30	0	660	264	8	0	3	3	35	0	1150	460

6. Determine the weight of excess air per lb. of coal burnt from the following data :—

Flue Gases.		Coal.	
$\text{CO}_2 = 7.0$ per cent.	} by vol.	$\text{C} = 85$ per cent.	} by weight.
$\text{O} = 11.3$ „		$\text{H} = 3$ „	
$\text{N} = 81.7$ „		$\text{O} = 4.8$ „	
		Ash, etc. = 7.2 „	

(Univ. of London, B.Sc. Engineering.)

7. Having given the results stated below for a boiler trial, calculate—

- Air admitted per lb. of coal.
- Excess air „ „
- Heat carried away by flue gases expressed as a percentage of heat value of fuel.

Fuel.		Flue Gases.	
$\text{C} = 83.97$ per cent.	} by weight.	$\text{CO}_2 = 8.20$	} by vol.
$\text{H} = 5.58$ „		$\text{CO} = 0.00$	
$\text{O} = 7.49$ „		$\text{O} = 11.17$	
Ash, etc. = 2.96 „		$\text{N} = 80.63$	

Temperature of inlet air, 61°F .

„ at gas exit, 452°F .

CHAPTER XV.

THE ENGINE AND ITS CALCULATIONS.

Engine Power.—The power of an engine, or the rate at which it can do work in terms of the engineer's unit (33,000 ft.-lbs. per minute) is expressed in two ways:—

(1) By determining the **actual horse-power developed** by the working substance in the engine cylinder or cylinders. This is called the **Indicated Horse-Power**, being so named from the fact that the mean effective pressure of the working substance during the cycle of operations is determined by means of an instrument called the indicator. Refer to p. 320 for a general description of this instrument.

(2) By finding the **horse-power actually available for work at the engine shaft**. This power is termed the **Brake Horse-Power**, because of the fact that it is actually determined by means of some brake dynamometer. It is obviously always less than the indicated horse-power—the **difference** between the two being that part of the indicated horse-power which is absorbed in overcoming the friction of the rubbing surfaces of the several working parts of the engine, such as the piston and cylinder, piston rods, valve rods in stuffing-boxes, cross-head in guides, connecting-rod ends on pins, crank-shaft in bearings, etc. We usually denote the indicated horse-power of an engine by the capital letters **I.H.P.**, and the brake horse-power by the capital letters **B.H.P.** The ratio of **B.H.P.** to **I.H.P.** is termed the **mechanical efficiency** of the engine or motor, considered as a machine, work being put in at one part (in the cylinders), and given out, less the friction, at another part (at the crank-shaft). We may represent these results symbolically as follows:—

$$\text{I.H.P.} - \text{B.H.P.} = \text{Horse-power lost in friction (H.P.)},$$

or

$$\text{I.H.P.} = \text{B.H.P.} + \text{H.P.},$$

$$\frac{\text{B.H.P.}}{\text{I.H.P.}} = \text{Mechanical efficiency},$$

which is usually represented by percentage thus :—

$$\frac{\text{B.H.P.}}{\text{I.H.P.}} \times 100 = \text{Percentage mechanical efficiency.}$$

From this it follows that

$$\frac{\text{B.H.P.}}{\text{Mechanical efficiency}} = \text{I.H.P.},$$

and $\text{I.H.P.} \times \text{Mechanical efficiency} = \text{B.H.P.}$

The higher the mechanical efficiency for a given load, the more perfect the working of the mechanism.

Further, the mechanical efficiency of an engine, like that of any other machine, increases with the load up to a definite maximum. For large engines, well made and in perfect adjustment, the mechanical efficiency at full load is often as high as from 90 per cent. to 92 per cent., and in some few cases it has reached even 94 per cent.; but it usually lies between 85 and 90, falling in very small engines to as low as 80 per cent. In gas engines the mechanical efficiency is lower than in steam engines.

Indicated Horse-Power.—The power developed in the cylinder of an engine is calculated from the following formula (see Chapter VIII.) :—

$$\text{I.H.P.} = \frac{P \times L \times A \times N}{33000},$$

where P = mean effective pressure on piston in pounds per square inch,

L = twice the length of stroke in feet (for double-acting engines only),

A = mean nett area of piston in square inches,

N = number of revolutions per minute.

Although this is the usual calculation for determining horse-power, it fails to take into account the diameter of the piston rod. In the case of a piston with a single rod, the steam pressure acts on the full area of one side of the piston, and on the other side on an area equal to the difference of the area of the piston and the sectional area of the rod. To allow for these two different areas, and to prevent unnecessary calculation, a piston area is taken in the formula = the full area of the piston - $\frac{1}{2}$ the area of the piston rod.

The cylinder horse-power then becomes—

$$\text{I.H.P.} = \frac{P \times \frac{\pi}{4} \left(D^2 - \frac{d^2}{2} \right) \times L \times N}{33000},$$

where D = diameter of piston, and d = diameter of piston rod. Sometimes, however, a tail-rod is carried from the piston through the other cylinder cover; then, if d_2 is the diameter of the tail-rod, the mean area to be taken in the formula is

$$\frac{\pi}{4} \left[D^2 - \frac{1}{2} (d_1^2 + d_2^2) \right].$$

For *single-acting engines* with an impulse only *once* per revolution, take L = length of *one* stroke in feet.

For *gas or oil engines*, working on the Otto cycle, *i.e.* with an impulse at its maximum only once in every two revolutions, take L = length of one stroke in feet, and N = number of explosions per minute.

For *compound or multi-cylinder engines*, calculate the horse-power developed in each cylinder separately, and add them together to find the total I.H.P.

Examples on the use of the formulæ:—

(1) Find the I.H.P. of a steam engine whose cylinder is 12 in. diameter, stroke 18 in., making 100 revolutions per minute, the mean effective pressure being 33 lbs.

Here $P = 33$ lbs.,

$L = 3$ ft. (two strokes, double-acting),

$A = \text{area of 12-in. piston} = \frac{12 \times 12 \times 11}{14} = 113$ sq. in.,

$N = 100$.

Then $\text{I.H.P.} = \frac{33 \times 3 \times 113 \times 100}{33000} = 33.9$ I.H.P.

(2) Find the I.H.P. of a 12 H.P. "nominal" Crossley gas engine, given the following particulars:—

Cylinder, 10 in. diameter.

Stroke, 16 in.

Revolutions per minute = 160.

Explosions per minute = 66.

Mean effective pressure = 80 lbs.

Here $\text{I.H.P.} = \frac{80 \times 1\frac{1}{2} \times 78\frac{1}{2} \times 66}{33000} = 16.8$ I.H.P.

Students should note that in this example the revolutions per minute are not required, the explosions per minute being used.

(3) Find the total I.H.P. of a compound engine, given the following particulars :—

Cylinders, 12 in. and 20 in. diameter.

Stroke, 2 ft.

Revolutions, 120 per minute.

Mean effective pressure = 44 lbs. in high-pressure cylinder.

" " " = 15 lbs. in low- " "

Here the power is divided into the two cylinders thus:—

$$\text{In H.P. cylinder, I.H.P.} = \frac{44 \times 4 \times 113 \times 120}{33000} = 72.3$$

$$\text{In L.P. cylinder, I.H.P.} = \frac{15 \times 4 \times 314 \times 120}{33000} = 68.5$$

Total I.H.P. = 140.8

Measurement of Brake Horse-Power (B.H.P.).—The B.H.P. of an engine or other prime mover is usually determined experimentally by providing some artificial resistance, to **absorb the power**, which is capable of being readily measured. The type of brake and the method of experimenting depend to some extent upon the power and speed of the engine. An early type of brake, still used in several forms, is represented in fig. 142. It consists essentially of two wooden blocks clamped round a small pulley on the engine shaft. A long lever is fixed to the top brake-block, carrying at the shorter arm of the lever a weight to balance the longer arm and the weight of the spring balance or scale-pan, whichever is applied. The balancing is done before the brake-blocks are made to grip the pulley and while the engine is stationary. In loading up the engine, the two thumb-screws are gradually tightened, giving more and more frictional grip between the rim of the pulley and the brake-blocks. After each adjustment of the thumb-screws, additional weight is placed in the scale-pan, or additional tension on the balance if it is used, until the lever is quite horizontal.

These adjustments will be repeated until the engine is running at its normal speed with the required load on the engine.

The whole of the work available at the engine shaft is absorbed in friction round the rim of the pulley.

Let F denote the resultant force of friction acting round the circumference of the pulley whose radius is R , and let N denote the number of revolutions per minute. Then the work done per minute against friction in ft.-lbs.

= force of friction, F , $\times$ circumferential velocity of pulley ($2\pi RN$)

or $\text{B.H.P. absorbed in friction} = \frac{2\pi RNF}{33000}$.

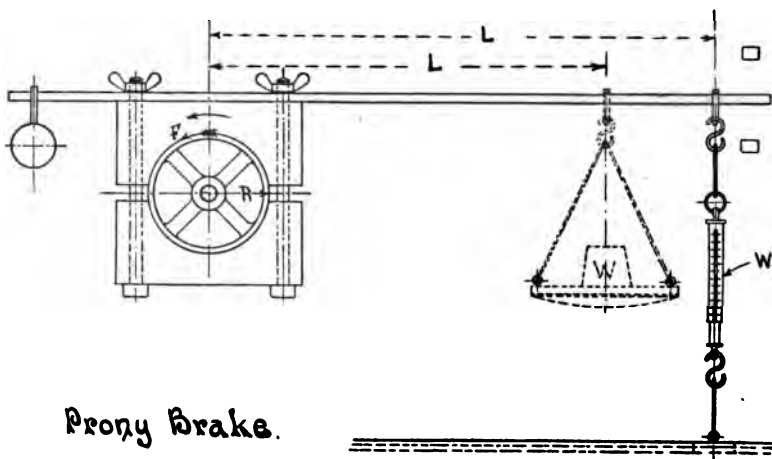


FIG. 142.

But $F \times R$, the moment of friction, = $W \times L$ (see fig. 142).

$$\therefore \text{B.H.P.} = \frac{2\pi LWN}{33000}.$$

This method is not suitable for large powers, as the heat developed, representing the equivalent of the work absorbed, would be quite sufficient to fire the blocks.

A more usual method of finding the B.H.P. is represented in fig. 143. A double rope is wound round the rim of the flywheel, the upper end of the rope being attached to a spring balance, and the lower end carrying weights, which are gradually increased until the engine is developing the power which it is required to test on

the brake. The rope is kept in position on the rim of the pulley by a number of wooden blocks at intervals, to which it is wired. If F denotes the pull in the spring balance, W the weights, R the distance in feet from the centre of the shaft to the centre of the rope;

$$\text{then B.H.P.} = \frac{(W - P) \times 2\pi R \times N}{33000}.$$

This type of brake is used for fairly large powers. For experimental work, where long and continuous running is sometimes required, the sides of the flywheel near the rim are flanged over, and a water circulation provided to keep the rim cool.

For finding the B.H.P. of an **electrical motor** the **Soame's dynamometer or brake** is largely used; it is represented in fig. 144, and needs very little description. It is carried by a tripod, and is therefore portable, and can be fixed over any motor which it is required to test from the ground-level. The sum of the tensions on the two sides of the band passing round the brake pulley is regulated by means of the screw and wheel in connection with the head of the tripod, the lower end of the screw being enlarged, and carrying the fulcrum of the lever. By means of the adjustment on the screw, and the addition of weights W and w regulating the difference of tensions, the required horse-power can be absorbed. Thus, referring to the figure, since we have equilibrium,

$$wl + T_2 r = T_1 r + WL,$$

or

$$\begin{aligned} (T_2 - T_1)r &= WL - wl, \\ 2\pi N(T_2 - T_1)r &= 2\pi N(WL - wl); \end{aligned}$$

and

$$\therefore \text{B.H.P.} = \frac{2\pi N(WL - wl)}{33000};$$

and if, as is usually the case, $L = l$,

$$\text{B.H.P.} = \frac{2\pi N L (W - w)}{33000}.$$

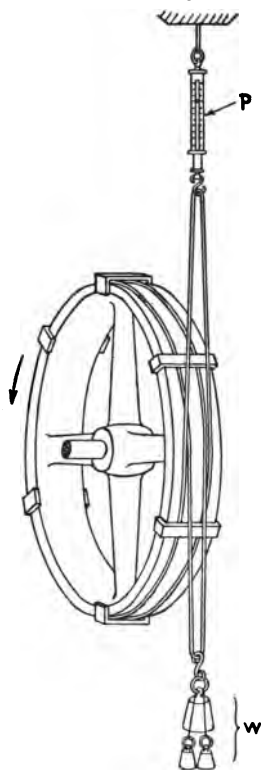
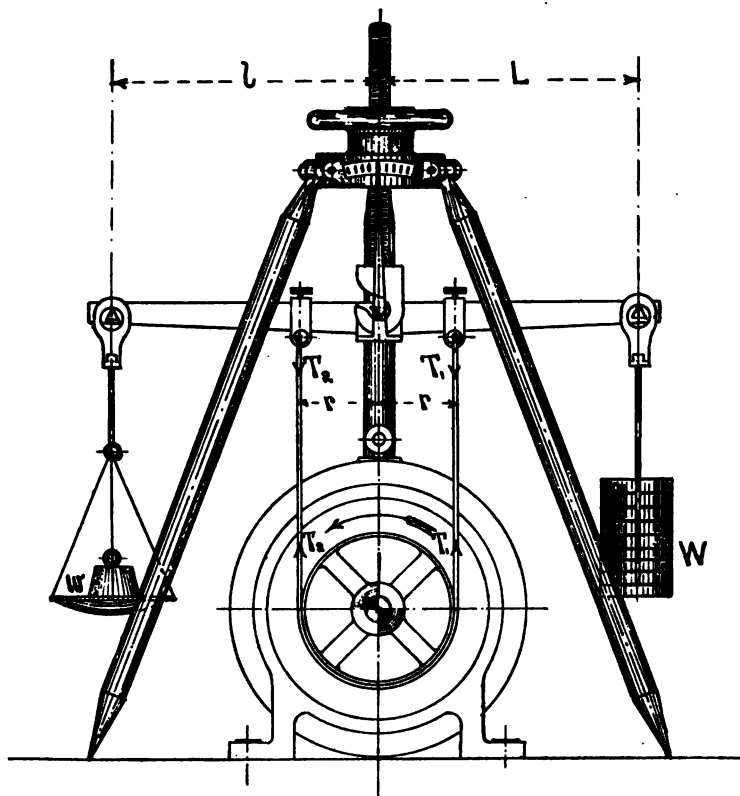


FIG. 143.

For high-speed petrol engines tested in the works, the brake represented in fig. 145 has been successfully applied. A detailed description of this is unnecessary. It will be sufficient if we point



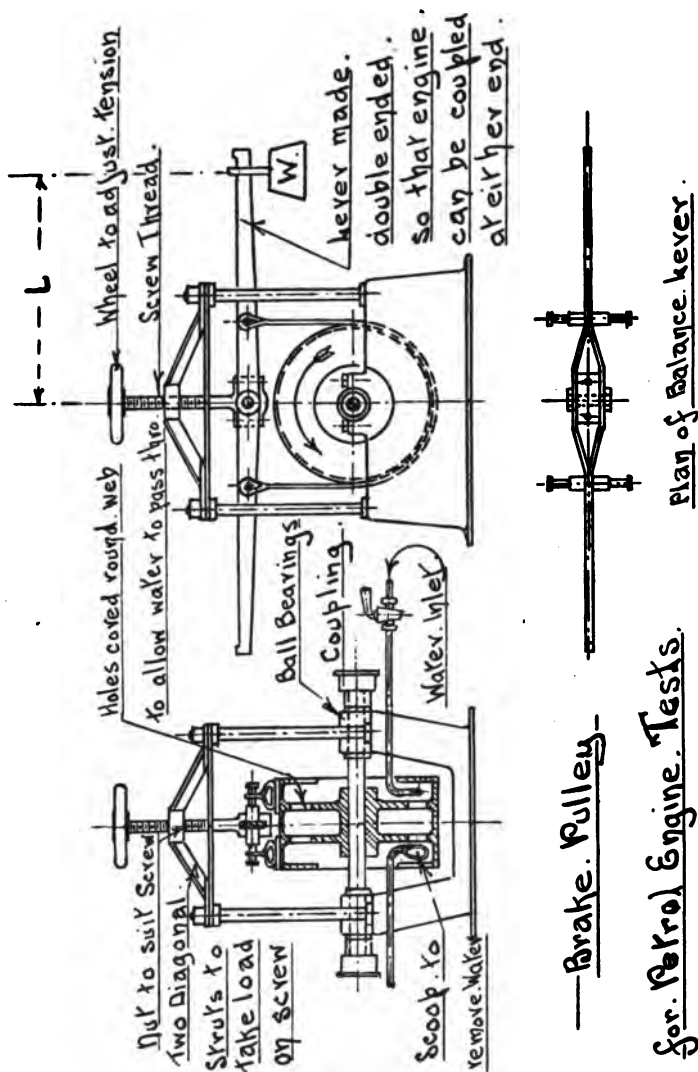
Soame's Dynamometer

FIG. 144.

out two or three of the special features, and give the calculation for the determination of B.H.P.

The brake pulley, which is flanged at both sides for a water circulation, is carried by a shaft running in ball bearings. At each end of the shaft is a special coupling by means of which a short flexible drive can be obtained from the motor to be tested. Two

wide brake straps are applied round the specially wide pulley and



attached to a lever carried by a vertical screw passing through a nut fixed in the cross-head at the top of the frame. The loading

of the brake is effected by means of a weight or of weights W , which can be moved into any position on the lever. After the final adjustment has been made, the lever being horizontal, the moment $W \times L$ = the force of friction around the circumference of the pulley $\times R$, and

$$\text{B.H.P.} = \frac{2\pi LNW}{33000},$$

i.e. if we neglect the additional friction introduced by the bearings of the dynamometer; but as a correction is applied, we may write—

$$\text{B.H.P.} = \frac{2\pi LNW}{33000} + \text{correction.}$$

A water circulation is provided to keep the pulley cool, water being run in over one flange of the pulley and scooped out near the other flange continuously. Centrifugal force ensures a fairly uniform depth of water round the entire rim.

Mean Effective Pressure.

Indicator Diagrams.—As already pointed out, when an engine is tested for power, diagrams are taken by means of an indicator (from each end of each cylinder, if the engine is double-acting) representing the varying pressures and volumes of the working substance as the piston moves through the cylinder. The indicator is shown in outline in its application in figs. 146 and 147. One indicator may be used for both sides of the piston, each in turn, as shown in fig. 147, a diagram being taken from each end alternately. For large engines with a high rotative speed an indicator is generally used at each end of the cylinder, with a separate cock. The stroke of the engine piston is reproduced on a much smaller scale on the indicator diagram by means of some special reducing motion actuated from some moving part of the engine. Fig. 146 represents the application of the indicator with a reducing gear to a single-acting combustion engine. In fig. 147 the indicator is shown with a reducing gear in the case of a double-acting steam engine. A full description of various types of indicators and reducing gears, and the methods of taking indicator diagrams, will be found in any good text-book on the steam engine. It will be sufficient for our present purpose if we describe the instrument in general terms. The indicator consists of a cylinder with a loosely fitting piston, the rod of which actuates the pencil tracing the diagram round the drum D , through suitable links giving a magnification of the

piston's vertical motion. The pressure on the under side of the indicator piston due to the working substance is balanced by the

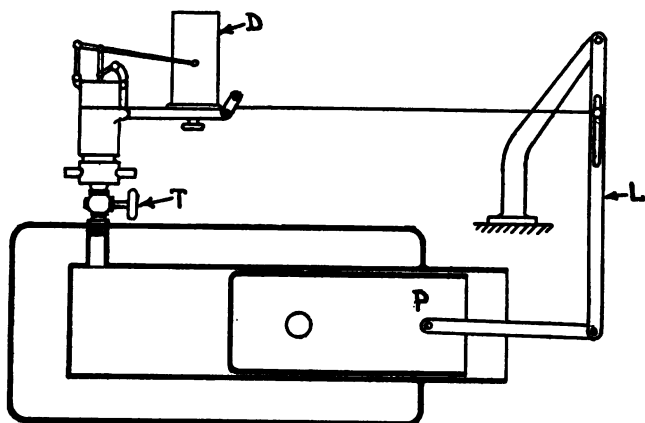


FIG. 146.

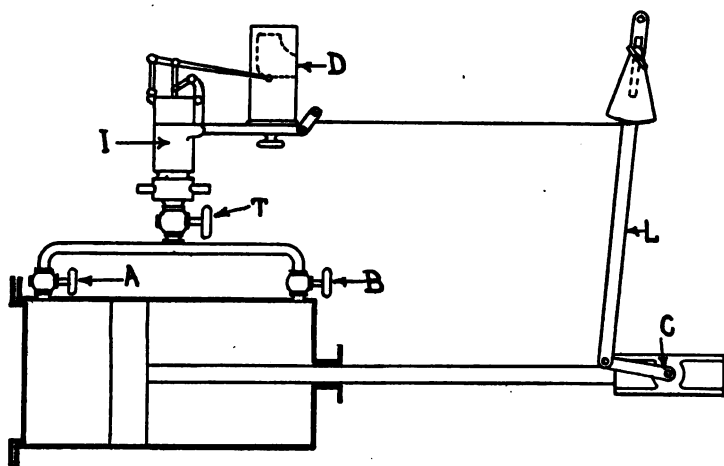


FIG. 147.

resistance of a helical spring on the upper side of the piston between it and the cover, the compression of the spring being proportional to the pressure of the steam or gas on the under

side of the piston. The strength of the spring is known in terms of the pressure, and is so stamped. Thus, for example, suppose the spring is stamped $\frac{1}{100}$, this would indicate that a vertical displacement of 1 in. by the pencil on the indicator card would correspond to 100 lbs. pressure per square inch of the gas on the under side of the indicator piston; and since compressions and extensions of a helical spring are proportional to load or pressure, it follows that each lb. pressure would be represented by a vertical displacement of $\frac{1}{100}$ in. The indicator drum D carrying the card turns on its own axis first in one direction, then in the other. On the outward stroke of the piston, the cord from the reducing

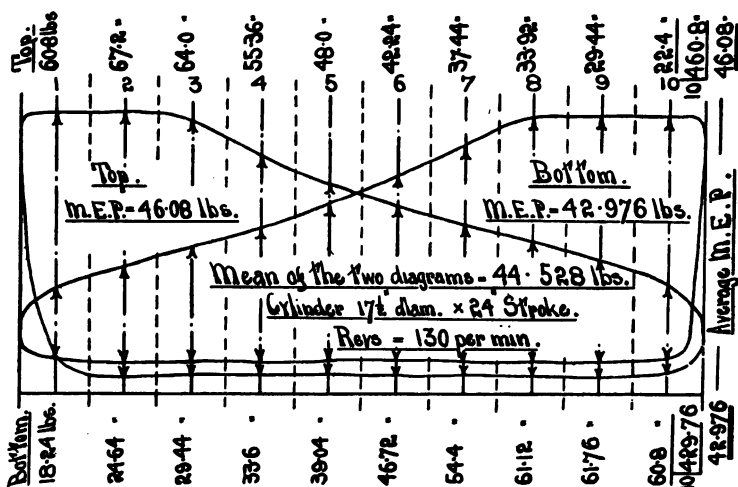


FIG. 148.

motion winding round the base of the paper drum rotates the drum against the resistance of a coiled spring within it. On the return stroke this spring rotates the drum in the opposite direction. The indicator cord and its attachment with the reducing gear must be so arranged that horizontal displacement around the drum with respect to the pencil should correspond to the displacements of the engine piston in the cylinder. Fig. 148 represents the indicator diagram taken one from each end of the double-acting engine. The mean effective pressure of steam for each side of the piston can be obtained from the corresponding diagram either by (1) the ten ordinates method, or (2) by means of the planimeter. The first method is indi-

cated on p. 36, and the second method, which consists in first finding the area of the diagram, is dealt with on pp. 37 to 40. It is hardly necessary to point out to the student who has read intelligently through Chapter VIII. that the area of an indicator diagram represents the work done in the cycle, and that, since the length of the diagram represents, on a reduced scale, the stroke of the engine, the average height of the diagram, expressed in terms of the pressure scale or scale of the spring, represents the mean effective pressure.

Each of the diagrams in fig. 148 is divided into ten equal divisions, and the mean height of each division is scaled to the pressure scale of the spring used. The sum of these ten ordinates for each diagram is then taken and divided by ten, thus giving the average or mean of all the ten ordinates or pressures. This average is the mean effective pressure for the diagram. Half the sum of the two mean pressures gives the mean effective pressure of the steam per revolution; and the I.H.P. can then be determined by calculation, as already explained. If the diagram is very irregular, and greater accuracy is required, it is advisable to divide it into twenty equal parts and take the mean of the twenty ordinates. If the planimeter is used—and this is by far the best method of working out the diagram—the area in square inches is first determined, and divided by the length of the diagram, which has been measured very carefully to two places of decimals. The quotient gives the average height in inches, which, multiplied by the pressure scale of the spring, furnishes us with the mean effective pressure.

The best form of the planimeter, known as the Amsler planimeter, gives a reading of four figures. This type of instrument is described on pp. 37 to 40. A useful addition to the planimeter consists in the addition of the two upper steel points as shown in the diagram, by means of which the average height of the diagram is given by the recording mechanism in fortieths of an inch, when the distance between the steel points is adjusted to give the exact length of the diagram; that is to say, assuming that the distance between the steel points is first adjusted to the length of the diagram, and the pointer traced round the diagram in the same way as in the determination of the area, then the instrument records directly, not the area of the diagram, but the average height in fortieths of an inch. A little reflection will show the student that this reading, multiplied by a certain factor, depending upon the scale of the diagram, will give directly the mean effective pressure. This factor is evidently $\frac{1}{40} \times$ pressure scale of spring. Thus, if the pressure scale is 100 lbs. to the inch, the factor for multiplying is $100 \times \frac{1}{40} = 2.5$.

The table given below shows the multiplying factors for a few springs of different strength :—

TABLE XXV.

Scale of Diagram, lbs. = 1 in.	Factor for Multiplying.	Scale of Diagram, lbs. = 1 in.	Factor for Multiplying
8	0.2	40	1.0
10	0.25	50	1.25
16	0.4	60	1.5
20	0.5	80	2.0
24	0.6	100	2.5
30	0.75	150	3.75
32	0.8	200	5.0

Probable Mean Effective Pressure.—In designing an engine, the cylinder dimensions will have to be calculated so that the engine may develop the required H.P., and so it becomes first of all necessary to estimate in some way the probable mean effective pressure. This may be done by one or other of two methods—(1) by graphics, (2) by calculation.

(1) **Mean Effective Pressure by Graphics.**—This method consists of drawing out the indicator diagram which it is expected will be obtained in actual practice, and then calculating its M.E.P. in the usual way. For a single-cylinder engine it is a comparatively simple operation, but it becomes more complicated for multi-cylinder engines.

(a) *For a Single-cylinder Engine.*—Collect all the data available, and tabulate, as in the following example :—

Steam pressure in boiler = 50 lbs.
 Drop in pressure to engine = 10 lbs.
 Steam pressure at engine = 40 lbs. gauge.
 „ „ = 55 lbs. absolute (+ 15 lbs. for atmospheric pressure).

Cut-off in cylinder, 0.25 of stroke.

Clearance (assumed), 10 per cent. of piston displacement.

Conditions (given) : condensing, jacketed.

Construct the diagram shown in fig. 149 by drawing the two axes OY and OX, the former representing pressure and the latter volumes. On the former construct a pressure scale, as shown, to represent absolute steam pressures, and, choosing any convenient scale to represent volumes, mark this scale along OX,

OA representing the clearance (assumed at 10 per cent.) and AX representing the capacity of the cylinder, or volume swept through by the piston in one stroke. At B, = 25 per cent. of stroke, erect the perpendicular BC to represent the fraction of the stroke at which the steam is cut off and expansion begins. Complete the rectangle OYZX by making OY = 55 lbs., the absolute pressure of the steam at the engine. Then draw in the hyperbolic curve CD representing an isothermal expansion at constant temperature according to the law $pv = a$ constant. To draw the hyperbolic curve CD, choose a number of points, such as E and F in the diagram, and draw from these points to the origin O. Where these lines intersect the vertical BC, project ordinates intersecting at G and H respectively, as shown, and the points of intersection so obtained, such as G and H, will lie on the hyperbolic curve,

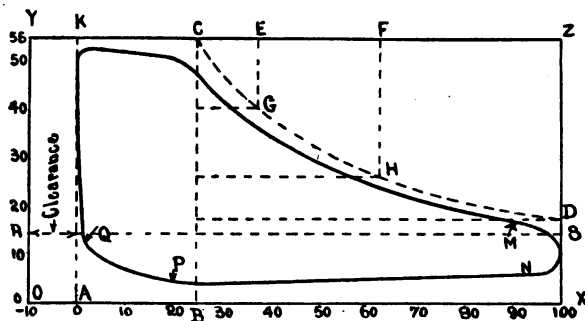


FIG. 149.

which can then be filled in. Having obtained this theoretical diagram, the probable diagram can be blocked in, allowing for the losses likely to be experienced. Starting with the piston at the commencement of its forward stroke, the steam pressure will be nearly up to the full 55 lbs., or say 52 lbs., allowing for 3 lbs. drop in getting into the cylinder. As the piston advances, this pressure is more or less maintained according to the type of valves employed. If it be an ordinary slide-valve engine, the pressure will probably fall some 4 or 5 lbs. before the point of cut-off is reached; but with a drop-valve the pressure would not be expected to fall so much. At the point of cut-off, when the steam valve is gradually closing, the pressure will fall much more, until, by the time the valve is quite closed, it will probably be some 10 lbs. under the expansion curve. As expansion proceeds, some of this loss will be returned in the shape of re-evaporation, until at the end of the expansion period the pressure will only be some

6 or 7 lbs. below the theoretical curve. If the engine be non-jacketed, this amount of re-evaporation will not be so marked, as the loss of 10 lbs. at the commencement of the expansion period would probably be maintained, if it did not increase, till the end of the expansion. At about 12 or 14 per cent. of the stroke before the end, say at M, the exhaust valve will begin to open, and the pressure will fall more rapidly, due to the connection from the cylinder to the condenser being opened. This forms what is known as the "toe" of the diagram, the shape of which will depend very much on the setting of the valve and the type of valve. From the commencement of the return stroke, the pressure will fall slightly, due to the exhaust valve opening wider, until it nearly reaches the pressure in the condenser. For ordinary engines this may be taken at about 26 to 27 in. of mercury or about 2 lbs. per square inch absolute pressure, so that the lowest pressure recorded on the diagram will probably be about 3 lbs. above the perfect vacuum line OX. At about 20 per cent. of the stroke before dead centre, at P, the exhaust valve begins to close, and the exhaust is throttled, thus raising the pressure a little, which becomes more marked after the valve is quite closed, until at Q, when the steam valve opens again, the pressure will have been brought up to the atmospheric line RS. Having completed the diagram, its area can be calculated in the usual way, and the M.E.P. obtained.

(b) *For a Compound Engine.*—The diagrams for a compound engine are a little more complicated. Take, for example, the case of a compound engine whose cylinders have a ratio of 1 to 3, with steam at 125 lbs. pressure in the boiler, or (say) 110 lbs. pressure (by gauge) at the engine, cutting off at 30 per cent. of the stroke in the high-pressure cylinder. The diagrams for such an engine are shown in fig. 150. Here the clearance is assumed at 10 per cent. of the piston displacement in the high-pressure cylinder, and 8 per cent. in the low-pressure cylinder. To draw the probable indicator diagrams, it is preferable to adopt common pressure and volume scales for the two diagrams, as shown in fig. 150. Here OY and OX represent the axes of pressure and volume, as before, only the volume scale for the high-pressure cylinder is somewhat compressed to include the volume of the low-pressure cylinder without unduly lengthening the diagram. The distance line OA represents the clearance of the high-pressure cylinder, being 10 per cent. of the volume swept through by the high-pressure piston, represented by the line AD. The distance OE represents the clearance of the low-pressure cylinder, being 8 per cent. of the low-pressure cylinder capacity, represented by the volume EH, being three times the

volume of the high-pressure cylinder AD. The point of cut-off is next fixed by drawing the vertical line KL such that AL is 30 per cent. of AD; and the curve KMN, constructed, as before, to represent the expansion of the steam from the point of cut-off in the high-pressure cylinder to the end of the low-pressure cylinder. The cut-off in the low-pressure cylinder is now fixed, so that the receiver pressure between the two cylinders will give approxi-

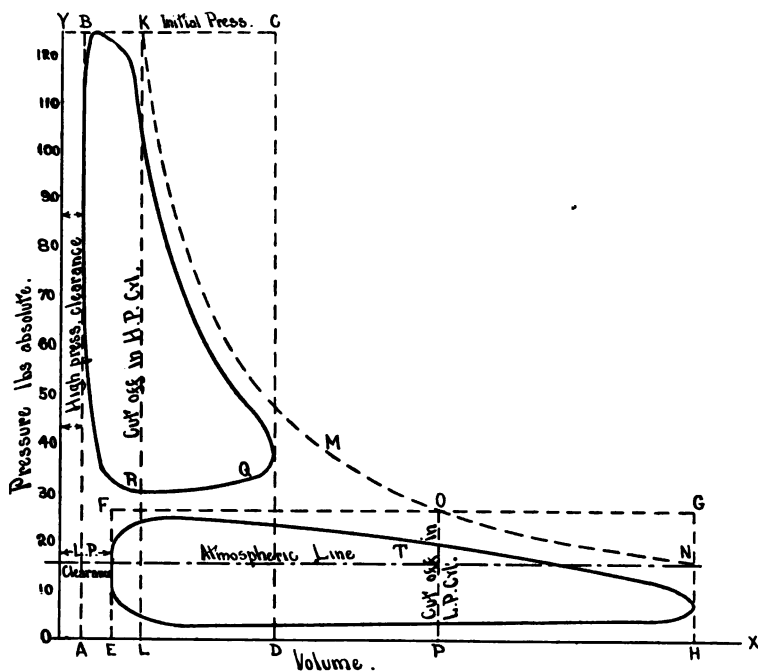


FIG. 150.

mately equal powers in each cylinder. In the diagram fig. 150 the cut-off in the low-pressure cylinder has been assumed at 55 per cent. of the stroke, and is shown by the line OP, the volume EP being 55 per cent. of EH. This will give a pressure in the receiver, represented by the line FG, at about 26 lbs. per square inch absolute. The two diagrams being now "blocked in," the indicator diagrams can next be drawn, allowing for the various losses or rounding off of the corners as in the case of the single-cylinder engine just described. The exact shape of the exhaust

line QR for the high-pressure diagram and the steam line for the low-pressure diagram will depend very much on the relative volume of the receiver and the position of the two pistons; the smaller the receiver volume, the greater the fall in the exhaust line QR. In the case under consideration, the receiver volume has been assumed rather large, equal to about $1\frac{1}{2}$ times the volume of the low-pressure cylinder, so that the fall indicated by the line QR is comparatively slight. Having drawn in the two diagrams, their mean effective pressures may now be calculated, either by the "ordinate" method or by means of a planimeter. In the example given, the M.E.P. is 49 lbs. for the high-pressure cylinder, and 15.8 lbs. for the low-pressure cylinder, giving a total mean effective pressure referred to the low-pressure piston of

$$\frac{49}{3} + 15.8 = (\text{approx.}) 32.1 \text{ lbs. per sq. in.}$$

The theoretical M.E.P. obtained, as calculated by the formula

$$p_m = p_1 \frac{(1 + \log_e r)}{r} - p_b$$

is

$$p_m = 125 \frac{(1 + 2.095)}{8.125} - 2.5 = 45.1 \text{ lbs.,}$$

and the diagram factor works out to

$$\frac{32.1}{45.1} = 0.71,$$

which corresponds fairly well with average practice for ordinary compound engines (see Table XXVI., p. 332).

(2) **Mean Effective Pressure by Calculation.**—Various tables are given for the mean pressure of steam according to the initial pressure and the point of cut-off, but most of these take no account of the amount of clearance or the losses in the cylinder by reason of the wire-drawing of the steam and rounding off of the corners of the diagram. To account for the latter, the theoretical M.E.P. expected may be corrected by multiplying by a certain factor known as the "diagram factor." Take, for example, the single-cylinder condensing engine just considered (see p. 324). Here the normal cut-off is at 25 per cent. of the stroke; but, assuming the clearance at 10 per cent., the real ratio of expansion becomes $\frac{110}{35} = 3.14$ instead of 4. For this ratio the hyperbolic logarithm

is 1.1346, and the theoretical M.E.P. expected, assuming a back-pressure of 3 lbs. absolute, is

$$\begin{aligned} p_m &= p_1 \times \frac{1 + \log_e r}{r} - p_b \\ &= 55 \times \frac{1 + 1.1442}{3.14} - 3 \\ &= (55 \times 0.683) - 3 \\ &= 37.56 - 3 = 33.56 \text{ lbs.} \end{aligned}$$

Assuming a diagram factor of 0.75, this will give an actual M.E.P. expected of $33.56 \times 0.75 = 28.17$ lbs.

A table of the hyperbolic logarithms of the more usual figures will be found on p. 85; or the hyperbolic logarithm may be found by multiplying the ordinary logarithm by 2.302585.

As regards the value of the diagram factor, this will depend very much upon the conditions under which the engine is running. Its normal value may be said to vary from 0.7 to 0.9, although under exceptional circumstances it frequently lies outside these limits. Generally, for high rotative speeds and high steam pressures its value is comparatively low, for obvious reasons; whereas, for slow-running engines well jacketed, and with steam ports well designed, short, and of ample area, its value more nearly approaches the higher figure quoted.

For very small non-condensing engines, cutting off steam at about three-quarter stroke, the M.E.P. may be taken approximately at about 60 per cent. of the boiler pressure.

The calculation of the M.E.P. of compound or triple-expansion engines is similar to that for single-cylinder engines, except that the number of expansions " r " should be the *total* number of expansions from the point of cut-off in the high-pressure cylinder to the end of the stroke in the low-pressure cylinder; and the value of the diagram factor should be correspondingly reduced to allow for losses between the cylinders. The M.E.P. so obtained will then be the total M.E.P. referred to the low-pressure piston. The value of the diagram factor will vary for compound and triple-expansion engines from about 0.6 to 0.75, being slightly higher for compound or two-stage expansion, owing to there being but one loss between the two cylinders. Its value will also depend very considerably upon the arrangement of the receivers between the cylinders, whether they are jacketed or protected from radiation or not. The mean value of the diagram factor for the three compound engines (0.714), was some 5 per cent. higher than the mean value (0.663) for three triple-expansion engines.*

* *Proceedings of the Institution of Mechanical Engineers*, Feb. 1894.

M.E.P. referred to L.P. Piston.—It is useful for certain calculations to refer the mean effective pressure in the several cylinders of a compound, triple- or quadruple-expansion engine to an equivalent pressure on the low-pressure piston. That is to say, to find the value of the pressure which, if acting on the low-pressure piston alone, would give the same amount of work that is done conjointly in the two or more cylinders of the expansion engine. As an example, consider the case of a triple-expansion engine, and denote the areas of the high-pressure, intermediate, and low-pressure pistons by A_h , A_i , and A_l respectively, and the corresponding mean pressures by P_h , P_i , and P_b , and let the equivalent or referred pressure be denoted by P_E . Then assuming, as is usually the case in practice, that the three strokes are equal, we obtain an equation of work—

$$P_E A_L S = P_h A_h S + P_i A_i S + P_b A_l S$$

$$\text{or } P_E = \frac{P_h A_h + P_i A_i + P_b A_l}{A_L}.$$

The student will have no difficulty in modifying the calculation necessary to find the referred pressure in a compound or quadruple-expansion engine, or to an engine in which the strokes are not all equal.

Determination of the Size of an Engine to develop a given Horse-Power.

Having defined the H.P., and having explained carefully the method of determining it, we now proceed to notice the practical problem which every engineering draughtsman who has to deal with engine design has to solve, viz. the determination of the cylinder dimensions of an engine to develop the given H.P. In the case of a single-cylinder engine the problem is very simple indeed. Taking the formula

$$\text{I.H.P.} = \frac{PLAN}{33000},$$

we have a simple equation to solve for the determination of A . The number of revolutions N will be determined by the type of the engine; the length of the stroke will in some way be dependent upon the diameter of the cylinder and other practical considerations; while the pressure P will be estimated in the manner already explained.

For a compound engine, knowing the type of engine that it is

required to design, we should probably first of all fix the boiler pressure and the piston speed, and next determine in some way the probable mean effective pressure for the two cylinders referred to the low-pressure cylinder. Such a table as that given on p. 332 may be constructed by the methods already explained, which will give this referred pressure for given cut-offs and ratios of expansion. The other way of determining the probable M.E.P. is to use a graphical diagram similar to the one described on pp. 326 to 328. An example will make clear the method of dealing with a problem of this character.

Assume a working pressure of 100 lbs. per square inch in the boiler and an average piston speed of 400 feet per minute; let it be required to find the cylinder dimensions and stroke to develop 115 H.P. Then take the formula

$$\text{I.H.P.} = \frac{\text{PLAN}}{33000},$$

and, solving for A, obtain

$$A = \frac{33000 \times 115}{38.4 \times 400} = 247 \text{ sq. in.}$$

This area corresponds to a diameter of $17\frac{3}{4}$ in., or, to take even figures, say 18 in. This gives the diameter of the low-pressure cylinder. The M.E.P., 38.4, is taken from the table given for a cut-off at 0.4, the cut-off in the high-pressure cylinder.

We have just found the diameter of the low-pressure cylinder on the assumption that the whole of the work or power is developed in that cylinder with a M.E.P. of 38.4. The actual M.E.P. in the low-pressure cylinder may, of course, be something quite different from this figure.

The size of the high-pressure cylinder can now be determined when we fix the ratio of the area of the high-pressure cylinder to the low-pressure cylinder. Assume this ratio, which ratio agrees fairly well with the average practice for condensing engines working up to 120 lbs., to be as 1 to 2.75, then the area of the high-pressure piston = $\frac{247}{2.75} = 90$ sq. in., corresponding to a diameter of about $10\frac{3}{4}$ in.; and, to be on the safe side, 11 in. may be taken as the suitable diameter for the high-pressure cylinder.

Having fixed upon the diameters of the cylinders, the stroke and rotative speed of the engine must next be determined. For slow-running engines, the stroke is usually about $1\frac{1}{2}$ times as much as the diameter of the low-pressure cylinder, or (say), in the case quoted, let the stroke be equal to 2 ft. This will give a rotative

speed of $\frac{400}{2 \times 2} = 100$ revolutions per minute, the 2-ft. stroke being multiplied by 2 because there are two strokes to each revolution.

With reference to Table XXVI., it should be borne in mind that the total mean effective pressures quoted are those for condensing engines of ordinary speeds, say from 300 to 450 ft. per minute. For non-condensing engines, the mean effective pressures will be about 12 lbs. per sq. in. less than those quoted; and for higher speeds they should be still further reduced by some 10 to 15 per cent. according to the speed. The pressures in heavy type are those for the cut-off nearest to that given by the formula for the most economical rate of expansion.

TABLE XXVI.—HORSE-POWER OF COMPOUND CONDENSING STEAM ENGINES. Total mean effective pressure referred to low-pressure piston.

Cut-off in H.P. Cylinder.	Boiler Pressure in lbs. (Gauge).								Total No. of Ex- pansions.
	50	60	70	80	100	120	140	160	
	Mean Effective Pressures.								
0.2	11.5	13.6	15.7	17.8	22.1	26.4	30.5	34.7	10.0
0.25	14.0	16.6	19.1	21.6	26.7	31.9	37.0	42.1	8.5
0.3	16.2	19.2	22.1	25.1	31.0	36.9	42.8	48.7	7.5
0.35	18.3	21.6	25.0	28.3	35.0	41.7	48.4	55.0	6.7
0.4	20.1	23.7	27.4	31.0	38.4	45.7	53.0	60.2	6.0
0.45	21.5	25.5	29.4	33.3	41.2	49.1	57.0	64.8	5.5
0.5	23.0	27.2	31.3	35.5	43.9	52.2	60.7	69	5.0
0.55	24.4	28.8	33.2	37.6	46.6	55.3	64.2	73	4.6
0.6	25.5	30.2	34.9	39.5	48.8	58.2	67.5	76.8	4.3
0.65	26.6	31.5	36.4	41.2	51.0	60.8	70.3	80	4.0
0.7	27.5	32.5	37.5	42.5	52.5	62.5	72.5	82.5	3.75
0.75	28.4	33.5	38.6	43.7	54.1	64.3	74.8	85	3.5
Most Economical Expansion	3.75	4.25	4.75	5.25	6.25	7.25	8.25	9.25	
Terminal Pressure, lbs. abs.	17.3	17.6	17.9	18.1	18.4	18.6	18.8	18.9	

Assumptions on which above table is based:—

- (1) All engines condensing. If non-condensing, take off 12 lbs. M.E.P.

- (2) Drop of pressure 10 lbs. between boiler and engine.
- (3) Clearance assumed, 10 per cent. in high-pressure cylinder,
9 per cent. in low-pressure cylinder.
- (4) Ratio of cylinders, 1 to 2.75.
- (5) Formula used for most economical expansion,

$$r = \frac{p_1 + 10}{20}.$$

- (6) Terminal pressure assumed in law, $pv = \text{constant}$.

Thermal Efficiency of Steam Engine.

The actual thermal efficiency of a steam engine is the ratio of the heat utilised as work on the engine piston or pistons to the heat supplied to the engine by the steam between the two working limits of temperature. In making this calculation, first find the heat utilised per minute as work on the piston, by determining the heat equivalent of the I.H.P. Then calculate the heat to be debited against the engine per minute, by taking the total heat of the steam entering the engine stop-valve, less the heat of the same weight of water at the temperature of the engine exhaust.

The Carnot Heat Engine of Comparison.—In order to realise the comparative value of the performance of an actual heat engine, its efficiency is compared with that of an ideally perfect engine working between the same limits of temperature on an ideal cycle of operations. The working substance of such an engine is supposed to be a perfect gas.

The French philosopher Carnot devised a cycle of operations for a perfect heat engine, and showed that the efficiency of such an engine was given by the expression $\frac{T-t}{T}$, where T is the upper limit of temperature, and t the lower limit of temperature, T and t both being expressed in terms of absolute temperature. Further, he showed that no other heat engine could do more work from a definite quantity of heat with the same limits of temperature. The expression just given represents, therefore, not only the thermal efficiency of the engine working on this ideal cycle, but is also the measure of the greatest possible utilisation of heat energy in any engine when working between the same limits of temperature.

Example 1.—Suppose a number of trials are taken on a condensing steam engine with varying boiler pressures of 50, 100, 150, 250, and 300 lbs. per sq. in. absolute, and that the temperature of the condenser in each case is 100° F. Calculate the efficiencies of an ideal heat engine on the Carnot cycle, working between the

same limits of temperature. Taking the engine working with steam at 200 lbs. pressure, and referring to the steam tables, we find that the corresponding temperature is 381.5°F . The absolute temperature for the upper limit is therefore $381.5 + 461 = 842.5^{\circ}\text{F}$. The absolute temperature for the lower limit is $100 + 461 = 561^{\circ}\text{F}$. The efficiency of an engine working between these two limits

$$= \frac{T - t}{T} = \frac{842.5 - 561}{842.5} = 0.334 \text{ or } 33.4 \text{ per cent.}$$

It will be noticed that the same result is arrived at by simply subtracting 100, the lower limit, from 381.5, and dividing this difference by the absolute temperature, $381.5 + 461$. The other results are worked out in a similar manner and should be expressed in tabular form.

Thus it will be seen that in the high ranges of temperature it is only possible, even theoretically and in an imaginary engine, to utilise about one-third of the heat-drop in passing through the engine.

Example 2.—Assuming the engine referred to in the previous exercise to be worked as a non-condensing with a back-pressure of 3 lbs. by gauge or 17 lbs. per sq. in. absolute, calculate the efficiencies of an ideal heat engine working between the same limits of temperature. Referring to the steam table, we find the temperature of steam at rejection to be 219.5 , and this is our lower limit of temperature. Hence, for the engine of 200 lbs. pressure, the efficiency

$$= \frac{381.5 - 219.5}{381.5 + 461} = 0.192 \text{ or } 19.2 \text{ per cent.}$$

The other results are calculated in like manner and given in column 5 in the above table.

Example 3.—The temperature immediately after explosion in a gas engine cylinder may reach 2000°C ., and may be exhausted at about 300°C . Calculate the efficiency of an ideal heat engine working between the same limits of temperature.

$$\text{Efficiency} = \frac{2000 - 300}{2000 + 273} = 0.748 \text{ or } 74.8 \text{ per cent.}$$

The student will note how very much larger this efficiency is than those obtained for engines working between the limits of temperature used in some engine practice.

The Rankine Standard Engine of Comparison.—Although an engine working, or imagined as working, on the Carnot cycle between two assigned limits of temperature gives the greatest

efficiency and therefore the greatest utilisation of heat as work on the piston, yet it is impossible for steam engines to work, even theoretically, on this cycle of operations. This being so, Rankine was led to define an ideal standard engine of comparison working on a definite cycle (called the Rankine cycle), in which the working substance was steam produced at constant pressure, either saturated or superheated, which would give the greatest utilisation of the heat supply.

First, taking the case of a steam engine using saturated steam—

$$\left. \begin{array}{l} \text{The thermal efficiency of an} \\ \text{ideal steam engine working} \\ \text{on the Rankine cycle} \end{array} \right\} = \frac{(T_a - T_e) \left(1 + \frac{L_a}{T_a} \right) - T_e \text{ hyp. log } \frac{T_a}{T_e}}{L_a + T_a - T_e},$$

where T_a = absolute temperature of saturated steam at admission
to engine stop-valve,

T_e = " " " " steam at exhaust,

L_a = the latent heat of steam at the temperature T_a .

The expression in the denominator represents the amount of heat in 1 lb. of steam, measuring above the exhaust temperature T_e . The numerator gives the maximum heat available for work in 1 lb. of saturated steam entering the engine stop-valve. Obviously the latter expression as the ratio of the former gives the ideal efficiency of the engine.

In all heat engines, practical or otherwise, it is advisable to state the performance in terms of the number of B.Th.U. of heat per minute employed to develop 1 I.H.P.

Now, 1 I.H.P. = 33,000 ft.-lbs. of work per minute, and 1 B.Th.U. is capable of doing 778 ft.-lbs. of work.

$\therefore$ 1 I.H.P. or 33,000 ft.-lbs of work per minute requires

$$\frac{33000}{778} = 42.4 \text{ B.Th.U.}$$

Further, since the thermal efficiency (e) is the ratio of

Heat utilised as work
Heat supplied above the lower limit of temperature',

we have

$$\frac{42.4}{\text{B.Th.U. per H.P. per minute}} = e,$$

$$\therefore \text{B.Th.U. per H.P. per minute} = \frac{42.4}{e}.$$

Applying this to the expression for efficiency given above for the standard engine, we obtain—

$$\left. \begin{array}{l} \text{B.Th.U. per I.H.P. per} \\ \text{minute for engine on} \\ \text{Rankine cycle using satu-} \\ \text{rated steam} \end{array} \right\} = \frac{42.4(L_a + T_a - T_c)}{(T_a - T_c)\left(1 + \frac{L_a}{T_a}\right) - T_c \text{ hyp. log } \frac{T_a}{T_c}}.$$

For an engine using superheated steam—

The thermal efficiency of ideal engine on Rankine cycle

$$= \frac{(T_a - T_c)\left(1 + \frac{L_a}{T_a}\right) + 0.48(T_s - T_a) - T_c\left(\text{hyp. log } \frac{T_a}{T_c} + 0.48 \text{ hyp. log } \frac{T_s}{T_c}\right)}{\{L_a + T_a - T_c + 0.48(T_s - T_a)\}},$$

T_s being the temperature of the superheated steam at the engine stop-valve. The B.Th.U. per I.H.P. per minute theoretically required for this cycle is found by dividing 42.4 by the value of this expression.

Efficiency Ratio.—The performance of an actual steam engine is compared with the above results for the ideally perfect steam engine on the Rankine cycle by expressing the number of B.Th.U. of heat per I.H.P. per minute used by the actual engine as a fraction or percentage of B.Th.U. required by the ideal engine to work between the same limits of temperature, or by expressing the actual thermal efficiency as a percentage of the efficiency of the ideal steam engine. Thus,

$$\text{Efficiency ratio} = \frac{E}{e} \times 100 \text{ per cent.}$$

The performance of an engine should also be stated in terms of the B.Th.U. per minute per brake horse-power; and when the engine is employed driving dynamo electric machinery and generating current, the B.Th.U. per electric horse-power and per Board of Trade unit should also be given.

Steam Consumption and Willans' Law.*—Mr P. W. Willans called attention to the fact that, if the powers developed by an engine running at varying loads, but under otherwise similar conditions, are plotted out as abscissæ, with the corresponding weights of steam used per hour as ordinates, a straight line is the result, with an equation of the form $W = w + w_1(\text{H.P.})$, where W represents the weight of steam per hour, H.P. the horse-power

* In this paragraph the authors have followed closely the method given by Professor Perry (*The Steam Engine*, Macmillan).

developed, w the intercept on the axis of W , giving the constant consumption of steam for engine friction, losses, etc., and w_1 the increase in the weight of steam per unit increase in power. Thus, suppose a number of trials are taken on an engine running under similar conditions from light load to full load, but with varying boiler pressures, and for each trial the steam consumption is measured, we should find, on representing these results graphically, that there was a linear law connecting the steam used per hour with the power developed by an equation of the form given above. This relationship, known as Willans' Law, is true for single engines, compound, triple, single- or double-acting engines, condensing and non-condensing, or for engines working on a constant throttle or a constant expansion, etc., etc.*

Further, a similar result is obtained if the brake horse-powers or electrical horse-powers are plotted along the base, with the steam consumption per hour as ordinates, or even if the mean effective pressures are plotted as abscissæ, with the steam consumption as ordinates.

Willans' Law can be shown to be true from theoretical considerations. Consider the following example:—

Example.—Assume a double-acting single-cylinder steam engine running a number of trials, each at 100 r.p.m., under similar conditions, except that in each trial a different initial steam pressure is taken, commencing with 50 lbs., and increasing by 10 lbs. to 150 lbs. per sq. in. absolute. Calculate for each trial the mean effective pressure, the indicated horse-power, the weight of steam used per hour, the weight of steam per indicated horse-power per hour, and then plot out the results obtained. Assume a cut-off at one-third of the stroke and hyperbolic expansion, a back-pressure of 17 lbs. per sq. in. absolute, piston diameter 10 in., stroke 12 in.

On p. 201 it is given that the mean pressure (p_m) = the initial pressure (p_1) $\times \frac{1 + \text{hyp. log } r}{r}$, so that this result must be calculated for each initial pressure. The mean effective pressure (M.E.P.) is found by subtracting the back-pressure (17 lbs.) from the result. The calculation of the I.H.P. requires no explanation. In order to determine the weight of steam, we first refer to the steam tables given on p. 290 for the volume (v) of 1 lb. steam at each of the given pressures. Then the weight of steam in one stroke is found by dividing the volume up to the point of cut-off by v . The result, multiplied by the number of strokes per hour, gives the weight (W) of steam per hour. Finally, the steam per I.H.P. is given by $\frac{W}{\text{I.H.P.}}$. The whole of these calculations are given in

tabular form below. Fig. 151 represents the results plotted out as rectangular co-ordinates. The line representing the relationship between the steam used per hour has an equation $W = 128 + 14.4$ (H.P.). The graphical relationship between the indicated horse-power and the steam used for each unit of power is shown by the curved line. An inspection of this curve, and also the last column in the table, shows that the steam consumption per I.H.P. increases rapidly as the initial pressure and indicated horse-power diminish. Hence we may say that *high-pressure steam is more economical than steam at lower pressures, and also that it is uneconomical to work an engine at light loads.* Two other curves are given, which may speak for themselves.

TABLE XXVII.

Initial Pressure, lbs. $\square$ "abs. P_1 .	Mean Pressure, lbs. $\square$ " $P_m = 7P_1$.	Mean Effective Pressure = $P_m - 17$ lbs. = M.E.P.	I.H.P.	Vol. in cu. ft. of 1 lb. of steam. v .	Weight of steam in 1 stroke. w_1 .	Weight of steam per hour. W .	Steam per I.H.P. per hour $\frac{W}{\text{I.H.P.}}$.
150	105	88	41.888	2.98	.061	732	17.475
140	98	81	38.556	3.18	.057	684	17.740
130	91	74	35.224	3.41	.053	636	18.055
120	84	67	31.892	3.67	.049	588	18.437
110	77	60	28.545	3.99	.045	540	18.917
100	70	53	25.220	4.36	.041	492	19.508
90	63	46	21.896	4.81	.037	444	20.227
80	56	39	18.564	5.38	.033	396	21.331
70	49	32	15.232	6.09	.029	348	22.846
60	42	25	11.900	7.04	.025	300	25.210
50	35	18	8.568	8.35	.021	252	29.411

A number of practical examples on Willans' law are given in the exercises at the end of the next chapter. These exercises give the results of actual trials and should be carefully studied.

Heat Distribution and Losses in Engine and Boiler Plant.—As showing what becomes of the heat developed during the combustion of the coal in the boiler furnace, we give an example, the results of which are represented graphically in fig. 152, taken from the *Minutes of Proceedings of the Institution of Civil Engineers*. In the diagram 1 unit width represents 100,000 B.Th.U. of heat.

The heat developed by combustion is 183600 B.Th.U. per minute, and the heat equivalent of the work done on the brake is 25390 B.Th.U. per minute. The ratio of this last result to the

first = $\frac{25390}{183600}$, or about 13·8 per cent. gives the combined efficiency of the plant.

Now, if the student will sum up the total heat losses per minute (in this example this sum = 158210 B.Th.U.), and add the heat equivalent of the useful work done (25390 B.Th.U.), the result will equal the heat of combustion (183600 B.Th.U.), for in any heat installation the

$$\text{Heat supply} = \text{useful work} + \text{heat lost.}$$

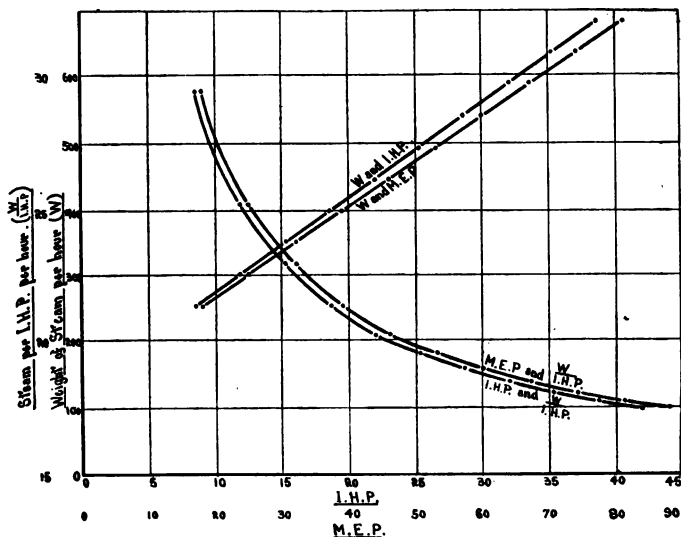


FIG. 151.

From Boiler—

Carried away by flue gases	=	20150 B.Th.U. per minute.
Radiation from boiler	=	10000 " "
Radiation from economiser	=	5000 " "
Radiation from flue gases on the way to economiser	} =	1000 " "

From Steam Pipes—

Radiation from steam pipes	=	3100 " "
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From Feed Pipes—

Radiation from economiser delivery	} =	250 " "
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Radiation from feed delivery to economiser	}	=	100 B.Th.U. per minute.
Radiation losses from hot well and pipes to feed pump suction			
Radiation from steam-jacket returns	}	=	150 " "
<i>From Engine—</i>			
Heat carried away by circu- lating water	}	=	110240 " "
Radiation from engine			
Heat equivalent of engine friction	}	=	1820 " "
Radiation between exhaust and condenser			
Total of losses		=	158210 " "

Credit.	Debit.
Heat supply = 183600 B.Th.U.	Heat utilised = 25390
	Heat lost = 158210
	Total . . . 183600

Electrical Horse-Power.—Electrical energy is developed chiefly by the use of steam engines driving electrical machines or dynamos producing a potential difference at the dynamo terminals, resulting in what is called a current of electricity. The dynamo therefore is a machine for the production of electrical energy by the expenditure of mechanical work, the former being measured by the product of the current produced (amperes) and the difference of potential in volts at the terminals. The unit of electrical energy, known as "the watt," is the rate of doing work in a circuit by 1 watt in 1 second. Thus the power developed by a dynamo giving 20 amperes at 100 volts to a circuit is $20 \times 100 = 2000$ watts, or, taking 1000 watts as 1 kilowatt (written "kwtt." for short) $= \frac{20 \times 100}{1000} = 2$ kwts. The connection between electrical and mechanical energy is given thus:—

$$\begin{aligned}
 1 \text{ watt} &= 44\frac{1}{2} \text{ ft.-lbs.} \\
 746 \text{ watts} &= 1 \text{ H.P. (33,000 ft.-lbs.).} \\
 0.746 \text{ kwtt.} &= 1 \text{ H.P.} \\
 1 \text{ kwtt.} &= 1.34 \text{ H.P.}
 \end{aligned}$$

(The student should here refer back to the notes on p. 195.)

Thus a dynamo giving 50 amperes at 200 volts is supplying $\frac{50 \times 200}{1000} = 10$ kwts. ; or an amount of electrical energy equivalent to $10 \times 1.34 = 13.4$ mechanical horse-power. To generate this amount of electrical energy the dynamo will require to be supplied with an excess of mechanical energy over 13.4 H.P., due to various losses, including those of friction. The efficiency of a dynamo varies very much according to its size, whereas large machines of 100 kwts. and over have an efficiency of 90 to 92 per cent. ; smaller machines (of, say, 5 to 10 kwts.) have an efficiency of only 80 to 85 per cent. The efficiency here referred to is the ratio of electrical energy developed to the mechanical energy expended. Thus, the previous dynamo giving 10 kwts., equal to 13.4 electrical H.P., might require, say, 16 H.P., as measured by a dynamometer on the engine shaft. In that case, its efficiency would be

$$\frac{13.4}{16} \times 100 = 83.75 \text{ per cent.}$$

If E.H.P. represents Electrical Horse-Power as registered by the voltmeter and ammeter (kwts. $\times 1.34$), and B.H.P. represents the Brake Horse-Power of the engine (if direct coupled), or mechanical horse-power transmitted by the belt or ropes, then

$$\text{Dynamo efficiency} = \frac{\text{E.H.P.}}{\text{B.H.P.}} \times 100,$$

or
$$\text{B.H.P.} = \frac{\text{E.H.P.} \times 100}{\text{Dynamo efficiency}}.$$

A number of practical exercises on the work dealt with in this chapter will be found on pp. 349 to 352.

CHAPTER XVI.

COMPARATIVE COSTS OF POWER PRODUCTION.

Various Methods of Power Production.—There are several ways in which power or mechanical energy can be produced—steam, gas, oil, electric, hydraulic, etc.; and in order to compare the various systems, some unit of measurement is required on which to base the calculations, which shall take into account not only the different conditions under which the engine or motor may be working, but also the variations in the cost of the medium employed. For example, with steam engines and other heat motors generally, we measure the number of heat units used by the engine or motor per I.H.P. or per B.H.P. per minute, as recommended by a Committee of the Institution of Civil Engineers some few years back. Such a figure is extremely useful, and comprehensive in its character, being quite independent of the medium employed, whether it be steam (saturated or superheated), gas (lighting or power), oil, or hot air; and it possesses a number of advantages over the old method of expressing the result of steam engine tests in lbs. of coal per I.H.P. per hour, or lbs. of steam used per I.H.P. per hour, as these latter expressions are incomplete without knowing the quality of the coal used, the kind of steam produced, and the pressure at which it is generated. The weight of steam used per H.P. per hour in an engine using saturated steam is not comparable with that used by an engine working with superheated steam, owing to the increased quantity of heat per lb. which the latter medium contains. Neither can the weight of best steam coal used by an engine and boiler be compared with the weight of coal-dust, coke, breeze, or other inferior fuels, without knowing the heating value of the fuel employed. Consequently there arose a necessity for some term which would, in its expression, combine the varying heating powers of different fuels, and heat-capacity of different qualities of steam. Also the adoption of such a term as **thermal efficiency** for the comparison of engine performances lends itself to comparing the steam engine with the gas engine, oil engine, or other energy motor.

But although the thermal efficiency of any heat motor forms a most convenient basis for expressing the efficiency, it does not say anything whatever about the **cost** of producing the power. It does not enter into the £, s. d. of the question. A question of great importance for the practical engineer to consider is, "**How much work can be obtained for so much money?**" It is more a **commercial** question than a scientific one.

Before going further it is desirable to briefly consider "thermal efficiency" in its relation to what we may call commercial efficiency. The power developed in the cylinder or cylinders of any motor is obtained from the heat value of the working fluid or medium. In the case of the steam engine, a certain quantity of steam at a given pressure is admitted to the cylinders, performs a given amount of work, and is discharged, either through the exhaust pipe into the atmosphere or into the hot well of the condenser. In its passage through the engine it loses a certain amount of heat, and develops a definite amount of power. Heat and mechanical energy being mutually convertible, the one can be expressed as a certain percentage of the other. If the work done, expressed in *heat* units, be divided by the quantity of heat received in the same time, the percentage is the thermal efficiency of the engine. Take, for example, the case of a compound condensing steam engine of ordinary type, using saturated steam of 120 lbs. pressure, developing 100 I.H.P., and consuming (say) 18 lbs. of steam per I.H.P. per hour. As each lb. of steam at the given pressure gives 1130 B.Th.U. of heat (found from steam tables) above the temperature of discharge, and the engine uses $\frac{18}{60} = 0.3$ lb. of steam per I.H.P. per minute, it gives 1 I.H.P. for 1 minute for an expenditure of $0.3 \times 1130 = 339$ B.Th.U. of heat. The work done is 33,000 ft.-lbs. or $\frac{33000}{778} = 42.41$ heat units per minute, so that the thermal efficiency is equal to

$$\frac{42.41}{339} \times 100 = 12.51 \text{ per cent.}$$

This is quite an ordinary performance for an average compound condensing engine of the size named. With a higher class of engine, with drop-valves or automatic expansion gear, the consumption of steam would probably be reduced to 15 lbs. per I.H.P. per hour, corresponding to a thermal efficiency of 15 per cent. on the same basis.

Now compare these results with those of a gas engine of equal power, running on *lighting gas*, and using (say) 16 cubic feet per

I.H.P. per hour. Assuming the gas to have a calorific value of 650 B.Th.U. per cubic foot, the thermal efficiency will be

$$\frac{42.41}{\frac{16}{60} \times 650} \times 100 = 24.5 \text{ per cent.},$$

or nearly double the thermal efficiency of the steam engine of the same power. If we take power gas, of lower calorific value, and assuming a consumption of 1 lb. of anthracite of 15,500 B.Th.U. calorific value per I.H.P. per hour, we have a thermal efficiency of engine and generator

$$= \frac{42.41}{\frac{1}{60} \times 15500} \times 100 = 16.4 \text{ per cent.}$$

For an oil engine, using (say) 0.7 lb. of oil at 18,000 B.Th.U. heating power per I.H.P. per hour, we have the thermal efficiency of engine equal to

$$\frac{42.41}{\frac{0.7}{60} \times 18000} \times 100 = 20.1 \text{ per cent.}$$

These results are average and approximate figures only, obtained in actual practice, and can be exceeded with special arrangements; but they will serve for the purpose of this chapter.

Commercial Efficiency.—We have already considered the necessity for some expression which shall represent the cost of power production for comparing performances on a commercial basis, and such a unit is found in the cost of fuel for producing 1000 H.P. hours. This term, as generally understood, is based on brake or effective horse-power, and therefore includes the mechanical efficiency of the engine. In the case of the steam engine, it includes the boiler used for steam generation, and takes into account the varying price of the fuel as well as the efficiency of the engine. The price of the same fuel varies very much in different localities, the coal which in London costs 20s. per ton may cost only one-half that amount at the pit's mouth; and it is certainly open to question whether engines in London should be debited with the higher-priced fuels due simply to the locality. The question depends entirely upon the standard of comparison required. If we only wish to compare the performance of a certain engine in London with one working (say) in South Wales, then a comparison of their thermal efficiency is all that is required; but if

we wish to compare the actual *costs* of producing the power in the two places, then we must take into account the prices paid for the fuel in the two places, and the performance of the London engine will suffer in consequence. Let us now consider a few typical cases of cost of producing power, basing our calculations on the cost of fuel per 1000 H.P. hours. For convenience of reference, we will take the examples already given, and assume average London prices.

For the *steam engine* using 18 lbs. of steam per I.H.P. per hour, assuming an evaporation in the boiler of 9 lbs. of water per lb. of coal, and fuel cost at 15s. per ton, the mechanical efficiency of the engine being 88 per cent., the fuel cost per 1000 H.P. hours will be:—

$$\begin{aligned}
 18 \times 1000 &= 18,000 \text{ lbs. of steam per 1000 I.H.P. hours.} \\
 18000 \div 0.88 &= 20,400 \text{ lbs. of steam per 1000 B.H.P. hours} \\
 20400 \div 9 &= 2270 \text{ lbs. of coal} && \text{ " } && \text{ " } \\
 2270 &&& && \\
 2240 &\times 15\text{s.} = 15\text{s. 3d., cost of fuel} && \text{ " } && \text{ " }
 \end{aligned}$$

The calculation can be worked more directly by substituting

$$C = \frac{w \times 1000 \times p}{M_e \times e \times 2240},$$

where C = cost of fuel per 1000 H.P. hours, in shillings ;
 w = weight of steam used per I.H.P. per hour, in lbs. ;
 p = average price per ton of fuel, in shillings ;
 M_e = mechanical efficiency of engine ;
 e = evaporation in boiler, lbs. of water per lb. of fuel.

For the *more economical type* of steam engine, using only 15 lbs. of steam per I.H.P. per hour, using the same figures for boiler and fuel, we have

$$C = \frac{15 \times 1000 \times 15}{0.88 \times 9 \times 2240} = 12\text{s. 8d. per 1000 H.P. hours.}$$

For the *gas engine* using *lighting gas* at (say) 2s. 3d. per 1000 cub. ft., assuming a mechanical efficiency of 0.85 per cent., we have

$$C = \frac{16 \times 1000 \times 2.25}{0.85 \times 1000} = 42\text{s. 5d per 1000 H.P. hours.}$$

For the *gas engine* using *power gas*, taking the price of anthracite at 22s. per ton, we have

$$C = \frac{1 \times 1000 \times 22}{0.85 \times 2240} = 11\text{s. 7d. per 1000 H.P. hours.}$$

For the *oil engine*, using oil at 4d. per gallon, with a specific gravity of 0·8, the cost will be

$$\frac{0\cdot7 \times 1000}{0\cdot85 \times 10 \times 0\cdot8} \times \frac{4}{12} = 34\text{s. 4d. per 1000 H.P. hours.}$$

We thus see that the cost of fuel will vary very much—from 12s. to 42s. in the cases considered—and that the cost does not follow closely the thermal efficiency; that is to say, the engine with the highest thermal efficiency is not necessarily the cheapest. Compare now these results with the *cost of electrical energy* and hydraulic power. Taking the former at 1½d. per Board of Trade unit, we have the cost per 1000 H.P. hours, assuming a motor efficiency of 92 per cent.,

$$C = \frac{1000}{0\cdot92 \times 1\cdot34} \times \frac{1\frac{1}{2}\text{d.}}{12} = 101\text{s. per 1000 H.P. hours.}$$

For *hydraulic power*, taking the cost of supply at 1s. per 1000 gallons, and the pressure at 700 lbs. per sq. in., and assuming an efficiency of water motor of 0·8, we have

$$C = \frac{33000 \times 60 \times 1000 \times 6\cdot25}{700 \times 144 \times 1000 \times 0\cdot8} = 155\text{s. per 1000 H.P. hours.}$$

Total Cost of Power Production.—Hitherto we have only considered the fuel cost of the power produced. The total cost includes a large number of other items which have always to be carefully considered. Such items include—

- (a) Interest on capital outlay.
- (b) Depreciation of plant.
- (c) Repairs and maintenance of plant.
- (d) Labour and attendance.
- (e) Oil, waste, and stores account.

These are more or less establishment charges, and will vary very considerably with the size of the plant and duration of working. Assuming the plant to drive a factory or mill for 300 working days per annum at 10 hours per day, working at full power, the total establishment charges would be from £4 to £6 per effective H.P. per annum. Table XXVIII., p. 348, gives an approximate comparison between the various costs, but the figures must be taken by way of example only, as they would differ very much according to size of plant and locality and other factors.

TABLE XXVIII.—COMPARATIVE COSTS OF VARIOUS POWER PLANTS FOR A 100 I.H.P.

		Steam.		Gas.	
Type of plant		Compound condensing engine.	Compound condensing engine.	Ordinary gas engine using lighting gas.	Power gas plant with generators.
Description of plant		Ordinary type, with Lancashire boiler.	Economical type, with Lancashire boiler.		
Consumption of steam, coal, gas, etc., assumed		18 lbs. steam per I.H.P. hour. 9 lbs. water per lb. coal. Coal at 15s. per ton.	15 lbs. steam per I.H.P. hour. 9 lbs. water per lb. coal. Coal at 15s. per ton.	16 cub. ft. gas per I.H.P. hour. Gas at 2s. 3d. per 1000 cub. ft.	1 lb. anthracite per I.H.P. per hour. Anthracite at 22s. per ton.
Thermal efficiency per cent. .		12·5	15	24·5	16·4
Cost of fuel { per 1000 B.H.P. hours per year of 3000 working hours		15s. 3d. £202	12s. 8d. £169	£2, 2s. 5d. £540	11s. 7d. £147
Cost of plant { Engine, pipes, and foundations Boiler or generator and chimney		£700	£1000	£600	£800
		500	450	...	400
Total cost .		£1200	£1450	£600	£1200
Interest on capital charge at 3½ per cent.		£42	£52	£21	£42
Establishment Charges per annum.	Depreciation { Engine (5%) £35 Generator (7½%) 37 Total	£72	(5%) £50 (7½%) 34 £84	(7½%) £45 ... £45	(7½%) £60 (8%) 32 £92
	Repairs and maintenance .	£30	£60	£40	£50
	Attendance, labour, etc. .	80	90	30	80
	Oil, waste, and stores .	25	25	25	25
	Total establishment charges	£249	£311	£161	£289
	Fuel cost per annum .	202	169	540	147
	Total cost per annum .	£451	£480	£701	£436

EXERCISES ON CHAPTERS XV. AND XVI.

1. Find the I.H.P. of a gas engine, given diameter of cylinder = 6 in., stroke = 10 in., revolutions per minute = 240, explosions per minute = 110, mean effective pressure on piston = 84 lbs. per sq. in.
Ans. 6.6.

2. Find the I.H.P. of a double-acting steam engine with cylinder 10 in. diameter, stroke 1 ft. 6 in., revolutions per minute = 120, and mean effective pressure on piston = 28 lbs.
Ans. 24.

3. Calculate the I.H.P. of a compound steam engine from the following data :—Cylinders 30 in. and 57 in. diameter, strokes 3 ft., revolutions per minute = 86, M.E.P. = 45.65 and 13.42 lbs. respectively.
Ans. 1042.

4. Calculate the I.H.P. of a single-cylinder double-acting steam engine having given—piston 110 $\square$ in. area, piston speed 500 ft. per min., M.E.P. = 65 lbs. $\square$ in.

5. A single-cylinder engine has a diameter of cylinder equal to its stroke. If the I.H.P. = 12.5 and M.E.P. = 22 lbs., find the size of the cylinder, the revolutions being 150 per minute.
Ans. 9.85 in.

6. At what piston speed must a compound engine be run to indicate 180 H.P., if the L.P. cylinder is 24 in. diameter and the total M.E.P. referred to L.P. cylinder = 33 lbs. ?
Ans. 398 ft. per minute.

7. Find the diameters of the two cylinders of a compound engine to indicate 120 H.P. at 80 revolutions per minute, the common stroke being 2 ft. 3 in., the total M.E.P. referred to L.P. cylinder = 27.5 lbs., if the ratio between the two cylinder volumes be as 1 : 3.
Ans. 13 in. and $22\frac{1}{2}$ in.

8. Find the diameters of the cylinders of a triple-expansion engine to indicate 300 I.H.P., the cylinder volumes being in the ratio of 1 : 2.6 : 7.0, the common stroke is 3 ft., revolutions per minute = 75, and the total M.E.P. referred to L.P. cylinders = 38.5 lbs.
Ans. $10\frac{1}{4}$ in., $16\frac{1}{2}$ in., 27 in.

9. Find the total I.H.P. of a triple-expansion engine with cylinders 12 in., 20 in., and 30 in. diameter respectively, all strokes 2 ft., revolutions per minute = 120, and mean effective pressures of 42 lbs. in H.P. cylinder, $17\frac{1}{2}$ lbs. in I.P. cylinder, and $10\frac{1}{2}$ lbs. in L.P. cylinder.
Ans. 257.

10. In testing a gas engine for its brake horse-power, the following observations were made:—

Diameter of brake wheel	= 5 ft. 6 in.
Total load on brake, including weight carrier	= 135 lbs.
Pull on spring balance	= 25 lbs.
Revolutions per minute	= 160

Find the brake horse-power.

Ans. 9·21.

11. If the above engine uses 200 cub. ft. of gas per hour, costing 3s. per 1000 cub. ft., find the cost of gas per brake horse-power per hour.

Ans. 0·78d.

12. Find the approximate mean effective pressures in the following cases:—

(A) Single-cylinder steam engines—

(a) Initial pressure in cylinder, 45 lbs. above atmosphere; steam expanded, $2\frac{1}{2}$ times (including clearance); back pressure = 16 lbs., diagram factor = 0·72.

(b) Initial pressure in cylinder = 55 lbs. absolute; cut-off, 30 per cent. of stroke; clearance, 10 per cent. of piston displacement; back pressure = 3 lbs. absolute; diagram factor = 0·68.

(c) Initial pressure in cylinder = 65 lbs. absolute; cut off, $\frac{1}{4}$; clearance, $\frac{1}{8}$ of piston displacement; back pressure = 17 lbs. absolute; diagram factor = 0·75.

13. (B) Multi-cylinder steam engines:—

Find the total M.E.P. referred to L.P. piston—

(a) Initial pressure, 120 lbs. absolute; total number of expansions (including clearance) = 10; back pressure = 2·5 lbs.; combined diagram factor = 0·68.

(b) Initial pressure, 180 lbs. absolute; back pressure = 3 lbs.; total No. of expansions = 12; combined diagram factor = 0·65.

(c) Initial pressure = 140 lbs. absolute; ratio of L.P. cylinder volume to H.P. = 3·25 to 1; cut-off in H.P. cylinder = 30 per cent.; clearance = 10 per cent. in both cylinders; back pressure = 3 lbs.; diagram factor = 0·7.

14. An engine indicating 100 H.P. drives a dynamo giving 450 amperes at 110 volts. What is the combined efficiency of the engine and dynamo?

Ans. 66·3.

15. If the above engine uses 18 lbs. of steam per I.H.P. per

hour, and the steam costs 2s. per ton to generate it, find the fuel cost of electrical energy per Board of Trade unit. *Ans.* 0·248d.

16. Having given I.H.P. = 60, piston speed = 600 ft. per minute, absolute initial pressure = 90 lbs. sq. in., back pressure = 15 lbs. per sq. in. absolute, ratio of expansion (excluding clearance) = $4\frac{1}{2}$, clearance volume = 7 per cent. of volume swept through by piston, and that the effect of early release, compression, etc., reduces the actual effective mean pressure to 90 per cent. of the theoretical, calculate the diameter of the cylinder. (Univ. of London, B.Sc. Engineering.)

17. A gas-engine cylinder is 9 in. diameter, its stroke 16 in., mean pressure 75 lbs. per sq. in., revolutions 180, explosions 85, diameter of fly-wheel 5 ft., friction of brake band 160 lbs., gas used per hour 250 cub. ft. of a thermal value of 600 B.Th.U. per cub. ft.; find the I.H.P., B.H.P., thermal efficiency, and mechanical efficiency. *Ans.* 16·4, 13·7, 27·8, 83·5 per cent. (I.C.E., Feb. 1906.)

18. A steam engine requires 310 B.Th.U. per H.P. per minute when working between the temperature limits of 380° F. and 125° F. What is the ratio of its thermal efficiency to that of an ideal engine working between the same temperature limits (a) on the Carnot cycle, (b) on the Rankine (Clausius) cycle?

Ans. 45·15 per cent., 50·35 per cent. (Univ. of London, B.Sc. Engineering.)

19. Calculate the actual thermal and mechanical efficiencies of a steam engine from the following data :—

Pressure at stop valve, 120 lbs. per sq. in. absolute. (Temperature, 341° F.)

Moisture in steam at engine stop-valve, 4 per cent.

Back pressure, 4 lbs. per sq. in. absolute. (Temperature, 153° F.)

I.H.P. = 50·8; revolutions per minute = 183.

Weight of steam used per hour, 1016 lbs.

Diameter of brake wheel = 75 in., rope = $\frac{3}{4}$ in.

Tension on ends of rope, 440 lbs. and 40 lbs. respectively.

(Univ. of London, B.Sc. Engineering.) *Ans.* 12·3 and 86·7.

20. The steam consumption of a small engine at different loads was as follows :—

B.H.P.	2·2	4·0	5·4	7·0	9·25
Steam (W) per hour (in lbs.)	100	152	190	235	300

Plot the weight of steam used per hour to a base of B.H.P. and determine the law in the form

$$W = a (\text{B.H.P.}) + C.$$

21. An engine uses the following weights of steam per hour at different powers :—

B.H.P.	90	67½	45	22½
W. lbs.	2600	2050	1480	900

Plot a curve showing (a) total steam used, (b) steam used per B.H.P. on a base of B.H.P., and determine the law of curve (a). *Ans.* $25.5 \text{ (B.H.P.)} + 326\frac{1}{2}$.

22. A compound engine and dynamo, direct coupled, gave the following results :—

I.H.P.	15	23	37	48	62
Output (kwts.)	5.4	8.8	16	22.5	31
Total steam per hour (lbs.)	435	580	822	1025	1290

Plot the above output and steam used to a base of I.H.P. and add curves showing :—

(a) Mechanical efficiency of combined plant $\left(\frac{\text{Electrical H.P.}}{\text{Indicated H.P.}} \right)$.

(b) Steam used per I.H.P. per hour.

(c) „ „ „ kwtt. „

23. Find the fuel cost of generating electricity per B.T. unit from the above curves, if the average output be 25 kwts., the boiler using coal at 17s. per ton and evaporating $8\frac{1}{2}$ lbs. of water per lb. of coal.

24. From the results given in fig. 152, and on pp. 339 and 341, calculate :—

- The percentage ratio of each of the losses given to the heat value of the fuel.
- The percentage ratio of the total net losses debited to the boiler to the heat value of the fuel.
- The percentage ratio of the total net losses debited to the engine as a ratio of the heat value steam supplied to the engine.
- The overall losses from furnace to work done on the brake as a percentage ratio of heat value of fuel.

25. A steam boiler evaporates 9 lbs. of water per lb. of coal used, and the coal costs 12s. 6d. per ton. Find the cost of producing 1 ton of steam. *Ans.* 1s. 4.6d.

26. Find the cost of evaporating 1000 gallons of water, if the coal costs 8s. 6d. per ton, and each lb. of coal makes $7\frac{1}{2}$ lbs. of steam. *Ans.* 5s. 1d.

SECTION IV.

CHAPTER XVII.

THE COMMERCIAL SIDE OF ENGINEERING.

Introductory.—As an introduction to the following chapters dealing with the calculation of weights, the preparation of estimates, the remuneration of labour, and the use of squared paper, the authors offer a few general or introductory remarks on the commercial side of mechanical engineering. They do so in the hope of inducing students and practical men engaged in engineering work to take a real interest in these sections, and in order to show them that the work dealt with is of great practical value. At the same time, a survey of the subject will afford, as it were, an indication of the lines upon which further study should be based. They would urge the young engineer on no account to neglect the study of the commercial side of his profession, for, although very little is published dealing with this subject, yet by keeping this aspect of engineering constantly in view, picking up here a little and there a little, he will learn something and save himself much valuable time in the future. Training and study are as necessary in this as in any other branch of engineering, and as the days go by it is more and more noticeable that it is the trained engineer who is called upon to occupy the positions of trust and influence. In former days this was not so; but at the present time, when prominent positions in engineering become vacant, those are chosen to fill them who are skilled not only in practical but also in the commercial parts of the business, and who, as well as knowing *how* the work ought to be done, *can also estimate the cost* of doing it. It is in this connection, as in other respects, that our technical schools have scope for further usefulness. Sight is sometimes lost of the fact that, under existing social conditions, one of the great aims of almost all business is to get a fair return for the capital expended. We are as yet—although we may be sorry that it is so—far from realising the

dream of the idealist, "When no one should work for money and no one should work for fame, but each for the joy of the working." Many know, too often to their cost, that it is quite possible to become skilful mechanical engineers without knowing much about the commercial side of the business. For want of this knowledge many men who are well up in theory and practice find themselves utterly disqualified for some enviable posts, and suffer the chagrin of seeing them put into the hands of men who in other respects are their inferiors.

The limits of space will not allow us to deal at any great length with the *personnel* and *management* of an engineering concern, but a few remarks will be valuable in the way of suggestion.

The personnel of an engineering establishment will be regulated by the nature of the concern itself, as to whether it is a *company* or private firm. If the former, it will have a *board of directors*, with a *chairman* and *secretary*, in addition to a manager—the directors, chairman, and secretary not necessarily being engineers. If a *private firm*, there may be *one or more principals* in addition to the manager. In any case, however, the number of the staff will be to some extent regulated by the state of trade and the remunerative nature, or otherwise, of the business. In the case of a large company, there will very often be a *general manager*, *assistant manager*, and various under managers or *works managers*, in addition to *foremen of departments*. In the case of a private firm, if the principal is a man of engineering ability he will probably take over the oversight of the financial and commercial department, and leave the works to the manager, or the work may be divided equally between them by mutual agreement, such an understanding often being found necessary to prevent friction and the orders of the one conflicting with those of the other. The other members of the staff will be found in various departments, such as the *general office*, with its *cashier*, or invoice clerks, etc.; the drawing office, with the *head draughtsman* and under draughtsmen; the several *classes of labour*, with their respective foremen, such as pattern-makers, brass and iron moulders, smiths, boiler-makers, fitters, turners, erectors, and machine-men; the stores, with its *storekeepers*; the time office, with the *timekeeper*; general labourers, etc.

The *qualifications of the manager* at the head of the concern should be such as to enable him to discharge his duties in a business-like, scientific, and speedy manner. It is of the utmost importance that he should combine in himself *theoretical*, *commercial*, and *practical knowledge*. He should be a man of sound and ready judgment, thoroughly abreast of the times, and posted up in a good general knowledge of the trade and *fluctuations*

of the market, for upon the art of buying will in a great many instances depend whether profit or loss shall be made. In some cases there is so little margin between the actual cost of the material and the market prices of the finished articles after the necessary labour has been expended, that, unless the material be well bought, loss is inevitable. He should also be a man able to adapt himself to special circumstances, and able to grapple with crises or exigencies which may arise. Much thought will be required to effectively co-ordinate and administer the work in the several departments of the concern so that the business can be carried on smoothly, without unnecessary friction between the different responsible heads. In short, he should be *facile princeps* a leader of men, and must exercise great care in the selection of his staff and foremen, and in addition to being a man of character, should be able to control men. After being as careful as possible, however, in the selection of his under staff, there will still be great need on all technical points for personal supervision, the old adage often proving itself true, that, if you want work done well, you must either do it yourself, or watch its progress all along the line.

The duties and responsibilities of the manager are many, and are often such as to tax the strongest constitutions. First and foremost, he will be impressed with the necessity of system in the administration of the business. This system will be one that is *uniform* throughout. It will begin at the office, in a first-class style of book-keeping. In the time office, an absolutely reliable method of time-keeping will be adopted. In the stores, inward and outward goods will be invariably recorded. This system will be manifested in a proper registration of all loose tools used in and out of the work. In some firms, system has almost been reduced to a formula, and men are kept on the same class of work and *specialities*, and intricate machines are turned out rapidly at a minimum cost. All tools and other machinery, including the prime movers, are kept in a state of good repair and efficiency. If a tool becomes out of date, it often becomes necessary to scrap it, in order that the efficiency of the work may be maintained. Suitable gear will be employed for lifting heavy machinery, and placed under the charge of some competent person. All unnecessary waste of scrap and material will be avoided. Consistent with sound design, all superfluous material and labour will be done away with. Any up-to-date manager realises that it is *economy in design* and the *cheapening of production* to which he must look for more remunerative results. To this end the addition of *labour-saving appliances* will not be overlooked. In addition to *fostering existing trade*, the manager will endeavour by all

legitimate means to *develop and obtain new*, and in order to do this he will cultivate a good business address.

As far as the *quality of the work* turned out is concerned, he will strive to show, from the work done, that the articles and machinery he has supplied have been of a high order of material and workmanship. It is a matter of vital importance to get a name for superior manufacture at reasonable and marketable prices, and if this is accomplished, the probability is that orders will ensue without so much labour and expense in advertising. Many firms obtain a run of business of a durable character as the result of splendid produce, while many others who have resorted to extensive advertisement fail to keep up the demand for their goods because the articles they have supplied have been styled "cheap and nasty."

One of the most important of the manager's duties arises in connection with the *preparation of estimates*. Many he will deal with himself, as the result of his special knowledge acquired from actual experience and from the cost of similar work recorded in his books. The details and working out of some he will leave to others whom he may trust; but in any case, as the responsibility comes upon his shoulders, he will personally look over them from time to time, to make sure of their correctness, and will often apply general and commercial considerations to bear upon them which may be known to himself alone. Over and above the costs of material and labour, he will have a knowledge of the cost of office and other general expenses, and of the marketable rates or prices of similar classes of work, and will eventually fix the amount of his tenders accordingly. The duty of *signing all contracts* very often rests with him, and, in addition, the general oversight of the work during process of manufacture. Further, he will have to consider the best means of securing the completion of the contract within the specified periods, as frequently *penalty clauses* are inserted for failure in this direction. The existence of a penalty clause is not always a regrettable item, as it limits or fixes the contractor's liability. Again, in the exercise of his duties the manager will have to interview customers and inspectors, and attend to travellers, or, at least, as many of them as he wishes to see. Those in the office under him will usually know or ascertain whom he desires to interview, and try to prevent his time being trespassed upon unnecessarily. Interviewing inspectors is generally a matter of such great importance that he will discharge this duty himself, and not relegate it to anyone else. Large contracts are usually carried out under independent inspection, to make certain that the work satisfies the requirements of the specification and embodies the best material and workmanship. These inspectors, for example, may be from the War

Office, Lloyd's, Board of Trade, railway and city engineers, or private inspectors for various companies.

One phase of managerial experience which may be noted is the trying nature of the many things which constitute the manager's career, and perhaps one of the chief causes of harassment is that springing out of the conflict between capital and labour. Another source of irritation is that arising from the severe competition of the present day, the like of which was not realised a few decades ago, and which seems to be increasing in intensity as the days go by. Further, a manager sometimes has to deal with unreasonable customers and inspectors, some of whom are quite ignorant of the value of the work done. Then, again, worry is often caused by the carelessness or negligence of the members on the staff or workmen.

A full treatment of the method of keeping costs in well-conducted engineering concerns, for work in process of construction, is beyond the scope of the present work; but we may in passing notice a few points in connection with them. When a contract is made, the quantities of the several parts are taken out, and the necessary material which is not already in stock is at once ordered from outside. On arriving at the works, it is entered into a rough inward book to the particular work for which it is intended. If any materials are taken from the stores, they are charged up in a similar way to the particular contract. In taking out weights, it must not be overlooked that where the sizes are taken from drawings, these are finished dimensions, and allowance must be made in estimating the weight of the raw material required so as to allow for machining. The material used in the preparation of the patterns for any castings must not be overlooked. The material and cost of same for each contract or piece of work is regularly entered up into a material cost-book, and the actual cost of material used arrived at as the work progresses.

Similarly a wages book, showing the money paid in wages on each contract, is kept properly posted up. From these two books cost-book summaries are finally prepared. These two abstracts together show the *nett cost of material and labour* on each piece of work, and the contractor is enabled to arrive at the cost of work in any or all its stages, and can usually form some idea as to whether a contract is likely to be a financial success or not even before it reaches completion.

In arriving at the *total cost of any work*, however, there are two other very important factors to be taken into account—namely, those due to *indirect expenses* and *establishment charges*. Establishment charges have to do with expenses common to all departments

alike, whereas indirect expenses or shop charges are largely departmental. Under establishment charges would be included such items as salaries paid to managers, office expenses, drawing-office ditto, insurance, rates and taxes, coal, water, gas, maintenance of building, plant, etc. Indirect expenses are numerous, and peculiar to each department. In no two firms are these charges exactly alike, for they vary in proportion to equipment, facilities, environment, staff, etc., and in dealing with them different firms adopt different methods, though the end arrived at is much the same. In large works, where costs are carefully kept, establishment charges and indirect expenses are taken separately; but in a large number of small firms the two are taken together. When taken together, they are located by spreading a percentage over each department in proportion to amount of wages. With many firms the indirect expenses are covered by putting them down to the cost of the various work done. The method adopted of dealing with these expenses depends on the efficiency of the management. These charges are finally totalled up at the end of the business year, when the annual stock-taking is finished, and are finally expressed as a percentage of the year's turnover. Now, it is obvious that, if a contract or piece of work is to be remunerative, it would have to cover a proper percentage of establishment charges and indirect expenses in addition to the cost of material and labour, and to leave a margin we call *profit*. The charges apportioned to the different jobs are often, at the discretion of the manager, spread a little out of proportion, more being put down to a good paying job, and less to one which has not been so remunerative. We should note that the percentage of these charges is always a variable quantity, and is regulated by the amount of work done in the year. For instance, if the total costs for the year are £10,000, and the establishment charges £2000, then these charges form 20 per cent. of the costs. If, however, in another year the works are busy, and the turnover is £20,000 instead of £10,000, which turnover could possibly be executed without much increase in staff, then the charges spread over the increased turnover would diminish the percentage from 20 to 10. If any new machinery is bought, the cost would come out of the *capital in the bank*, and would not be put down to the above charges. This expenditure represents merely a change in the form of capital. In making an annual balance, *depreciation* would not be overlooked.

Let us now for a short time consider one or two points in connection with specifications and *contracting*.

A *specification*, as is well known, is a written or printed docu-

ment of agreement between the two parties. This document sometimes states not only the general outline of the machinery required, but also the enumeration and nature of the various parts of which the whole is composed. Sometimes, in addition, quantities and weights are given, which of course simplifies matters for the contractor very much—though these ought in every case to be checked, as it is stated that the contractor must be responsible for any inaccuracy that might be found. Contracts may be *complete* or *incomplete*; and the specifications are prepared accordingly. Great care must be exercised in dealing with incomplete specifications, for unless the situation is clearly defined litigation may follow. We are constantly hearing of engineering disputes which have to be settled by *arbitration*. In order to give some idea of the difference between complete and incomplete contracts, we may take for the purpose of illustration the manufacture of a pair of marine engines and boiler. Where the contract is a complete one, the specification would express that these have not only to be made and supplied, but also fixed, with all their appurtenances, in the boat for which they were designed, ready for sea. The specification will set forth the type and power of the engines, and will enumerate the various parts and fixings; and, lest anything should be left out, a clause will be inserted stating that, should anything be omitted therein that is usually supplied by engineers to make a complete and perfect job, the same must be understood to be included; and should any dispute after this arise between the two contracting parties, *engineering custom or usage* would decide the points at issue.

An incomplete contract in reference to the manufacture of the same pair of marine engines and boiler would be constituted by the machinery having simply to be made and delivered by the makers to the purchasers, but not fixed, the fixing in the boat being done either by the purchasers or by others to whom they might entrust the work. In this case the specification should be prepared with great care to specify *exactly* the particulars of fittings and fixings or outfit to be supplied with the machinery. Many of these fixings, such as stokehole plates, ladders, casings, gratings, etc., could not be made until the machinery was placed in the boat, and are not usually supplied by the makers, unless they also have the machinery to fix. The contract having been made definite by specification, and *signed by both parties*, is alone the recognised bond between the two, and such only would be recognised in a court of law.

The specifications issued for tenders by public bodies, such as the Admiralty, railway and dock companies, gas or water companies, corporations, etc., are usually of a very explicit

character, and leave little room for difference of opinion. These companies, as a rule, only invite firms to tender in whom they have confidence, and often as the result the lowest tender is accepted. In many cases, where the quantities and weights are clearly expressed, the tenders received are very close together, large contracts being lost by very small amounts.

In conclusion, we will notice briefly one phase of the subject which must often have been a matter of surprise and even amusement to those who have watched from time to time the various estimates for public works. That is the great *disparity in tenders* which are sent in by different firms for the same contract. While some have been remarkably near together, no doubt as the result of a thorough knowledge of the requirements and careful calculation, others have sent in very low tenders difficult to understand. On the other hand, some tenders have been so high as to lead us to infer either ignorance or a wish to avoid securing the work.

It is a well-known fact to those who have been behind the scenes that other reasons may be adduced for these great differences in tenders than ignorance, want of judgment, or the lumping or guessing system. One reason for high tenders is that many firms, on getting the specifications of work, find that it is out of their line, and not adapted to their plant and tools, and so, to get back the *deposit* they had to lay down in order to secure a copy of the specification, they quote a big price, and thus make sure the order will not come their way. Others determine, on account of *stringent clauses* in the specification, that if they get the work at all they will only take it at such a figure as will cover all risks and at the same time bring them in a reasonable profit. Others, again, quote high because the work to be done is so far away from their base of operations that they would be involved in many expenses to which those who are near would be strangers. Many high tenders spring from the fact that some firms have very expensive plant and a large staff to uphold, and stiff prices alone are sufficient to uphold them. Reasons for low quotations may be advanced. Some firms are comparatively young in business, and so, to get a connection, will often quote little more than cost price. *Bad trade* is undoubtedly the back upon which the blame of many low tenders ought to be placed. When trade is bad, cost price, and even less, may be quoted to keep an establishment together, though such a policy cannot be pursued indefinitely. It is well known that contractors in large contracts are paid by *instalments* as the work progresses; they receive the amount due to them in two, three, four, or more instalments, as may be specified in the agreement, and the profit or loss on the trans-

action is not always fully realised until it comes to the last instalment. Firms who go on quoting in this way find very often that they can only finish a preceding large contract by drawing upon the instalment of another; and so they go on securing orders at even less than cost price, until a day comes when further orders are not forthcoming, and we hear of a collapse for so many thousands, which leads us to wonder how they could have gone to the bad to such an extent.

CHAPTER XVIII.

CALCULATION OF WEIGHTS.

THE engineer is frequently required to calculate the weight of different pieces of machinery, and the student should therefore make himself familiar with the various methods of estimating the weight of different articles expeditiously, even sometimes at the expense of a great degree of accuracy, bearing in mind that a weight calculated quickly, within 5 per cent. of the truth, is of more importance than a result expressed to three or four places of decimals arrived at very laboriously, with great loss of time.

Weight of Square Plates.—A very elementary exercise in the calculation of weights is that of estimating the weight of plates of various sizes and shapes. Take, for example, a plate of cast-iron 1 ft. square and 1 in. thick. It is usual in many offices to take the weight of cast-iron at 0·264 or 0·265 lb. per cub. in., but in actual practice this has been found to be rather low, due to the increase in size by rapping the pattern in the mould, etc., and the value taken in these exercises will be 0·27 lb. per cub. ft. A short table of weights per cub. in. and per cub. ft. of some materials used by engineers is given in the following table:—

TABLE XXIX.—WEIGHT OF MATERIALS.

Metals.	Lb. per cub. in.	Lbs. per cub. ft.
Iron, cast	0·27	460
„ wrought	0·28	480
Steel	0·29	490
Brass	0·295	510
Gun-metal	0·305	530
Phosphor bronze . .	0·31	535
Copper	0·32	550
Aluminium	0·093	160
Zinc	0·26	450
Tin	0·27	460
Lead	0·41	710
Mercury	0·49	850

In the above example the area of the plate is 1 sq. ft. or 144 sq. in.; and its thickness being 1 in., the volume of metal in the plate will be $144 \times 1 = 144$ cub. in.

The weight of the plate will be found by multiplying its volume in cub. in. by the weight of the material per cub. in., thus:—

$$144 \times 0.27 = 38.88 \text{ lbs.}$$

For this exercise the weight would have been found more expeditiously by taking all the dimensions in feet. Thus 1 sq. ft. area $\times \frac{1}{12}$ ft. thick = $\frac{1}{12}$ cub. ft., which, at 466 lbs. per cub. ft.,

$$= \frac{466}{12} = 38.83 \text{ lbs.}$$

If the plate has a hole in it, say 2 in. diameter, as shown in

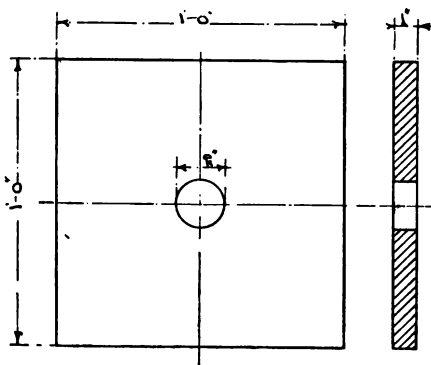


FIG. 153.

fig. 153, the nett area of the plate will be $12 \times 12 = 144$ sq. in., minus the area of the hole $= 2 \times 2 \times \frac{11}{14} = 3.14$ sq. in., or a nett area of

$$144 - 3.14 = 140.86 \text{ sq. in.}$$

This multiplied by the thickness gives 140.86 cub. in. or 38 lbs. (approximately).

If the plate has to be machined on both sides to a finished thickness of 1 in., the weight as cast, allowing $\frac{1}{4}$ in. on each side for tooling, will be

$$144 \times 1\frac{1}{4} \times 0.27 = 48.6 \text{ lbs.}$$

If the plate has a facing 1 in. wide $\times \frac{1}{4}$ in. thick all round

on one side, as shown in fig. 154, the weight will be calculated thus:—

- Volume of plate (as before) = 144 cub. in.
 + Volume of facing, which is $44 \times 1 \times \frac{1}{4} = 11$ cub. in.
 Total volume = $144 + 11 = 155$ cub. in.
 Weight = $155 \times 0.27 = 41.85$ lbs.

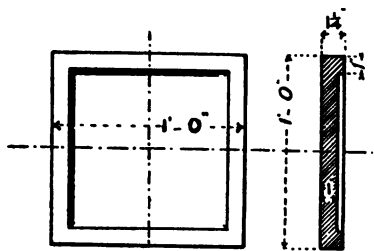


FIG. 154.

If the plate has a raised centre thickening piece, say $\frac{1}{2}$ in. thick $\times 4$ in. square, as shown in fig. 155, its weight will best be cal-

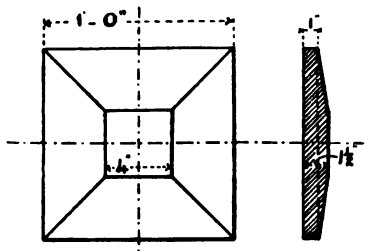


FIG. 155.

culated by taking the raised part by itself, and afterwards taking the four sides, thus:—

$$\text{Centre, } 4 \times 4 \times 1\frac{1}{2} \quad . \quad . \quad = 24 \text{ cub. in.}$$

$$\text{Average length of each side} = \frac{12 + 4}{2} \quad . \quad = 8 \text{ in.}$$

$$\text{,, width} \quad . \quad . \quad . \quad = 4 \text{ ,,}$$

$$\text{,, thickness} = \frac{1 + 1\frac{1}{2}}{2} \quad . \quad . \quad = 1\frac{1}{4} \text{ ,,}$$

$$\text{Volume} = 4 \times 8 \times 4 \times 1\frac{1}{4} \quad . \quad . \quad = 160 \text{ cub. in.}$$

$$\text{Total volume} \quad . \quad = 184 \text{ ,,}$$

$$\text{Weight} = 184 \times 0.27 = 49.68 \text{ lbs.}$$

If, in addition, it is provided with a square beading, as shown

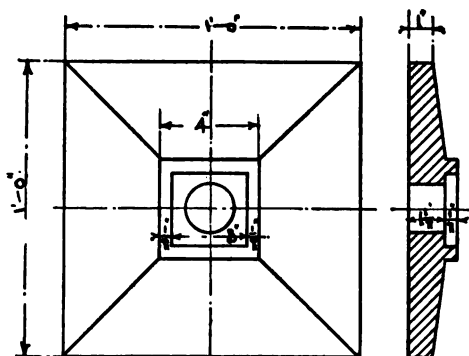


FIG. 156.

in fig. 156, frequently used for foundation bolt washer plates, its weight can be calculated thus:—

Volume of plate, as in previous example = 184.0 cub. in.

Beading, $4 \times 3\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ = 3.5 „

187.5 „

Less 2-in. hole = $2 \times 2 \times 1\frac{1}{4} \times 1\frac{1}{2}$ = 4.7 „

Nett volume = 182.8 „

Weight = $182.8 \times 0.27 = 49.356$ lbs.

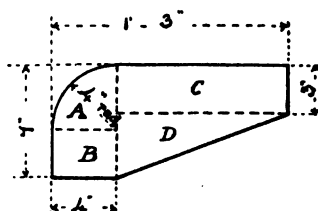


FIG. 157.

Similarly, plates of complicated shapes should be divided up into convenient sections, and the area or volume of each section calculated separately and the results added together.

Take, for example, a cast-iron plate of the shape shown in fig. 157. To calculate its weight, divide it into four parts, A, B, C, and

D, as shown by the dotted lines, and calculate the area of each part separately thus :—

A. Quadrant of a circle—

$$\text{Radius} = 4 \text{ in.} \quad \text{Area} = \frac{\pi r^2}{4}$$

$$= \frac{22}{7} \times \frac{16}{1} \times \frac{1}{4} = \frac{88}{7} \quad . \quad = 12.57 \text{ sq. in.}$$

B. Rectangle, 4×3 . Area . . . = 12.00 "

C. Rectangle, $(15 - 4) \times 3$. Area . . = 33.00 "

D. Triangle—

$$\text{Base} = 11 \text{ in.} \quad \text{Altitude} = 4 \text{ in.}$$

$$\text{Area} = \frac{11 \times 4}{2} \quad . \quad . \quad = 22.00 \quad "$$

$$\text{Total area} \quad . \quad = 79.57 \quad "$$

$$\text{Volume} = 79.57 \times 1 = 79.57 \text{ cub. in.}$$

$$\text{Weight} = 79.57 \times 0.27 = 21.48 \text{ lbs.}$$

Weight of Iron and Steel Plates.—The weight of iron and steel plates of simple dimensions can be calculated very simply by finding the area in square feet, and multiplying by the weight of a square foot of the plate of required thickness. Thus, a wrought-iron plate 1 in. thick weighs 40 lbs. per superficial foot, so that the weight of wrought-iron plates often can be calculated mentally by taking the product of area (in square feet) by the thickness (in inches) and by the constant 40. Thus a wrought-iron plate 8 ft. 0 in. $\times \frac{3}{8}$ in. thick will weigh

$$8 \times 2\frac{1}{2} \text{ (sq. ft. area)} \times \frac{3}{8} \text{ (in. thick)} \times 40 \text{ (lbs.)} = 300 \text{ lbs.}$$

Cast-iron plates weigh 38 lbs. per 1 in. thick per sq. ft., and brass plates 42½ lbs. The following table will be found useful :—

TABLE XXX.—WEIGHT OF PLATES IN LBS. PER SQ. FT.
FOR 1 IN. THICK.

Material.	Weight.	Material.	Weight.
Cast iron . .	38	Copper . . .	45
Wrought iron . .	40	Brass (average) . .	42
Mild steel . .	41	Lead . . .	60
Cast steel . .	41	Aluminium . .	13½

Weight of Circular Covers and Discs.—To calculate the weight of a plain circular disc, as shown in fig. 158, 1 ft. diameter $\times$ 1 in. thick, the process is this :—

$$\text{Volume} = \text{Area} \times \text{thickness}$$

$$= 12 \times 12 \times \frac{11}{14} \times 1 = 113.1 \text{ cub. in.}$$

If in cast iron, weight (solid) = $1131 \times 0.27 = 30.54$ lbs.

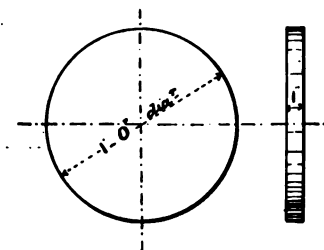


FIG. 158.

If, however, with 6 bolt-holes, as for a blank flange, holes say 1 in. diameter :—

Gross area of plate . . . = 113.1 sq. in.

Less 6 holes = $6 \times 1 \times 1 \times \frac{1}{4} = 4.7$ „

Nett area of plate . . . = 108.4 „

Weight = $108.4 \times 1 \times 0.27 = 29.27$ lbs.

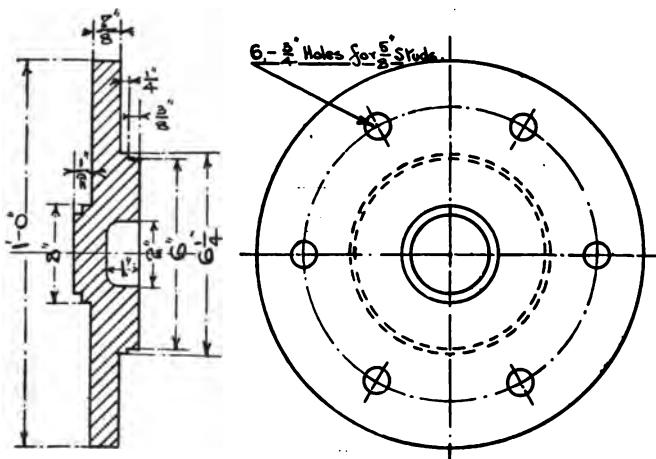


FIG. 159.

If the disc be more complicated, as in the case of a cylinder cover for a small steam engine, as shown in fig. 159, proceed as

follows, calculating the volumes of the various discs according to their sizes, thus :—

		Cub. in.
Flange, $12 \times 12 \times \frac{11}{16} \times \frac{7}{8}$	.	= 99.0
Registers, $6\frac{1}{4} \times 6\frac{1}{4} \times \frac{11}{16} \times \frac{1}{4}$	.	= 7.67
+ $6 \times 6 \times \frac{11}{16} \times \frac{3}{8}$	.	= 10.61
Boss, $3 \times 3 \times \frac{11}{16} \times \frac{1}{2}$	.	= 3.53
		120.81
Less recess, $2 \times 2 \times \frac{11}{16} \times 1$	.	= 3.14
„ bolt-holes, $6 \times \frac{3}{4} \times \frac{3}{4} \times \frac{11}{16} \times \frac{7}{8}$	=	2.32
		5.46
Nett volume		= 115.35

$$\text{Weight} = 115.35 \times 0.27 = 31.14 \text{ lbs.}$$

Conical Covers or Plates.—If the plate be conical or “dished,” as with some cylinder covers similar to that represented in fig. 160, proceed step by step, first with the flange, then the register or spigot, and finally calculate the volumes of the outside and inside of the conical portions, tabulating the quantities as under :—

			Cub. in.
Flange, 30 in. diam.	Area = 706.9 sq. in.		
20 „ „	„ = 314.1 „		
	Difference = 392.8 „	$\times 1$ (thick)	= 589.2
Register, 23 in. diam	Area = 415.5 sq. in.		
20 „ „	„ = 314.1 „		
	Difference = 101.4 „	$\times \frac{1}{2}$ „	= 50.7
22 $\frac{1}{4}$ in. diam.	Area = 388.8 „		
20 „ „	„ = 314.1 „		
	Difference = 74.7 „	$\times 1$ „	= 74.7
			714.6

Conical Outside.—Divide this up into sections, as shown by fig. 160, and take each section separately, thus :—

A. Disc, 20 (in. diam.) $\times 1\frac{1}{2}$ (in. thick)—

$$\text{Volume} = 314.1 \times 1\frac{1}{2} = 392.6 \quad \text{Cub. in.}$$

B + C. Conical, 20 (in. diam.) $\times$ 5 (in. high)— Cub. in. Cub. in.

$$\text{Volume} = \frac{314 \cdot 1}{3} \times 5 = 523 \cdot 5$$

$$\text{Sum A + B + C} = 916 \cdot 1$$

C. Conical, 9 (in. diam.) $\times$ $2\frac{1}{4}$ (in. high)—

$$\text{Volume} = \frac{63 \cdot 6}{3} \times 2\frac{1}{4} = 47 \cdot 7$$

$$\text{Sum A + B} = 868 \cdot 4$$

D. Cylindrical (boss), 9 (in. diam.) $\times$ $2\frac{1}{2}$ (in. high)—

$$\text{Volume} = 63 \cdot 6 \times 2\frac{1}{2} = 159 \cdot 0$$

$$\text{Total volume of outside, A + B + D} = 1027 \cdot 4 \quad \text{Cub. in.}$$

Conical Inside.—Divided into sections as shown in fig. 160.

E + F. Conical, 20 (in. diam.) $\times$ $4\frac{1}{2}$ (in. high)— Cub. in.

$$\text{Volume} = \frac{314 \cdot 1}{3} \times 4\frac{1}{2} = 471 \cdot 1$$

F. Conical, 6 (in. diam.) $\times$ 1.35 (in. high)—

$$\text{Volume} = \frac{28 \cdot 2}{3} \times 1 \cdot 35 = 12 \cdot 7$$

$$\text{Difference} = \text{E} = 458 \cdot 4$$

G. Cylindrical (recess), 6 (in. diam.) $\times$ 2 (in. high)—

$$\text{Volume} = 28 \cdot 2 \times 2 = 56 \cdot 4$$

$$\text{Sum} = \text{total volume of inside E + G} = 514 \cdot 8$$

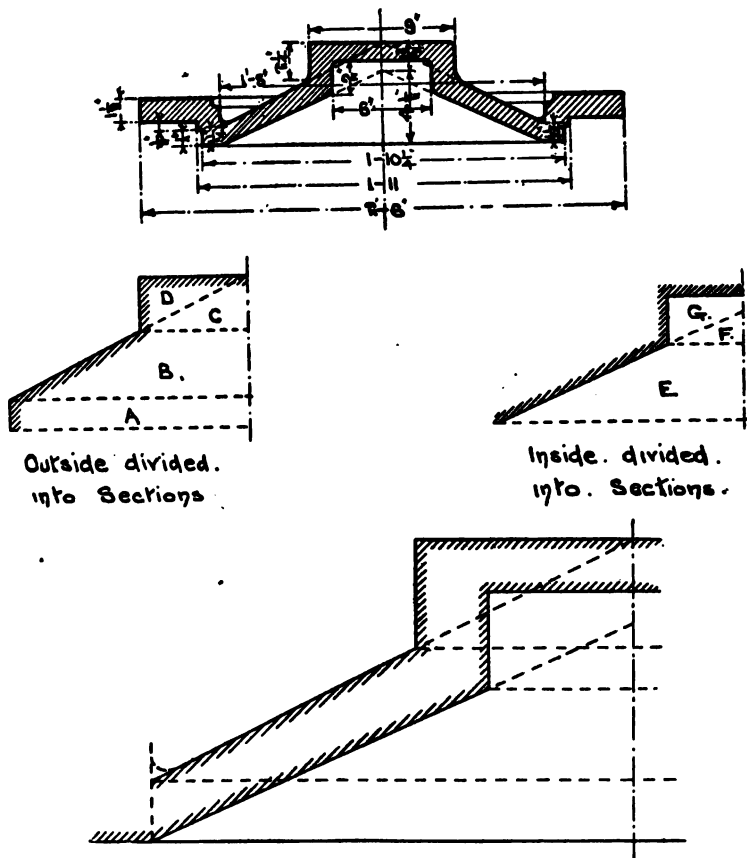
$$\text{Difference between outside volume and inside volume} = \text{nett volume of metal in centre} = 512 \cdot 6$$

$$\text{Total volume of metal in cover} = 1227 \cdot 2$$

Assuming holes to balance weight of "fillets,"

Weight of cover = $1227.2 \times 0.27 = 331.3$ lbs., or (practically) 3 cwt.

(Note.—The actual weight of this cover was 3 cwt. 0 qrs. 11 lbs.).



Conical Cylinder Cover

FIG. 160.

Weight of Steel Rolled Joists.—The calculations required in estimating the weights of steel joists, angle or tee bars,

etc., are usually of a simple character. Take, for example, the 10 in. $\times$ 5 in. rolled joist 15 ft. long represented in section in fig. 161. Taking the flanges and central web to have a mean thickness of $\frac{3}{8}$ in., the area of the cross section will be—

$$\text{Flanges, } 5 \times \frac{3}{8} \times 2 = 3.75 \text{ sq. in.}$$

$$\text{Web, } 9\frac{1}{4} \times \frac{3}{8} = 3.47 \text{ ,,}$$

$$\text{Total} = 7.22 \text{ ,,}$$

The volume of metal, therefore, will be

$$7.22 \times 180 = 1300 \text{ cub. in. (nearly).}$$

The weight, taking 1 cub. in. as weighing
0.29 lb. = $1300 \times 0.29 = 377$ lbs.

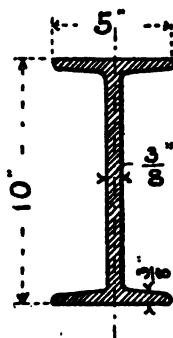


FIG. 161.

Weight of Pipes or Hollow Cylinders.—In estimating the weight of pipes or tubes, there are two principal methods which may be adopted. The first is to subtract the cross-sectional area of the inside of the pipe or tube from that of the outside, and multiply the difference, which gives the cross-sectional area of the metal, by the length. This method is preferable, where a table of the area of circles is available; otherwise, it will be found simpler to multiply the circumference given by the mean diameter of the pipe or tube by the thickness, and then by its length.

Taking the first method, to find the weight per foot run of a 4-in. cast-iron pipe, $\frac{3}{8}$ in. thick. By reference to a table of areas of circles we get:—

$$\text{Area to outside of pipe (4}\frac{3}{4} \text{ in. diam.)} = 17.72 \text{ sq. in.}$$

$$\text{,, inside ,, (4 in. diam.)} = 12.56 \text{ ,,}$$

$$\text{Difference} = \text{area of metal} = 5.16 \text{ ,,}$$

$$\begin{aligned} \text{Volume of metal for pipe 12 in. long} \\ = 5.16 \times 12 = 61.92 \text{ cub. in.} \end{aligned}$$

$$\begin{aligned} \text{Weight of pipe per foot run} \quad . \quad . &= 61.92 \times 0.27. \\ &= 16.72 \text{ lbs. (nearly).} \end{aligned}$$

Using the same method to find the weight of a copper condenser

tube, $\frac{3}{4}$ in. outside diameter, No. 16 B.W.G. (about $\frac{1}{8}$ in. thick), 10 ft. long :—

$$\begin{array}{ll} \text{Area to outside of tube } \left\{ \frac{3}{4} \text{ in. diam.} \right\} & = 0.441 \text{ sq. in.} \\ \text{,, inside ,, } \left\{ \frac{1}{2} \text{ in. ,,} \right\} & = 0.306 \text{ ,,} \end{array}$$

$$\text{Difference} = \text{area of metal} = 0.135 \text{ ,,}$$

$$\begin{array}{l} \text{Volume of metal for tube 10 ft. long} \\ = 0.135 \times 120 = 16.2 \text{ cub. in.} \end{array}$$

$$\text{Weight of tube} = 16.2 \times 0.32 = 5.184 \text{ lbs.}$$

As illustrating the second method of calculation, take the case of a cast-iron flanged pipe, 10 in. internal diameter, 1 in. thick,

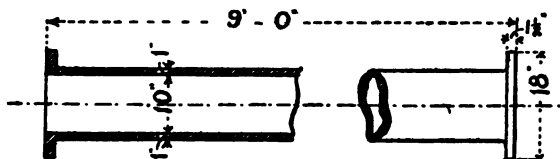


FIG. 162.

9 ft. long from face to face of flanges (see fig. 162). Here the mean diameter of the pipe will be $\frac{10+12}{2} = 11$ in., and the calculations are given below :—

$$\text{Weight of pipe, } 11 \times 3\frac{1}{4} \times 1 \times 108 \times 0.27 = 1008 \text{ lbs.}$$

Two flanges—

$$\begin{array}{ll} \text{Area to outside of flange} = 18 \times 18 \times \frac{1}{4} & = 254.5 \text{ sq. in.} \\ \text{,, ,, pipe} = 12 \times 12 \times \frac{1}{4} & = 113.1 \text{ ,,} \end{array}$$

$$\text{Difference} = 141.4 \text{ ,,}$$

$$\text{Volume of each flange} = 141.4 \times 1\frac{1}{4} = 212.1 \text{ cub. in.}$$

$$\text{Weight of ,,} = 212.1 \times 0.27 = 57.27 \text{ lbs.}$$

$$\text{,, two flanges} = 57.27 \times 2 = 114.5 \text{ lbs.}$$

$$\begin{array}{l} \text{Total weight of pipe} = 1008 + 114.5 = 1122.5 \text{ lbs.} \\ = 10 \text{ cwt. 0 qrs. 2.5 lbs.} \end{array}$$

A useful approximation may be given, i.e. that with cast-iron flanged pipes of average proportions, the weight of a pair of flanges is approximately equal to the weight of a foot extra length of pipe. Had the pipe in the above exercise been taken as a plain

pipe with spigot ends 10 ft. long (the extra foot representing the pair of flanges), the calculation would have been—

$$\text{Weight} = 11 \times 3\frac{1}{4} \times 1 \times 120 \times 0.27 = 1120 \text{ lbs.,}$$

a result within $\frac{1}{4}$ of 1 per cent. of the more elaborate calculation.

Weight of Shafting.—The calculation of the weights of shafting, both solid and hollow, forms a useful exercise. Take the case of a 3-in. mild steel shaft 12 ft. long. Here the weight will be

$$3 \times 3 \times \frac{11}{16} \times 144 \times 0.29 = 295.3 \text{ lbs.}$$

If the shaft be provided with one or more enlargements for

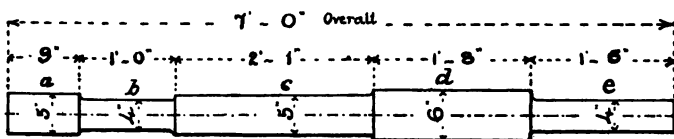


FIG. 163.

wheel-seats, as represented in fig. 163, the calculation for each section of the shaft may be taken separately, thus:—

$$\begin{aligned} A + C &= 5 \times 5 \times \frac{11}{16} \times 34 = 667.8 \text{ cub. in.} \\ B + E &= 4 \times 4 \times \frac{11}{16} \times 30 = 377.1 \text{ ,,} \\ D &= 6 \times 6 \times \frac{11}{16} \times 20 = 565.7 \text{ ,,} \\ \hline \text{Total} &= 1610.6 \text{ ,,} \end{aligned}$$

In this case it will be simpler to find the cubic contents of each piece of equal diameter, as shown, and multiply the total cubic contents by the weight factor or density thus:—

$$\begin{aligned} \text{Weight} &= 1610.6 \times 0.29 = 467.07 \text{ lbs.} \\ \text{Add 2 per cent. for radii in corners} &= 9.34 \text{ ,,} \\ \hline \text{Total} &= 476.41 \text{ ,,} \\ &= 4 \text{ cwt. 1 qr. 0.4 lb.} \end{aligned}$$

Hollow Shafting.—This can usually be treated as a pipe, making allowance for any enlargements or journals as explained previously.

For example, calculate the weight of a propeller stern shaft,

18 in. diameter $\times$ 10 ft. long, with solid welded flanges and 8-in. hole (see fig. 164).

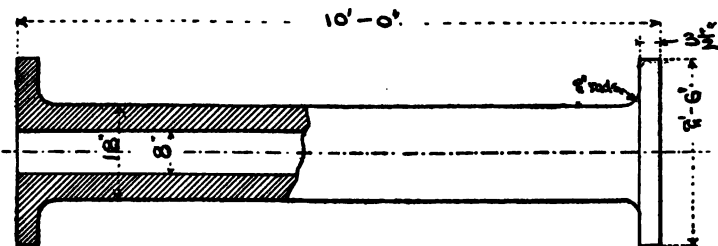


FIG. 164.

Cub. in.

Body of shaft = Area of 18-in. circle = 254.5 sq. in.

Minus 8-in. hole. = 50.3 "

Nett area . . . = 204.2 "

Volume = 204.2 $\times$ 120 . . . = 24,504

Two end flanges 30 in. diam.—Area = 707.1 sq. in.

Minus 18 in. (shaft) = 254.5 "

Nett area . . . = 452.6 "

Volume = 452.6 $\times$ 7 . . . = 3,168

Fillets or 2-in. radii under flanges—

Area of 4-in. square = 16.0 sq. in.

„ 4-in. circle = 12.56 "

Take $\frac{1}{4}$ = 4 | 3.44

Area of fillet . . . = 0.86 "

Length of fillets = circum. of 20-in. circle $\times$ 2.

Volume of fillets = 62.8 $\times$ 2 $\times$ 0.86 . . . = 108 (nearly)

Total volume . . . = 27,780

Less bolt-holes in flanges, say 10 bolts 3 in.

diam. in each flange—Volume = 20 $\times$ 3

$\times$ 3 $\times$ $\frac{1}{4}$ $\times$ $\frac{1}{4}$. . . = 495

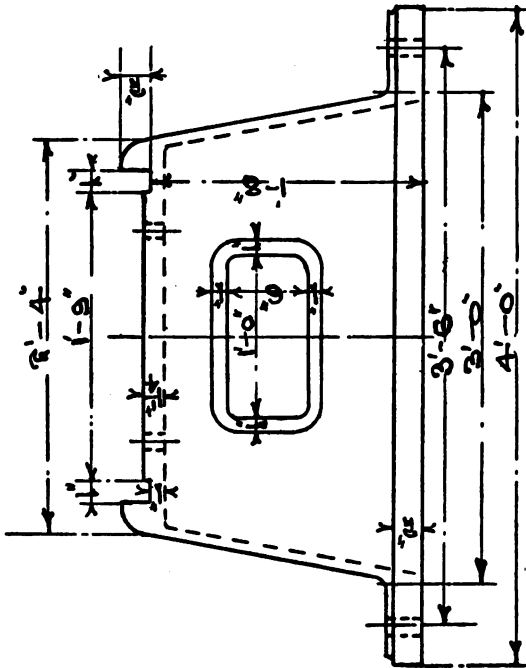
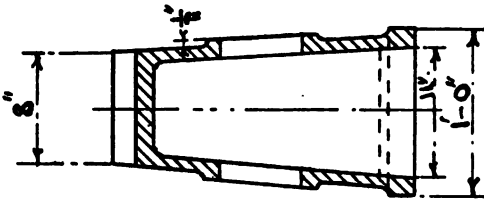
Total nett volume . . . = 27,285

Weight = 27,285 $\times$ 0.29 = 7913 lbs. (nearly)

= 3 tons 10 cwt. 2 qrs. 17 lbs.

Weight of a Cast-iron Pedestal.—As a progressive example in calculating the weight of castings, take the pedestal or box section

frame represented in fig. 165. The details of the calculations required are given below :—



Cast. Iron Pedestal.

FIG. 165.

Top plate, $28 \times 8 \times 1$	Cub. in.	= 224
Facing on ditto, $21 \times 8 \times \frac{1}{4}$	=	42
Joggles on ditto, $2 \times 2 \times 2 \times 8$	=	64

Front and back plates (average width $\frac{28+36}{2} = 32$ in.),	Cub. in.
$(2 \times 32 \times 17 \times \frac{3}{4}) - (2 \times 12 \times 6 \times \frac{3}{4})$	708
Beadings on ditto, $2 \times 40 \times 0.39$ (sq. in.)	= 31
Side plates (average width $\frac{6\frac{1}{2} + 9\frac{1}{2}}{2} = 8$ in.), $2 \times 8 \times$	
$17\frac{1}{2} \times \frac{3}{4}$	= 210
Bottom plate, $2 \times 7\frac{1}{2} \times 12 \times 2$	= 360
Bottom beadings, $2 \times 34\frac{1}{2} \times 2 \times 1\frac{1}{4}$	= 173
Bosses on base = area 4 in. diam. = $12.5 + 4(42 - 36)$	
$= 12.5 + 32 = 44.5$ sq. in. $\times \frac{1}{2}$	= 22
Total net volume (assuming bolt-holes to counterbalance the fillets)	1834
Weight = $1834 \times 0.27 = 494$ lbs. (approx.)	
$= 4$ cwt. 1 qr. 18 lbs.	

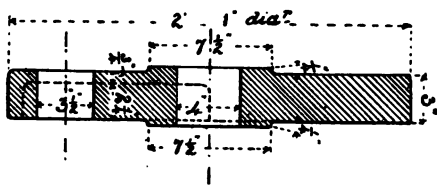
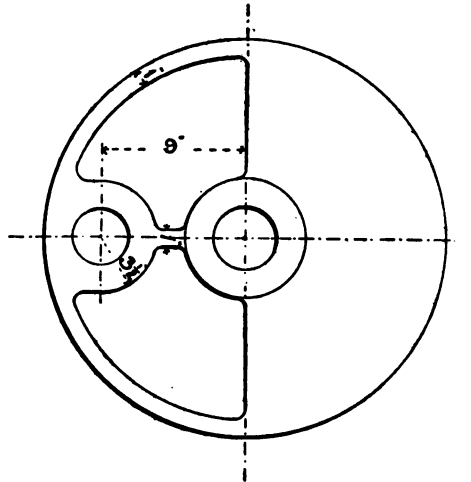


FIG. 166.

Weight of Cast-iron Balanced Disc.—As illustrating the method of calculating the weight of a piece of machinery with a number of holes or recesses, take the case of a cast-iron balanced disc, as represented in fig. 166. In this case it will be convenient to arrange the calculations in two columns, positive and negative, as shown, the former representing the volume of the whole, and the latter that of the holes and recesses, thus :—

	Positive.	Negative.
	Cub. in.	Cub. in.
<i>Positive—</i>		
Disc, 25 in. diam. $\times$ 3 in. thick—		
Volume = $25 \times 25 \times \frac{11}{16} \times 3$. . .	1473.2	
2 bosses, each $7\frac{1}{2}$ in. diam. $\times$ $\frac{1}{2}$ in. thick—		
Volume = $\frac{11}{16} \times \frac{15}{2} \times \frac{15}{2} \times 2 \times \frac{1}{2}$. . .	22.1	
<i>Negative—</i>		
Shaft hole, 4 in. diam. $\times$ $3\frac{1}{2}$ in. deep—		
Volume = $\frac{1}{2} \times \frac{1}{2} \times \frac{11}{16} \times \frac{7}{2}$. . .	44.0	
<i>Balancing recess—</i>	Sq. in.	
23 in. diam., $23 \times 23 \times \frac{11}{16} \times \frac{1}{2}$. . .	207.8	
Less central half boss $7\frac{1}{2}$ in. diam.—		
Area = $\frac{15}{2} \times \frac{15}{2} \times \frac{11}{16} \times \frac{1}{2}$. . .	22.1	
Less pin half boss = $7 \times 7 \times \frac{11}{16} \times \frac{1}{2}$ = 19.25		
+ $7 \times 2\frac{1}{2}$ = 17.5		
Less 1-in. rib in centre = $1 \times 1\frac{3}{4}$ = 1.75		
	60.6	
Nett area of recess = 147.2		
Volume of recess = $147.2 \times 2\frac{1}{4}$. . .	331.2	
	Totals = 1495.3	375.2
Less holes and recesses . . .	375.2	
Nett volume of metal . . .	1120.1	
Weight = 1120.1×0.27 = 302.4 lbs.		
	= 2 cwt. 2 qrs. 22 lbs.	

Weight of Fly-wheel.—Another useful exercise on the calculation of weights is that of the ordinary fly-wheel, such as that

shown in fig. 167. Taking first the rim, and using a table giving areas of circles, proceed as follows:—

	Cub. in.
<i>Rim</i> —96 in. diam. = 7238 sq. in. area.	
92 „ = 6648 „ „	
Difference = 590 „ „ × 12 in. thick gives	7080
92 in. diam. = 6648 „ „	
84 „ = 5542 „ „	
Difference = 1106 „ „ × 5 in. thick gives	5530
84 in. diam. = 5542 „ „	
81½ „ = 5217 „ „	
Difference = 325 „ „ × 9 in. thick gives	2925
<i>Barring cleats</i> —100 × 4 × 1½ × 1½	750
<i>Boss</i> ,—14 in. diam. = 154 sq. in. area	
Bore 7 „ = 38 „ „	
Difference = 116 „ „ × 12 in. thick .	1392
6 <i>Arms</i> , each 33¾ in. long—	
Area at rim = 2 × 3 × ¼ = 4.71 sq. ins.	
„ boss = 3 × 4½ × ¼ = 6.75 „	
Mean = 5.73 × 33¾ × 6 . . .	1160
Total	18837
Weight = 18837 × 0.27 = 5086 lbs.	
= 2 tons 5 cwt. 1 qr. 18 lbs.	

Very frequently, in calculating weights, the volume of metal required, and therefore the weight, is some function of the square or cube of one of its dimensions. In such cases the volume and weight can be quickly arrived at by a comparison. Take, for instance, the case of cast-iron balls used for governors. Here the volume of metal (and therefore the weight) varies directly as the cube of the diameter. It is stated on p. 55 that the volume of a sphere is equal to $d^3 \times 0.5236$, where d = diameter of sphere; consequently, if we have two balls or spheres of diameters d and d_1 , respectively, their volumes (and therefore weights) will be in the ratio of $d^3 : d_1^3$. Therefore, if we know the weight of a cast-

iron ball of any diameter (say 1 in.), we can find the weight of cast-iron balls of any diameter by simply multiplying by the ratio of the third powers of the diameters,

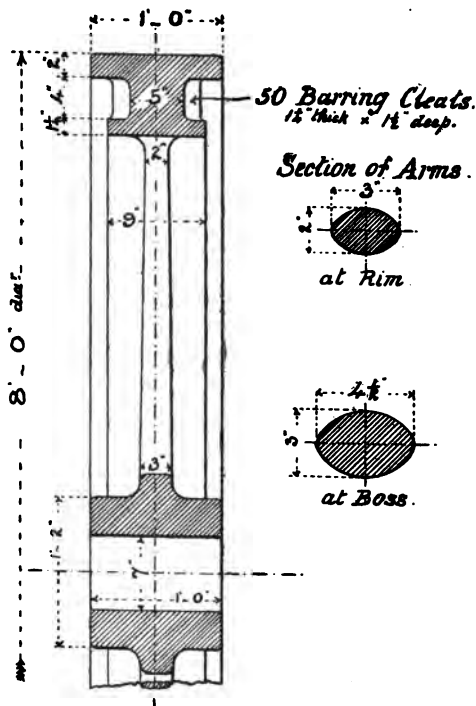


FIG. 167.

For example, a cast-iron ball 1 in. diameter will weigh

$$1^3 \times 0.5236 \times 0.27 = 0.141 \text{ lb.}$$

Therefore a ball 2 in. diameter will have a weight which can be found directly by means of proportion, thus:—

$$\text{As } 1^3 : 2^3 :: 0.141 : x,$$

$$\text{or } x = \frac{0.141 \times 8}{1} = 1.13 \text{ lbs.}$$

Or a cast-iron ball $2\frac{5}{8}$ in. diam. will weigh

$$\frac{0.141 \times (2\frac{5}{8})^3}{1^3} = 2.55 \text{ lbs.}$$

We now give an example of the method of calculating the weight of, and setting out the work for, a *complex casting* such as a cast-iron steam-jacketed pan with spherical base (see fig. 168),

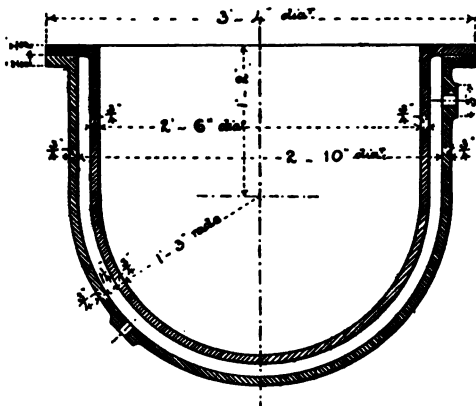


FIG. 168.

as used for melting tallow, wax, etc. Here the calculations required would be tabulated somewhat as follows:—

WEIGHT OF CAST-IRON STEAM-JACKETED PAN.

Cub. in.

Inner Pan—

Flange 40 in. diam. outside, 30 in. diam. inside,
 $\frac{7}{8}$ in. thick—

$$\begin{aligned}\text{Volume of metal} &= (40^2 - 30^2) \times \frac{\pi}{4} \times \frac{7}{8} \\ &= (1600 - 900) \times \frac{1\frac{1}{4}}{8} \times \frac{7}{8} \\ &= 700 \times \frac{1\frac{1}{4}}{8} \times \frac{7}{8} \quad . \quad . \quad . = 481\end{aligned}$$

Upper part (cylindrical)—

$$\begin{aligned}\text{Volume} &= \{(31\frac{1}{2})^2 - 30^2\} \times \frac{\pi}{4} \times 13\frac{1}{8} \\ &= (992\frac{1}{4} - 900) \times \frac{1\frac{1}{4}}{4} \times \frac{105}{8} \quad . \quad . \quad . = 951\end{aligned}$$

Bottom (hemispherical)—

$$\begin{aligned}\text{Volume} &= \frac{[(31\frac{1}{2})^3 - (30)^3]}{2} \times 0.5236 \\ &= \frac{(31256 - 27000)}{2} \times 0.5236 \quad . \quad . \quad . = 1114\end{aligned}$$

Outer Pan—

Cub. in.

$$\begin{aligned}\text{Flange} &= (40^2 - 34^2) \times \frac{\pi}{4} \times \frac{7}{8} \\ &= (1600 - 1156) \times \frac{1}{16} \times \frac{7}{8} = 305\end{aligned}$$

$$\begin{aligned}\text{Upper part} &= [(35\frac{1}{2})^2 - 34^2] \times \frac{\pi}{4} \times 12\frac{1}{4} \\ &= (1260\frac{1}{4} - 1156) \times \frac{1}{16} \times \frac{49}{2} = 1003\end{aligned}$$

$$\begin{aligned}\text{Bottom} &= \frac{[(35\frac{1}{2})^3 - (34)^3] \times 0.5236}{2} \\ &= \frac{(44739 - 39304 \times 0.5236)}{2} = 1423\end{aligned}$$

$$\text{Bosses, 2 in. diam.} = 3.14 \times \frac{1}{2} \times 2 = 3.14$$

$$\text{Less holes } \frac{3}{4} \text{ in. diam.} = 0.44 \times$$

$$1\frac{1}{4} \times 2 = 1.1$$

$$\text{Nett volume} = 2.0 \quad 2$$

$$\text{Total volume of metal} = 5279$$

Estimated weight of both pans

$$= 5279 \times 0.27 = 1425 \text{ lbs.}$$

$$= 12 \text{ cwt. 2 qrs. 25 lbs.}$$

The actual weight of these two pans was 12 cwt., thus showing that the thickness of metal was less than that given. They were struck up in loam, which will probably account for this difference.

Weight of Forgings.—Very frequently, in calculating the weight of forgings made directly from bar steel—as, for example, the double-throw crank represented in fig. 169—the weight can be very quickly approximated from the known weight of bar iron or steel of the size used. For example, in the crank referred to, the webs are $3\frac{1}{4}$ in. diameter, circular in section, and the bearings and pins all 3 in. diameter finished, so that the stock size used would be $3\frac{1}{4}$ in. diameter bar steel. The length of bar used would be the over-all length of the crank, plus the length of each crank arm, or 6 ft. plus 4 times 6 in. (the throw), equal to 8 ft.; and allowing 6 in. for

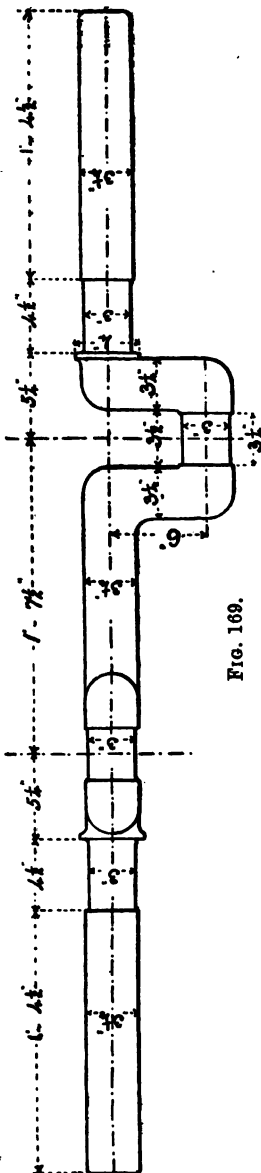


FIG. 169.

jumping up to form the two collars and parting off at the ends, we have the total weight of the forging equal to the weight of

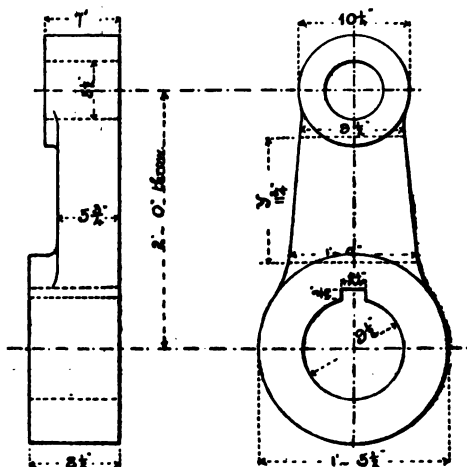


FIG. 170A.

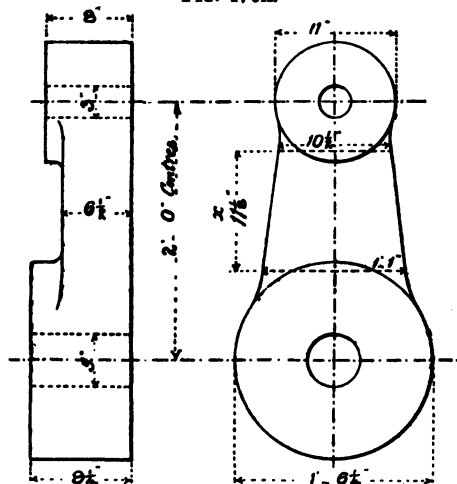


FIG. 170B.

a plain bar of mild steel $3\frac{1}{2}$ in. diameter $\times$ 8 ft. 6 in. long. The weight will be found thus:—

$$8.3 \times 102 = 847 \text{ cub. in.}$$

$$847 \times 0.29 = 245 \text{ lbs.} = 2 \text{ cwt. 0 qr. 21 lbs.}$$

The actual weight of the forging for this crank was 2 cwt. 0 qr. 20 lbs., practically the same figure.

Where, however, the forging is somewhat complicated, a very liberal allowance of excess material has to be made in calculating the weight; particularly is this the case if the forging has to be made by another firm and is bought by the cwt. For example, the mild steel crank arm shown in fig. 170A weighed 9 cwt. 2 qrs. 14 lbs. in the rough, and only 6 cwt. 2 qrs. 12 lbs. when finished. That is, the forging weighed 45 per cent. heavier than the finished crank. The calculations for the weight of the forging for this crank are given as follows (see fig. 170B, giving approximate forging sizes):—

Large boss, $18\frac{1}{2}$ in. diam. outside	= 261.6 sq. in.	Cub. in.
5 " " hole	= 19.7 "	
Metal	= 241.9 "	$\times 9\frac{1}{4}$ = 2240
Small boss, 11 in. diam. outside	= 95.1 "	
3 " " hole	= 7.1 "	
Metal	= 88.0 "	$\times 8$ = 704
Web (13 in. to $10\frac{1}{2}$ in. wide)	= $11\frac{3}{4} \times 11\frac{1}{8}$ (see x) $\times 6\frac{1}{2}$	= 850
Total volume	.	= 3794

$$\text{Calculated weight} = 3794 \times 0.29 = 1100 \text{ lbs.} \\ = 9 \text{ cwt. 3 qrs. 8 lbs.}$$

The actual weight was 9 cwt. 2 qrs. 14 lbs.

Difference = 22 lbs. = 2 per cent. heavy.

The calculation for the weight of the finished crank would be:—

Large boss, $17\frac{1}{2}$ in. diam. outside	= 240.5 sq. in.	Cub. in.
$9\frac{1}{2}$ " " hole	= 70.9	
Keyway, $2\frac{1}{4} \times \frac{3}{4}$	= 1.7	72.6
Metal	= $167.9 \times 8\frac{1}{2}$	= 1425
Small boss, $10\frac{1}{2}$ in. diam. outside	= 82.5 sq. in.	
$5\frac{1}{2}$ " " hole	= 23.7 "	
Metal	= 58.8 "	$\times 7$ in. = 412
Web (12 in. to $9\frac{1}{2}$ in. wide)	= $10\frac{3}{4} \times 11\frac{3}{4}$ (y) $\times 5\frac{3}{4}$ in. thick	= 728
Total volume	.	= 2565

$$\begin{aligned}\text{Calculated weight} &= 2565 \times 0.29 = 744 \text{ lbs.} \\ &= 6 \text{ cwt. 2 qrs. 16 lbs.}\end{aligned}$$

Actual weight was 6 cwt. 2 qrs. 12 lbs.

$$\text{Difference} = \frac{1}{2} \text{ per cent.}$$

In estimating the weight of the forging for a **solid marine type crank-shaft**, where the crank is slotted out of the solid, an extra allowance for excess weight should be made, as cranks of this type are seldom forged as "clean" as they might be. An instance of this occurred in estimating the weight of the double-throw (marine type) crank represented in fig. 171A, where the estimated weight of the forging was $12\frac{1}{4}$ cwt., but the actual weight was over 14 cwt. The estimated weight was obtained from the (assumed) forged sizes (given in fig. 171B), as follows:—

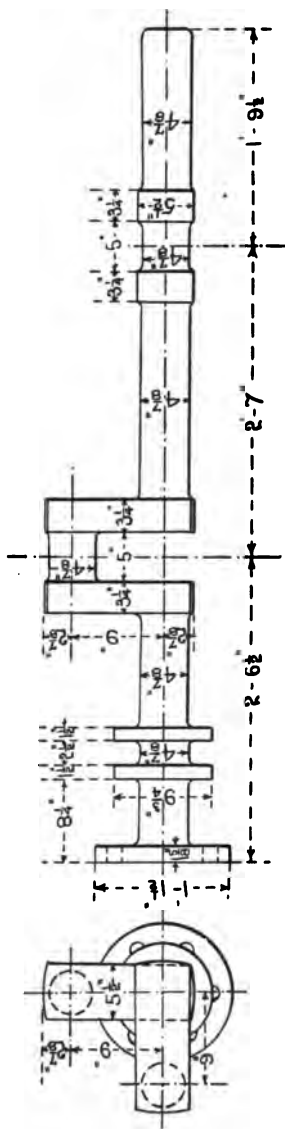
	Cub. in.
Two crank webs, each $16 \times 12\frac{1}{2} \times 6\frac{1}{2}$ thick	= 2600
Thrust collar, $10\frac{1}{2}$ in. diam. = $86.6 \times 6\frac{1}{2}$	= 562
Coupling, $14\frac{1}{2}$ " " = $165.1 \times 2\frac{1}{2}$	= 371
Shaft, $5\frac{1}{2}$ " " = $23.75 \times 50\frac{1}{2}$	= 1200
Total volume	= 4733

$$\begin{aligned}\text{Estimated weight of forging} &= 4733 \times 0.29 = 1373 \text{ lbs.} \\ &= 12 \text{ cwt. 1 qr. 1 lb.}\end{aligned}$$

Actual weight (bought out) = 14 cwt. 0 qr. 16 lbs.

Here the actual weight was some 15 per cent. heavier than the calculated, although the draughtsman had allowed (as he thought) sufficient excess over the finished dimensions. The calculation of the weight for the finished crank is given as follows:—

	Cub. in.
Four crank webs, each $14\frac{1}{2}$ in. (mean length) $\times 5\frac{1}{2}$ in. wide $\times 3\frac{1}{4}$ thick	= 1040
Two crank pins, each $4\frac{7}{8}$ in. diam. = 18.7 (sq. in.) $\times 10$	= 187
Two thrust collars, each $9\frac{3}{4}$ in. diam. = 74.7 (sq. in.) $\times 3$	= 224
Coupling, $13\frac{1}{2}$ in. diam. = 143.1 (sq. in.) $\times 1\frac{5}{8}$	= 233
Shaft, $4\frac{7}{8}$ in. diam. = 18.66 (sq. in.) $\times (20\frac{1}{8} + 19\frac{1}{2} + 15\frac{1}{4})$	= 1033
Gross volume	= 2717
Less 6 holes for coupling bolts $1\frac{3}{4}$ in. diam. = 2.4 (sq. in.) $\times 1\frac{5}{8} \times 6$	= 24
	2693
Plus 20 per cent. on nett volume to allow for radii on shaft	54
Total nett volume	2747



Estimated weight of finished crank = $2747 \times 0.29 = 797$ lbs.
 = 7 cwt. 0 qr. 13 lbs.

Actual weight was 7 cwt.

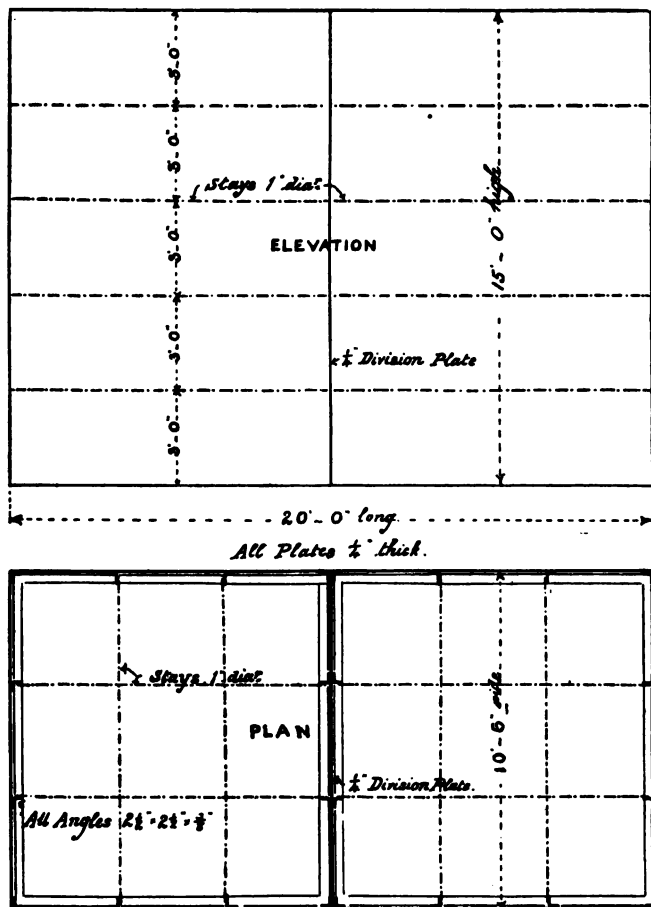


FIG. 172.

In this instance the weight of the rough forging was *double* the weight of the finished article.

Plate and Sheet-Iron Work.—In calculating the weight of plate work, it will generally be found quicker to take the

superficial area of all plates of equal thickness, and multiply by the weight per square foot, according to the method explained on p. 366. The weight of angle-irons, tees, etc., can also be obtained quicker by taking the total feet run of a given section and multiplying by the weight per foot run. Similarly for stays in tanks, etc.; while the weight of the rivets can be approximated at about 5 per cent. of the total weight of the job.

As an example of plate work, make the calculations for the weight of the **wrought-iron plate tank**, 20 ft. $\times$ 10 ft. 6 in. $\times$ 15 ft. high, as represented in fig. 172. Keep the weights of plates, angle-irons, stays, etc., separate, and tabulate as follows:—

	Rate per ft.	Item Weights.				Total Weights.			
		T.	C.	Q.	lbs.	T.	C.	Q.	lbs.
Plates (all $\frac{1}{2}$ in. thick)—									
Base, $20 \times 10 \cdot 5$									
= 210 sq. ft.									
Sides, $20 \times 15 \times 2$									
= 600 „									
Ends, $10\frac{1}{2} \times 15 \times 2$									
= 315 „									
Division, $10\frac{1}{2} \times 15$									
= 157.5 „									
Total area = 1282.5 „	lbs. 10	5	14	2	0				
Lapping (allow 5 per cent., say)			5	3	0				
Total plates						6	0	1	0
Angle-Irons (all $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{3}{8}$ in.)									
Each half tank—									
Bottom frame, 41 ft. run									
12 verticals, 180									
Total = $221 \times 2 = 442$ sq. ft.	lbs. 5.79	1	2	3	12	1	2	3	12
Stays (all 1 in. diam.)—									
Each half tank—									
$10 \times 10 = 100$									
$10 \times 10\frac{1}{2} = 105$									
Total = $205 \times 2 = 410$ sq. ft.	lbs. 2.68		9	3	7		9	3	7
Rivets (assumed at 5 per cent., say)									
						7	2	14	
Estimated gross weight						8	0	2	5

The actual weights of the various parts in the above tank were:—

	T.	C.	Q.	lbs.
Plates	6	0	0	0
Angle-irons . . .	1	2	0	0
Stays	0	9	0	0
Rivets	0	8	0	0
Total	7	19	0	0

The calculated weights were thus $1\frac{1}{2}$ cwt. too heavy, or about 1 per cent. of the total.

Weight of Castings.—In calculating the rough weight of castings, allowance has to be made for the extra metal cast for machining purposes. Generally this is equal to $\frac{1}{8}$ in. extra thickness on each surface machined; but although this is the general rule, it must not be used indiscriminately. In certain cases the amount of machining allowed for is greatly in excess of $\frac{1}{8}$ in., in order that the casting may be set true to another part, and cleaned up all over, or to allow of taking out the holes and irregularities in important work. For example, the bore of a steam cylinder would allow for more than $\frac{1}{8}$ in. all round to come out, because it is of the utmost importance that the inside surface shall be perfectly clean throughout and free from all blowholes, etc. In large cylinders, therefore, an allowance of $\frac{1}{4}$ in. or even $\frac{5}{16}$ in. may be made, all round, for machining in the bore. In other cases the allowance may be less than $\frac{1}{8}$ in., especially where it is not important that the face under consideration shall be cleaned up all over. For example, in high-class engine practice, where the amount of clearance allowed is very slight, the inside faces of the cylinder covers and the sides of the piston may be “rough turned” to take off any lumps on the casting. In such cases, an allowance for tooling of $\frac{1}{32}$ in. in small cylinders, or $\frac{1}{16}$ in. in large ones, would be ample, as it is not important if the faces are not quite cleaned up.

For example, in calculating the rough casting weight of the 10-ft. flywheel for which finished sizes are given in fig. 173, tabulate the items as follows:—

Rim—120 $\frac{1}{2}$ in. diam. = 11404 sq. in.

Less 117 “ “ = 10751 “

Metal = 653 “ × 9 in. thick . = 5877

Plus 117 in. diam. = 10751 “

Less 99 “ “ = 7698 “

Metal = 3053 “ × 8 in. thick . = 24424

Cub. in.

Plus 99 in. diam. = 7698 sq. in.	Cub. in.
Less 96 " " = 7238 "	

Metal = 460	" × 9 in. thick .	= 4140
-------------	-------------------	--------

Boss—20 in. diam. = 314	"
8½ " " = 56	"

Metal = 258	" × 10½ in. thick	= 2709 cub. in.
-------------	-------------------	-----------------

Less parting plates (20 - 8½) × ⅜ × 10½ = 45	"
--	---

Nett metal = 2663	"	= 2664
-------------------	---	--------

Plus 23 in. = 415 sq. in.	
---------------------------	--

Less 20 " = 314	"
-----------------	---

Metal = 101	" × 3½ thick = 353	
-------------	--------------------	--

Less parting plates (23 - 20) × ⅜ × 3½ = 4	
--	--

Nett metal = 349	"	= 349
------------------	---	-------

6 Arms—Section at rim = 3 × 1½ × 2 = 7·5	sq. in.
+ 3½ × 1¼ = 4·37	"
11·87	"

Section at boss = 3½ × 1½ × 2 = 8·75	"
+ 4½ × 1¼ = 5·5	"
14·25	"

Mean section of arms =	$\frac{11·87 + 14·25}{2}$
	= 13·06 sq. in.

Volume =	$\left(\frac{96 - 23}{2} \right)$	× 13·06 × 6	= 2860
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Nett volume	= 40314
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Estimated (rough cast) weight = 40,314 × 0·27.

= 10,884 lbs.

= 4 tons 17 cwt. 0 qr. 20 lbs.

Actual weight = 4 tons 17 cwt. 3 qrs. 0 lb.

= (about) ½ per cent. heavier.

It will be seen that in the above weight $\frac{1}{4}$ in. was allowed on the rim for turning, and $\frac{1}{4}$ in. on each side of the rim for facing, because of the size and the importance of these faces being clean. Similarly, $\frac{3}{8}$ in. all round was allowed on the bore, in order that the casting should clean up internally. The slight excess of the actual weight over the calculated weight would probably be due to the weight of the "fillets" not being taken into account.

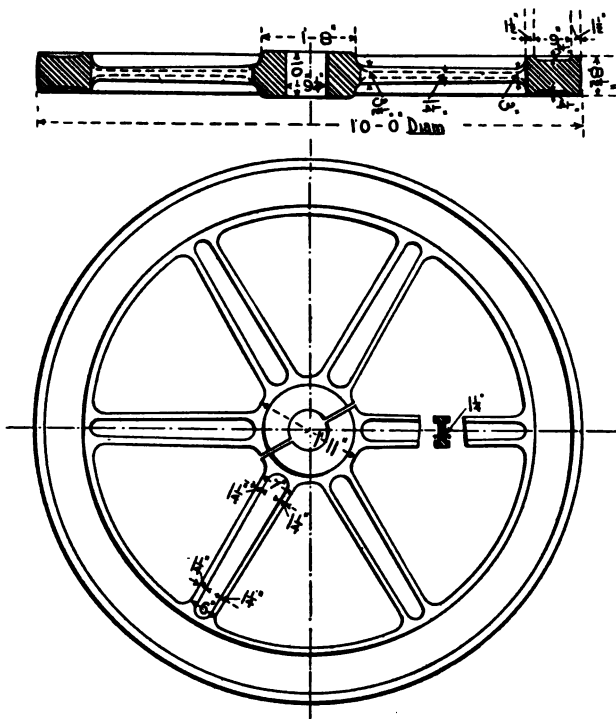


FIG. 173.

As a rather more complicated example, take the calculations for the determination of a cast-iron drum, shown in fig. 174, with flanged ends and corrugated body, the flanges being faced at the ends and over the top. In this case the volume of metal in the body of the drum has to be ascertained by setting out one of the corrugations to a large scale, and finding the actual length by stepping off with a pair of dividers or by using the opisometer. The length of one corrugation was found in this way to be

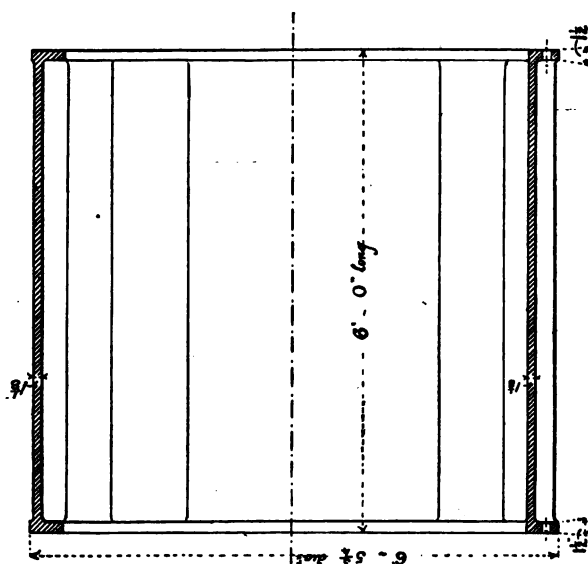
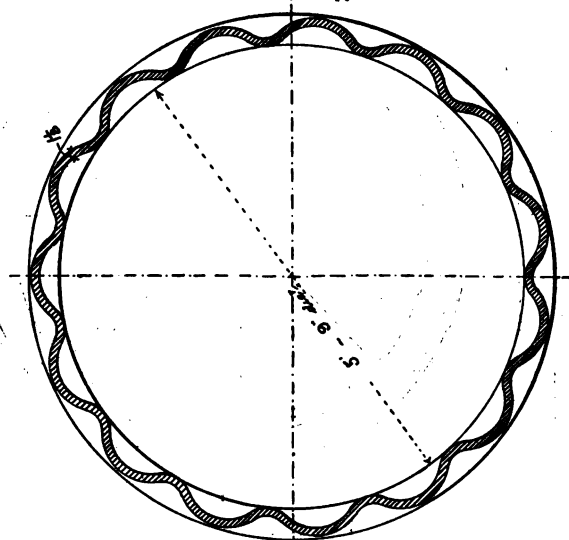


FIG. 174.



17.3 inches. Consequently, the circumferential length of the metal in the body is $17.3 \times 15 = 259.5$ inches.

**WEIGHT (AS CAST) OF CORRUGATED DRUM (finished sizes in
fig. 174).**

Cub. in.

Flanges—6 ft. 6½ in. diam. outside = 4839·8 sq. in. area
5 ft. 9 in. „ inside = 3739·2 „ „

$$\text{Volume of metal} = 1100 \cdot 6 \times 4 \quad . \quad = 4402$$

(Note.—Flanges taken at 2 in. thick each to allow
for machining.)

$$\text{Body—Volume of metal} = 259 \cdot 5 \times 1\frac{1}{8} \times 5 \text{ ft. 9 in.} \quad . \quad = 20140$$

Allowing fillets at ends to balance weight of bolt-
holes, nett volume of metal = 24542

$$\begin{aligned} \text{Calculated weight} &= 24,542 \times 0 \cdot 27 \times 6626 \text{ lbs.} \\ &= 2 \text{ tons } 19 \text{ cwt. } 0 \text{ qr. } 18 \text{ lbs.} \end{aligned}$$

$$\text{Actual weight} = 3 \text{ tons } 2 \text{ cwt. } 0 \text{ qr. } 0 \text{ lb.}$$

$$\text{Difference} = 4 \cdot 8 \text{ per cent. heavier than calculated.}$$

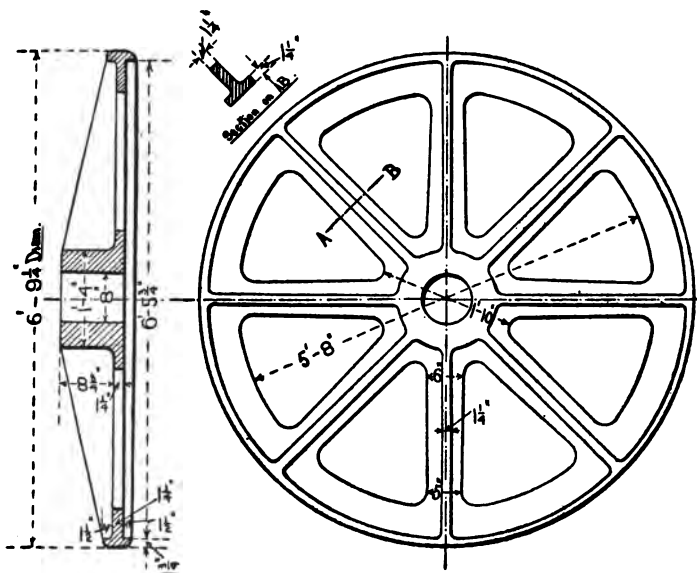


FIG. 175.

Here again the actual weight is considerably heavier than that calculated, the mould being struck up in loam. The weight of

the cast-iron ends for the above drum, the finished sizes for which are given in fig. 175, were calculated as follows :—

WEIGHT (AS CAST) OF ENDS FOR DRUM.			Cub. in.
Rim,	81½ in. diam. outside = 5184·8 sq. in. area		
	77¼ " " inside = 4747·7 " "		
	Volume of metal = 437·1 × 1½		= 656
Register,	81½ in. diam. outside = 5184·8 sq. in. area		
	77 " " inside = 4656·6 " "		
	Volume of metal = 528·2 × 1½		= 792
Web plate,	81½ in. diam. outside = 5184·8 sq. in. area		
	68 " " inside = 3631·6 " "		
	Volume of metal = 1553·2 × 1½		= 2329
Boss,	16 in. diam. outside = 201·1 sq. in. area		
	7 " " inside = 38·5 " "		
	Volume of metal = 162·6 × 9		= 1463
Boss plate,	22 in. diam. outside = 380·1 sq. in. area		
	7 " " inside = 38·5 " "		
	Volume of metal = 341·6 × 1¼		= 427
8 Arms, 23 in. long × 5½ in. mean width × 1¼ in. thick			= 1265
8 Ribs, 30½ in. long × 5½ in. mean depth × 1¼ in. thick			= 1582
	Nett volume of metal		= 8514
	Plus 5 per cent. for fillets		= 425
	Gross volume of metal		= 8939
<i>Estimated (rough cast) weight, each end = 2414 lbs.</i>			
	= 1 ton 1 cwt. 2 qrs. 6 lbs.		
<i>Actual (rough cast) weight</i>			
	= 1 ton 2 cwt.		
Difference = 2·1 per cent. heavier than calculated.			

Weight Conversions.

For convenience in converting weights from lbs. to tons, cwts., qrs., and lbs., the short table, No. XXXI., of weight conversions, given on p. 394, will be found very useful. This table gives the

TABLE XXXI.—WEIGHT CONVERSIONS.—LBS. TO TONS, CWTs. QRS. LBS.

Lbs.	0	100	200	300	400	500	600	700	800	900	Lbs.	T. C. Q. lbs.
0	C. qrs. lbs. 0 0 0	C. qrs. lbs. 0 3 16	C. qrs. lbs. 1 3 4	C. qrs. lbs. 2 2 20	C. qrs. lbs. 3 2 8	C. qrs. lbs. 4 1 24	C. qrs. lbs. 5 1 12	C. qrs. lbs. 6 1 0	C. qrs. lbs. 7 0 16	C. qrs. lbs. 8 0 4	1000	0 8 3 20
10	0 0 10	0 3 26	1 3 14	2 3 2	3 2 18	4 2 6	5 1 22	6 1 10	7 0 26	8 0 14	2000	0 17 3 12
20	0 0 20	1 0 8	1 3 24	2 3 12	3 3 0	4 2 16	5 2 4	6 1 20	7 1 8	8 0 24	3000	1 6 3 4
30	0 1 2	1 0 18	2 0 6	3 0 22	3 3 10	4 2 26	5 2 14	6 2 2	7 1 18	8 1 6	4000	1 15 2 24
40	0 1 12	1 1 0	2 0 16	3 0 4	3 3 20	4 3 8	5 2 24	6 2 12	7 2 0	8 1 16	5000	2 4 2 16
50	0 1 22	1 1 10	2 0 26	3 0 14	4 0 2	4 3 18	5 3 6	6 2 22	7 2 10	8 1 26	6000	2 13 2 8
60	0 2 4	1 1 20	2 1 8	3 1 24	4 0 12	5 0 0	5 3 16	6 3 4	7 2 20	8 2 8	7000	3 2 2 0
70	0 2 14	1 2 2	2 1 18	3 1 6	4 0 22	5 0 10	5 3 26	6 3 14	7 3 2	8 2 18	8000	3 11 1 20
80	0 2 24	1 2 12	2 2 0	3 1 16	4 1 4	5 0 20	6 0 8	6 3 24	7 3 12	8 3 0	9000	4 0 1 12
90	0 3 6	1 2 22	2 2 10	3 1 26	4 1 14	5 1 2	6 0 18	7 0 6	7 3 22	8 3 10	10000	4 9 1 4

equivalent in tons, cwts., etc., for weights from 10 lbs. to 1000, for step-by-step increases of 10 lbs., together with the weight of each 1000 lbs. up to 10,000. The method of using the table will be self-evident from an inspection. For instance, to convert 937 lbs. to cwts., etc.: under the vertical column headed 900, and level with line commencing 30 lbs., we read 8 cwt. 1 qr. 6 lbs., to which 7 lbs. has to be added, making 8 cwt. 1 qr. 13 lbs. For quantities greater than 1000 lbs., the weight given in the last column at the extreme right of the table has to be added to that given in the body of the table; thus 3575 lbs. = 1 ton 6 cwt. 3 qrs. 4 lbs. plus 5 cwt. 0 qr. 15 lbs. = 1 ton 11 cwt. 3 qrs. 19 lbs.

CHAPTER XIX.

PREPARATION OF ESTIMATES, DIVISION OF COSTS.

THE preparation of engineering estimates varies very much, according to the system of cost-keeping adopted, also with the nature of the various work to be carried out. Most engineering firms of importance are engaged in the manufacture of certain special classes of work, made in various sizes and forms, for which exact cost accounts are kept. The furnishing of an estimate for another size of some speciality is therefore merely a matter of judging what the probable nett cost of the article or job would be, based on the nett cost of similar articles previously made, but of a different size, and adding a percentage to cover establishment charges and profit. Sometimes the work to be estimated for will consist of one simple thing, such as a number of castings or mild steel forgings, when the estimate can generally be calculated direct from the weight of the materials. In some works this rough-and-ready method of estimating is carried on to a large extent, and includes whole machines or complex specialities of the firm, where the average cost per ton of various sizes has been previously determined. In many cases, however, the work to be estimated for is of an entirely new description, in which case the first step is to get out a complete list or schedule of all the various parts composing the machine or job, and calculate the weights and quantities of each part. In large works this would be done in the estimating department, by men specially trained to this class of work; but in smaller works the preparation of the list would probably be given to the draughtsman who has been engaged in getting out the drawings for the job. The manager or chief estimating clerk would then add the rates at which the material is to be charged, and the foremen of the various departments might be asked to supply estimates of the probable wages cost in their respective departments. The requisite shop charges would then be added in the form of percentages on the various wages costs of each shop separately, and the total sum for material and labour would receive an additional percentage to cover establish-

ment charges. The cost of painting, packing, and delivery would be charged according to the various railway or shipping rates, and erection *in situ* (if required) allowed for. Sometimes a contingency item of some $7\frac{1}{2}$ or 10 per cent. is added to allow for any sundries that may have been overlooked, when the whole receives a percentage as nett profit, giving the total sum to be charged or invoiced.

Division of Costs.—The total cost of any job can be roughly divided under four distinct headings:—

(a) Cost of material	(30 per cent.)
(b) „ labour	(40 „ „)
(c) Shop charges	(20 „ „)
(d) Establishment charges	(10 „ „)

The percentages in brackets represent very approximately what the relative costs of each of the four divisions were in a works employing about 600 hands. The actual percentages will differ very considerably according to the size of the works, its locality, and the nature of the manufactures. In some manufacturing works, the cost of (a) exceeds that of (b), and in large works the relative amount of (c) and (d) is very much lower than given above.

Each of these main headings is again divisible into sub-headings. For example, the cost of material must include such widely differing materials as wood, cast iron, wrought iron, steel, bronzes, white metals, etc. The cost of labour will include all wages paid on the job and to the job number, from that of the foreman to the sweeper-up. The shop charges must include foreman's wages, repairs, maintenance and depreciation of all tools in the department, loose and fixed, together with the cost of any excess material not used, less scrap value, and the cost of sundry incidentals for each and every department. The establishment charges will include such things as interest on capital charges, superintendence, advertising and outdoor travelling; power charges, gas, water, and insurance charges; maintenance of buildings, roads, and property. Generally speaking, the cost of item (c) can be reckoned as a percentage of (b) for each department, and (d) can be estimated as a percentage of (a + b + c). The profit required can then be determined and added to $a + b + c + d$, thus giving the contract price.

Relative Cost of Raw Material.—The cost of raw material varies very much with the fluctuations of the market from day to day. Average prices only are shown in Table XXXII., p. 398, which will serve to indicate the approximate relative cost of the various materials mostly used by engineers. The cost of cast iron

quoted includes cost of breaking, charging, and melting, and represents the **value of the metal in the ladle**. The prices of steel bars quoted are average market prices, delivered in 4-ton lots. For smaller quantities or special sizes an increased price is charged. The cost of cast steel varies enormously, from 6d. to 2s. per lb., according to the quality, special high-speed steels demanding the higher price named. The costs of brasses and bronzes stated are average costs in the ingot, and do not include cost of melting or mixing.

TABLE XXXII.

Relative Costs of Various Materials.

Cast iron	from	5s. to	6s. per cwt.
Mild steel (black)		6s. „	9s. „
„ (bright bars)		12s. „	15s. „
Shear steel		30s. „	40s. „
Cast steel		50s. „	100s. „
Yellow brass		50s. „	60s. „
Gun-metal		70s. „	80s. „
Bronzes		90s. „	110s. „
Anti-friction and white metals .		95s. „	120s. „
Aluminium		130s. „	140s. „

Wages Cost.—The second main heading in the estimate is wages cost or returns for labour of all kinds. In fixing the amount of this item, the person making the estimate might sometimes with advantage consult freely with the various shop foremen and sub-heads of departments interested. The foremen should be encouraged to make short summaries or notes of the wages cost of various pieces of work passing through their hands, especially if the work is done on the piece-work, bonus, or premium systems; and by referring to the actual cost of similar work and allowing for differences of design, size, and manufacturing processes, he is usually better able to form an estimate of the wages required to execute a certain job in his department. The foreman should not, however, be expected to extract the cost himself, as he has other duties far more important than those of a merely clerical nature; but, if the works are sufficiently large, it will be found worth while giving the principal foreman a clerical assistant who will be able to keep all the costs that he requires, summarise and post them up in a thumb-indexed book ready for reference when required. The actual cost under this heading will vary with the scale of wages paid, and here the question of locality again comes to the front. The scale of wages for London, Manchester, and other large engineering centres is higher than that in many

country and provincial towns, but generally the following may be taken as average prices per hour for various classes of work :—

TABLE XXXIII.

<i>Foundry—</i>		
Moulders	8d. to	9d. per hour.
Trimmers (usually tonnage)	6d. „	7d. „
Foundry labourers	5d. „	6d. „
<i>Pattern shop—</i>		
Pattern-makers	8d. „	9d. „
<i>Smithy and boiler shop—</i>		
Smiths	8d. „	8½d. „
Platers	8d. „	9d. „
Riveters (usually piece-work)	7d. „	8d. „
Holders, holders-up, and strikers and mates	6d. „	7d. „
<i>Machine shop—</i>		
Turners	8d. „	9d. „
Planers, shapers, and slotters	7d. „	8d. „
Drillers and screwers	6d. „	7d. „
Shop labourers	5d. „	6d. „
<i>Fitting and erecting shops—</i>		
Fitters	8½d. „	9½d. „
Fitters' mates	7d. „	8d. „
Charge hands	10d. „	1s. „
Labourers	5d. „	6d. „

The rates for lads and apprentices vary from about 4s. to 18s. per week, rising every six or twelve months of their time. There are various systems in use for the remuneration of labour, which may be generally classified as time, piece, bonus, or premium systems. These are treated more fully on pp. 407 to 415; but whatever system be used, the labour cost should include all remuneration, whether in the form of ordinary wages, bonus, or premiums. There is no fixed rule as to the wages of foremen: in some shops the foremen's wages are classified as salaries, and come under the heading of (c) shop charges, whereas in other shops the foreman fills up an approximate time-sheet showing the distribution of his time on the various jobs in progress in his particular shop, when his wages would naturally come under the heading of (b), labour costs. The labour costs should include all overtime at the increased rates, and usual outdoor expenses of erectors.

The average wages cost per hour of various shops taken collectively is sometimes of use in preparing estimates. It naturally varies somewhat according to the percentage of lads or apprentices,

but the following few examples that came under the authors' notice may be of some assistance in this connection. The prices refer to a London works of average size, employing some 300 hands, engaged on engine work and general mill gearing.

TABLE XXXIV.—AVERAGE WAGES COST PER HOUR IN VARIOUS SHOPS.

	Class of Work.		
	Light.	Medium.	Heavy.
Pattern shop pence	4·2	5·1	6·6
Iron foundry (moulders and core-makers only) „	6·8	7·2	8·3
Smithy „	...	5·8	...
Machine shop „	4·7	5·6	6·9
Fitting and erecting shop „	5·0	6·2	7·0

Shop Charges or Departmental Costs.—In apportioning the various amounts known as shop charges, it is necessary to distinguish between those items which are incidental to the shop and those which are general establishment charges. Taking, first, the general office, all wages paid here to clerks, etc., will appear in the establishment charges, as they are necessary adjuncts of the factory, and the cost must be spread over all running contracts. In the drawing office, however, the case is different. Here the draughtsmen's time is usually charged up to the job he is on, and as wages is included in item (b). There are, however, other expenses, such as chief draughtsman's time, cost of material, drawing paper, tracing paper and linen, printing paper, cost of cleaning office, and sometimes a comparatively large proportion of draughtsman's time spent on the preparation of drawings and designs which are not carried out, and therefore cannot be carried forward on to any particular job number. Such items will generally be classified under the heading of shop charges, and, totalled up at the end of the financial year, will be calculated as a percentage of the total wages paid in the drawing office which have been booked to job numbers. Similarly, in the pattern shop, there are the foremen's wages, repairs, maintenance and depreciation of the tools, such as band-saw, circular saw, lathe or drill, mortising machine. In some shops the cost of the wood is not booked to the job, so should be added to the shop charges, as also cost of screws, varnish and paint, and general incidentals of the shop. In the iron foundry, shop charges consist of depreciation and maintenance of machines, shop travellers, cupolas, cost of all shop

incidentals, such as foremen's time, foundry coke, sand and loam, blacking, chalk, hair, shovels, chaplets and sieves, gas coke for drying stoves, fettlers, brushes, etc. Usually a percentage of the cost of metal (about 8 per cent. to 12 per cent.) is allowed for wastage in melting, although this is sometimes included in the estimated cost of metal, when the latter is taken as cost of molten metal in the ladle, and includes cost of cupola sundries and wages of furnacemen. Very frequently the cost of the metal is taken to include all the incidentals of the foundry and wages of fettlers and labourers, but this is a misleading total, as it mixes up labour costs with shop charges. Similarly, in the brass foundry the shop charges will include depreciation and maintenance of machines and lifting tackle, coke for furnaces and drying sand, carbon pots and tongs, shovels, sieves, and wastage of metal. In the smithy or boiler shop, the departmental charges will consist of foremen's time, depreciation, and maintenance of all machinery, loose and fixed plant and tools, fuel for furnaces, and power, shortage of metal, and general shop charges. In the machine department, the depreciation and maintenance of machines forms a very considerable item, and in addition the shop charges should include foremen's time (when this is not charged to the various jobs), time of labourers and general tool-room charges (wages and material), belts and belting, oil, files, and waste, emery cloth and powder, etc. In the erecting shop, the charges will include depreciation and maintenance of tools and lifting tackle, overhead cranes, cost of steaming pit or testing department (if any), labour and crane-men's wages, upkeep of loose tools (often a very considerable item in this department), and wages of foremen if not included in wages cost.

The actual ratio which these various costs will bear to the nett wages booked to each particular job will naturally vary very much according to the size of the works and the system of cost-keeping in vogue. In small works the percentages will be very high as compared with large establishments, but as a general run the following list will approximately represent the percentage to be added to the nett wages cost to cover the shop charges:—

TABLE XXXV.

Drawing office	40 to	60 per cent.
Pattern shop (wood included in job number)	20 „	40 „
Iron foundry (exclusive of cost of cupola, labourers, and fettling)	10 „	15 „
Smithy and boiler shop	15 „	20 „
Machine department	80 „	120 „
Fitting and erecting department (exclusive of steaming pit or testing shed)	20 „	30 „

In some works, the cost of shop charges for various departments is not divided up, but an inclusive percentage is added to the wages cost to cover these charges for all the shops. This figure is generally put at 30 to 40 per cent. of the nett wages booked to the particular job.

Establishment Charges.—As already explained, these consist of such charges and costs as are common to the whole establishment, such as interest on capital charges, and cost of buildings, salaries of all officials, such as directors, secretary, manager, clerks, timekeeper, storekeeper, caretaker, cost of all advertising and outdoor travellers, rent, rates, and insurance of buildings and plant, maintenance and depreciation of buildings and property, cost of power plant (including fuel costs, wages, depreciation and upkeep, oil, waste, etc., involved in power production), water, expenses of estimating and cost departments. The total of all such charges as these is usually estimated for a year, and distributed as a percentage over the whole of the work expected during the year. In the event of such percentage being found insufficient to cover the charges, the balance should be debited to the profit and loss account, as otherwise the nett profit shown will be misleading. The actual amount of such charges will vary according to the size of the works and the management adopted, varying from 10 per cent. to 20 per cent. of the total expenses of the works.

Total Costs.—Having examined the various charges which have to be allowed for in framing an estimate, we now sum them up in tabular form, thus:—

Estimate or Contract Price.			
[100]			
Total Nett Cost.			Profit.
[85]			[15]
(a)	(b)	(c)	d
Material.	Wages cost.	Shop costs.	Establishment charges.
[25]	[35]	[15]	[10]

The ratio which these various items bear to each other is very different according to the nature of the work and the size of the establishment. The figures in brackets actually represented, for a small works of moderate output, the ratio of each item to the estimate or contract price. Very frequently the items (a) and (b) are referred to as the "prime cost" or "material and labour" cost of the article; and it should be borne in mind, when such an

expression is used, that the selling price is generally some 66 per cent. more than this, or the "prime cost" is about 60 per cent. of the selling price.

Approximate Estimating by Weight.—We have already referred to the system of furnishing approximate estimates of simple jobs according to the weight of the article or machine. Although this method of estimating is extremely crude, and should not be pushed too far, it often forms a convenient check upon the more elaborate calculations on which the estimate is based. For example, a firm who make a speciality of "castings for the trade" will know at what price per cwt. or per ton they can afford to sell such castings by an inspection of the drawings or patterns, without requiring from the foreman moulder an estimate of his wages for the job. If the work be "light," the price per cwt. is correspondingly higher than for heavier work; and should any intricate cores be required which are liable to cause defective castings or wasters, a higher rate will be charged than for castings of an average character. Also, if the metal has to be machined, or tested to fulfil some specified strength and stiffness, they will allow for such contingencies by including the value of a higher-priced mixture of metals. Similarly, firms who make specialities of, say, such articles as steel castings or steel forgings, gun-metal or brass castings, or complete finished articles, such as flywheels, frequently quote on a weight basis alone, formed from previous costs and years of experience.

As an example of approximate selling price for such runs of work, Table XXXVI. gives some idea of the prices of various articles which can usually be quoted on a weight basis by firms who make a speciality of such jobs. Generally the higher prices given refer to the smaller sizes.

Specimen Quantities Lists and Forms for Estimates.—As showing what an engineer's quantities list should contain, and approximately the method of building up an estimate, we give as an example the copy of an actual estimate for a small high-speed vertical engine of about 20 B.H.P. size, with a cylinder 9-in. bore and 6-in. stroke, the engine being of the semi-enclosed type; also a draft-sheet for an estimate of a boiler.

TABLE XXXVI.

Average Selling Prices of Standard Work.

Iron Castings—

Average weight, plain, not cored	8s. to 10s. per cwt.
Heavy, average cores	10s. „ 12s. „
Light or intricate, cored	12s. „ 18s. „
Plain cylinders or pipes (small)	16s. „ 18s. „
„ „ „ (medium)	14s. „ 16s. „
„ „ „ (large)	12s. „ 14s. „
Steam or hydraulic cylinders (small)	30s. „ 40s. „
„ „ „ (medium)	20s. „ 30s. „
„ „ „ (large)	15s. „ 20s. „
Extra, if jacket is cast in, 25 per cent. to 35 per cent.	
Base-plates or bed-plates (medium and plain)	10s. „ 12s. „
„ „ „ (large and cored)	12s. „ 15s. „
Brackets, standard patterns (light)	15s. „ 18s. „
„ (in large numbers, off machines)	10s. „ 12s. „

Constructional Work—

Girders and roof-work, heavy	12s. „ 14s. „
„ „ „ light	14s. „ 18s. „

Steel Forgings—

Plain and heavy	16s. „ 18s. „
Medium weight	18s. „ 20s. „
Light and complicated	20s. „ 25s. „

Brasses, etc.—

	Rough-cast.	Machined.
Gun-metal brasses or bearings, per lb. (small)	1s. 6d.	2s. 6d.
„ „ „ „ (medium)	1s. 3d.	2s. 3d.
„ „ „ „ (large)	1s. 0d.	2s. 0d.
„ „ „ „ with white		
metal liners (medium)	1s. 9d.	2s. 9d.

Flywheels—

Turned, bored, and slotted (heavy)	£16 to £18 per ton
„ „ „ (medium)	18 „ 20 „
„ „ „ (light)	20 „ 25 „

Boilers—

Cylindrical (exclusive of fittings)	£28 „ £35 „
Water-tube (including fittings)	30 „ 40 „

Machines Complete—

Plain standard patterns, such as planing, shaping, slotting, and shearing	£25 „ £30 „
Lathes, ordinary, sliding, surfacing, and screw-cutting	30 „ 35 „
Milling machines and automatic lathes	50 „ 80 „

Steam Engines—

Complete, including condenser and flywheel, { large (500-1000 H.P.)	£35 „ £40 „
average design and finish { medium (100-500 H.P.)	40 „ 50 „
„ { small (10-100 H.P.)	50 „ 80 „

ESTIMATE FOR ONE 9-IN. x 6-IN. SEMI-ENCLOSED VERTICAL ENGINE.

	Material.					Wages.			Shop Charges.			
	Weight.	Rate.	£	s.	d.	£	s.	d.	Rate.	£	s.	d.
Drawings (estimated)						7	0	0	30 %	2	2	0
Patterns "						20	0	0	25 %	5	0	0
Total drawings and patterns						27	0	0		7	2	0
Do. divided over 6 engines						4	10	0		1	8	8
<i>Moulding—</i>	T. C. Q. lb.											
Cylinder, etc.	0 1 2 0					0	15	0				
Trunk	0 4 2 14					0	9	0				
Base-plate	0 3 1 7					0	8	0				
Details	0 3 1 0					0	12	6				
Flywheel	0 7 1 7					0	7	6				
Total castings	1 0 0 0	4s. 8d.	4	5	0				50 %	1	6	0
<i>Forgings—</i>												
Crank (bought out)			2	15	0	0	10	0				
Connecting rod	0 1 0 0					0	3	6				
Eccentric rod	0 0 2 14					0	7	6				
Details	0 0 1 14											
Total forgings	0 2 0 0	9s.	0	18	0				50 %	0	10	6
<i>Machining—</i>												
Cylinder						1	5	0				
Piston and rings						0	15	0				
Piston rod						0	5	0				
Crosshead						0	7	0				
Trunk						1	6	0				
Base-plate						0	16	0				
Eccentric and straps						0	10	0				
Connecting rod						1	4	0				
Eccentric rod						0	10	6				
Valve spindle						0	4	6				
Valve and trunk covers						0	5	6				
Flywheel						0	15	0				
Crank						1	0	0				
Small details						0	10	0	100 %	9	13	6
Lining out						0	12	0	25 %	0	3	0
Fitting up complete						2	5	0	30 %	0	19	6
Steaming and testing						2	0	0	50 %	1	0	0
<i>Stores, etc.—</i>												
Governor (bought out)			4	0	0							
1 set of lubricators			2	10	0							
Sundry bolts and nuts	0 1 0 0	18s. 6d.	0	18	6							
<i>Extras—</i>												
1 set foundation bolts	0 0 3 0	6s.	0	4	6				100 %	0	4	0
Screwing foundation bolts						0	4	0	50 %	0	2	0
Plates for "	0 3 0 0	4s.	0	12	0	0	4	0				
Painting, packing, and delivery F.O.B.						1	10	0	33 1/3 %	0	10	0
Totals	1 6 8 0	..	16	8	0	25	11	6	..	15	13	2
Add—Shop charges						15	12	2				
Cost of material						16	8	0				
Total nett cost						57	6	8				
Add establishment charges, 15 per cent.						8	12	0				
Add for profit (say, 12 1/2 per cent.)						65	18	8				
Total estimate or selling price = (say)						74	8	8				
						75	0	0				

Price per ton = £56 2 0
 " " B.H.P. = £3 15 0

ESTIMATE FOR LANCASHIRE BOILER.

	Quantity or Weight.	Price or Rate.	Material.			Wages or Salaries.		
<i>Drawings</i>								
<i>Boiler shell, end plates, and joints—</i>								
Plating								
Straps								
Angles								
Rivets								
Bending								
Drilling								
Flanging								
Planing								
Erecting								
Riveting								
Caulking or fullering								
Additional								
<i>Furnace tubes and Galloway tube—</i>								
Plating								
Angles								
Rivets								
Bending, where not bought								
Drilling								
Planing								
Riveting								
Caulking or fullering								
<i>Stays—</i>								
Gusset								
Longitudinal								
Manhole frame and cover								
Fire grate { Bars								
Rings, doors								
Fusible plugs								
Blow-off pipe and stand								
<i>Boiler Mountings, etc.—</i>								
Stop valve								
Safety valves								
Check feed valve and pipe								
Water gauges								
Dampers								
Foot-plates								
<i>Miscellaneous stores not already accounted for.</i>								
<i>Testing</i>								
Total cost of material, less scrapping						Sum up wages here (A).		
			(B)			State cost of material (B) here.		
Estimated nett cost						(A+B)		
Charge for machinery, tools, other shop charges						C		
Establishment charges						D		
Estimated gross cost						A+B+C+D		
Profit						E		
			Price of tender			£ A+B+C+D+E		

CHAPTER XX.

REMUNERATION OF LABOUR.

THERE are four distinct methods of remunerating labour in use in the engineering trade, viz.:—(1) **payment by time only**, or so much money for so many hours' work; (2) **payment by piece-work**, or so much money for so much work done; (3) **payment by bonus**, or the giving of a certain proportion or bonus to the workman who does any job cheaper than the standard price; and (4) **payment by premium**, or payment of so many hours' extra money, for reducing the time taken to do any job. As regards the first two systems, time and piece-work, practically no calculations are required, as there are innumerable ready reckoners and wages tables published giving the wages for so many hours at so much per hour, and also giving the wages for so many articles at so much per piece. But as regards the third and fourth systems of payment a little calculation is necessary, and it is important that fairly full explanations of these methods be given.

Bonus System.—The bonus system of payment, as recently introduced into this country, consists in paying a workman so much extra beyond his day money for doing a certain job so much cheaper than the rate mutually fixed upon. It is, in effect, a compromise between day-work and piece-work. The way it is calculated is as follows. Before starting a job, the master (or his representative, the foreman) and the workman agree upon a certain basis price, called the bonus price, and the workman is then encouraged to do his work as quickly as he can by the offer as a reward of a certain portion of the difference between the bonus price and the day-work value of his time. The actual proportion of the difference which is offered to the workman differs in various systems, but is usually one-half. The result is that the workman uses all his endeavours to do the work as quickly as possible, so that the day-work value of his time may be as low as he can get it; and it is to the master's interest for the man to take a high bonus, as every shilling bonus earned by the workman is a shilling

saved to the master. The system therefore cuts both ways, and is mutually beneficial to both master and workman.

As showing the method of calculating the wages under this system, take the case of a machinist who planes a number of bed-plates (say) at a bonus price of 15s. each. If his rate is 7d. per hour, and the first one takes him 20 hours, he is paid his day-work time of 20 hours at 7d., or 11s. 8d., and in addition 1s. 8d. bonus, or one-half the difference between his day-work time and the bonus price agreed upon. As the bonus price is generally fixed at a fair day-work value for the job, the master saves 1s. 8d. on the job by the workman effecting it for so much less. If he is able to still further reduce his time, say to 16 hours, he is paid his day-time 16 hours at 7d., or 9s. 4d., plus one-half of the saving effected, viz. 2s. 10d. bonus. Thus the workman receives an increased pay per hour for his labour, and the master gets the job done at a reduced rate per piece, and both interests are benefited. In the workman's case his pay is 11s. 8d. plus 1s. 8d. for 20 hours' work, or at the rate of 8d. per hour when the job takes 20 hours, and still further increased to 9½d. per hour if he can reduce the time on the job to 16 hours. He thus has a very material incentive to working at his greatest output. On the other hand, the master gets the job done for 13s. 4d. in the first case, and 12s. 2d. in the second case, which would have cost him 15s. in wages if paying by day-work only. The accompanying Table, XXXVII., shows the effect of still further reducing the time taken, if that be possible, both in increased wages to the workman and decreased cost per piece to the master.

TABLE XXXVII.—EFFECT OF BONUS SYSTEM ON WAGES AND COST, ESTIMATED FOR PLANING BED-PLATES AT 15s. EACH BONUS PRICE, THE WORKMAN'S RATE BEING 7D. PER HOUR.

Time taken in hours.	Day-work wages at 7d. per hour.	Bonus earned = $\frac{1}{2}$ difference.	Total wages taken by workman.	Wages rate per hour.	Cost per piece.
	s. d.	s. d.	s. d.	d.	s. d.
25½	15 0	0 0	15 0	7·0	15 0
24	14 0	0 6	14 6	7·26	14 6
22	12 10	1 1	13 11	7·59	13 11
20	11 8	1 8	13 4	8·00	13 4
18	10 6	2 5	12 9	8·5	12 9
16	9 4	2 10	12 2	9·12	12 2
14	8 2	3 5	11 7	9·38	11 7
12	7 0	4 0	11 0	11·0	11 0
10	5 10	4 7	10 5	12·5	10 5

Premium Systems.—There are various premium systems in use for the remuneration of labour, but they are all similar to the bonus system just explained. The three systems best known are the Rowan system, the Halsey system, and the Weir system. In all three systems the workman is paid a premium or bonus varying in amount with the time saved as compared with the time allowed, over and above his usual wages for the time actually worked at hourly rates. These systems, as in the bonus system, have two chief advantages: (1) they *enable the workman to add to his ordinary wages* by increased industry and skill; (2) the *cost of production to the employer is reduced*.

Before starting the job, a certain *time limit* is allowed for completing the job, and in fixing this time an endeavour is made to approximate the time which would be taken by a mechanic of average ability when working at ordinary hourly rates. If the job takes the time allowed, or longer, the mechanic is still paid at the usual hourly rate. If the work is finished in less than the time allowed, a premium is given to the workman, varying in amount with the time saved, so that the total wage received for the time actually worked would then be—

Wages at hourly rates for }
time actually worked } + The premium earned.

We shall now consider briefly these three systems, and compare their remuneration to both workman and employer for various percentages of time saved as compared with the time allowed.

The Rowan System.—In this system the *premium in money paid to the mechanic bears the same proportion to his ordinary wages for the time actually worked on the job as the time saved is to the time allowed for completing it*. Suppose the time limit to be 20 hours, and the ordinary hourly rate of pay to be 8d. an hour; then, if the job is finished in 16 hours, the time saved is 4 hours; and therefore as 4 hours is $\frac{4}{20}$ or $\frac{1}{5}$, or 20 per cent. time saved, the premium will be 20 per cent. of the day-work wages. That is, the mechanic will receive payment of 16 hours at 8d. per hour, or 10s. 8d. plus 20 per cent. of 10s. 8d., equal to 2s. 1½d., making a total wage of 12s. 9½d. To put it symbolically—

let P = premium earned, in pence,
 T = time allowed, or time limit, in hours,
 t = time taken, in hours,
 r = day-work rate of pay, in pence per hour;

then $P : t \times r :: (T - t) : T$,

or $P = \frac{(T - t) \times t \times r}{T}$.

TABLE XXXVIII.—EFFECT OF PREMIUM SYSTEM ON WAGES AND COST. THE "ROWAN" SYSTEM.

Time Limit, T=20 Hours.								
Time taken.	Time saved.	Per cent. Time saved. Time allowed.	Day-work wages.	Premium earned.	Total wages.	Wages per hour.	Per cent. increase of wages.	Per cent. decrease in cost.
hours	hours		s. d.	s. d.	s. d.	d.		
20	0	0	13 4	0 0	13 4	8·0	0	0
19	1	5	12 8	0 7½	13 3½	8·4	5	0·3
18	2	10	12 0	1 2½	13 2½	8·8	10	1
17	3	15	11 4	1 8½	13 0½	9·2	15	2½
16	4	20	10 8	2 1½	12 9½	9·6	20	4
15	5	25	10 0	2 6	12 6	10·0	25	6
14	6	30	9 4	2 9½	12 1½	10·4	30	9
13	7	35	8 8	3 0½	11 8½	10·8	35	12
12	8	40	8 0	3 2½	11 2½	11·2	40	16
11	9	45	7 4	3 3½	10 7½	11·6	45	20
10	10	50	6 8	3 4	10 0	12·0	50	25
9	11	55	6 0	3 3½	9 3½	12·4	55	30
8	12	60	5 4	3 2½	8 6½	12·8	60	36

Table XXXVIII. shows the amount of premium, total wages, wages per hour, percentage increase in wages, and percentage decrease in cost for various amounts of time taken from 20 hours to 8 hours for this system; and the principal results are shown graphically in fig. 176, p. 411, plotted to a base of time taken in hours. Thus curve A shows the day-work wages, or the money the workman would ordinarily receive for the number of hours worked if paid by time alone; curve B shows the amount

of premium to be added ; curve C shows the total wages earned, or $C = A + B$; curve D shows the rate of wages earned, or $D = \frac{C}{t}$; curve E shows the percentage of increased wages, and curve F the percentage of decreased cost.

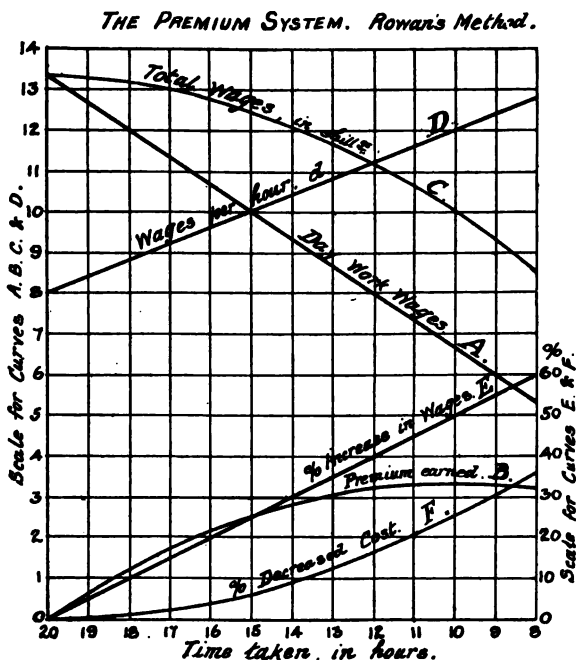


FIG. 176.

The Halsey System.—By this system the amount of premium earned is one-third of the amount of saving effected, independently of the percentage which it may bear to the day-work wage. Thus, taking the example given in illustration of the Rowan system, if the time limit be 20 hours, and the actual time taken be 16 hours, the saving effected is 4 hours at 8d. per hour or 2s. 8d., for which the amount of premium paid is one-third, or 10½d. The great drawback to this system is that it offers but slight pecuniary inducement to the workman in the earlier stages of time-saving, although it gets better at the higher rates of premium. This is clearly seen from an examination of Table XXXIX., p. 412,

where the individual columns are similar to those set out for the Rowan system. The amount of premium can be represented symbolically, thus—

$$P = \frac{(T - t) \times r}{3},$$

using the same notation as before.

TABLE XXXIX.—EFFECT OF PREMIUM SYSTEM ON WAGES AND COST. THE "HALSEY" SYSTEM.

Time Limit, T=20 Hours.								
Time taken.	Time saved.	Value of time saved.	Premium earned.	Day-work wages.	Total wages.	Wages per hour.	Per cent. increase of wages.	Per cent. decrease in cost.
hours	hours	s. d.	s. d.	s. d.	s. d.	d.		
20	0	0 0	0 0	13 4	13 4	8·0	0	0
19	1	0 8	0 2½	12 8	12 10½	8·1	1·6	3½
18	2	1 4	0 5½	12 0	12 5½	8·3	4	6½
17	3	2 0	0 8	11 4	12 0	8·5	6	10
16	4	2 8	0 10½	10 8	11 6½	8·7	8	13
15	5	3 4	1 1½	10 0	11 1½	8·9	11	16
14	6	4 0	1 4	9 4	10 8	9·1	14	20
13	7	4 8	1 6½	8 8	10 2½	9·4	18	24
12	8	5 4	1 9½	8 0	9 9½	9·8	22	27
11	9	6 0	2 0	7 4	9 4	10·2	27	30
10	10	6 8	2 2½	6 8	8 10½	10·6	32	33
9	11	7 4	2 5½	6 0	8 5½	11·3	41	36
8	12	8 0	2 8	5 4	8 0	12·0	50	40

The Weir System.—In this system *the amount of premium earned is one-half of the saving effected, or symbolically—*

$$P = \frac{(T - t) \times r}{2}.$$

TABLE XL.—EFFECT OF PREMIUM SYSTEM ON WAGES AND COST. THE "WEIR" SYSTEM.

Time Limit, T=20 Hours.									
Time taken.	Time saved.	Value of time saved.	Premium earned.	Day-work wages.	Total wages.	Wages per hour.	Per cent. increase of wages.	Per cent. decrease in cost.	
hours	hours	s. d.	s. d.	s. d.	s. d.	d.			
20	0	0 0	0 0	13 4	13 4	8	0	0	
19	1	0 8	0 4	12 8	13 0	8·2	2½	2½	
18	2	1 4	0 8	12 0	12 8	8·4	5½	5	
17	3	2 0	1 0	11 4	12 4	8·7	9	7½	
16	4	2 8	1 4	10 8	12 0	9·0	12½	10	
15	5	3 4	1 8	10 0	11 8	9·3	16½	12½	
14	6	4 0	2 0	9 4	11 4	9·7	21½	15	
13	7	4 8	2 4	8 8	11 0	10·1	27	17½	
12	8	5 4	2 8	8 0	10 8	10·7	33	20	
11	9	6 0	3 0	7 4	10 4	11·3	41	22½	
10	10	6 8	3 4	6 8	10 0	12·0	50	25	
9	11	7 4	3 8	6 0	9 8	12·9	61	27½	
8	12	8 0	4 0	5 4	9 4	14·0	75	30	

Table XL. shows the amount of premium earned and wages paid (total and per hour) under this system, together with the proportionate increase in wages and decrease in cost for the same actual times as before.

Comparison of the Three Premium Systems.—Comparing the three systems just described, there is something to be said in favour of each. The Rowan system undoubtedly affords the greatest inducement to the workman to put forward his best efforts and economise time to the best of his ability, when the amount of time that can be saved on the job or operation is comparatively low; but it is difficult for the average workman to

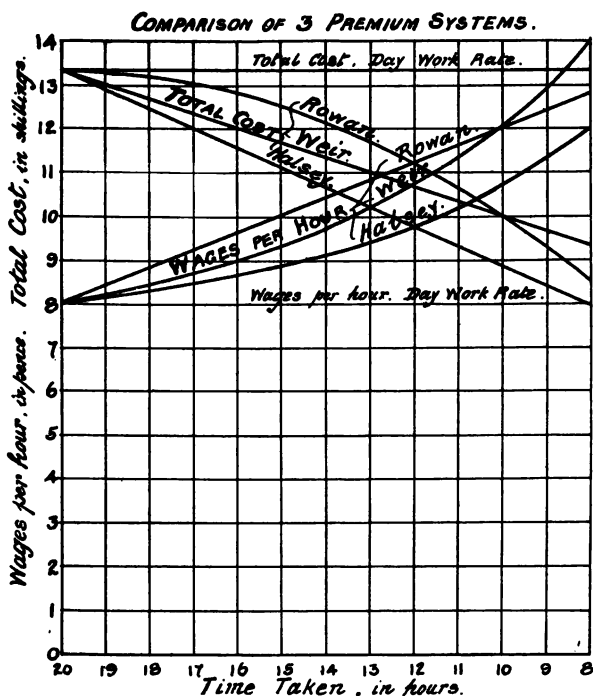


FIG. 177.

calculate the amount of bonus or premium due to him, owing to the fact that it is based on a percentage scale. The simplest way to state the calculation is,—the amount of premium saved is the same *percentage* of his day-work wages as the time saved is of the time limit. But where odd amounts of percentage have to be calculated on shillings and pence, it involves a little calculation which is generally beyond the average workman's capabilities. When the percentage comes out to 20 per cent. or $33\frac{1}{3}$ per cent., he is able to manage the calculation, but when it works out to,

say, 23 per cent. or 37 per cent., it is beyond him. A series of tables has been calculated and published* which affords a ready means of getting at the percentage of premium earned for all "times saved" up to 100 bonus on all "time limits" up to 200 hours, without any calculation whatever. The other two systems—Halsey's and Weir's—are both simpler for the purposes of calculation, but they both result in comparatively low amounts of premium unless the time-limit is fixed very generously.

By plotting the two principal results of each of the three systems on squared paper, as shown in fig. 177, p. 414, the relative amounts of wages per hour and cost of the job for each system is seen at a glance. Thus the wages earned per hour in the Rowan system follow a straight line law when plotted to a base of time taken, whereas the wages per hour for each of the other two systems fall on curves. *Vice versa*, the total cost of the job in Rowan's system follows a curve, but follows the straight line law in each of the other two systems.

* Golding's *Bonus Tables for Engineers, etc.*, London, Chas. Griffin & Co., Ltd., 1903.

CHAPTER XXI.

USE OF SQUARED PAPER IN AN ENGINEERING ESTABLISHMENT.

THE use of squared paper to the engineer in commercial work is exceedingly valuable. It should enter into his everyday life, for a graphical representation of variations will be found capable of affording information which a close study of columns of figures may not bring to light. At a glance may be seen the character of the variation which has taken place, whether up or down, the rate of the variation, and the period for which it lasted, and many other important deductions, some of which will be noticed in the following paragraphs.

For example, in most engineering works, the water used is supplied through a meter and charged at so much per 1000 gallons. Some day in a frost the main bursts, and the fracture being near a drain, the water runs away to waste without being noticed until the water rate demand note comes in. The manager then begins to think there must be something wrong with his meter. If the reading of the water had been taken every morning or every week-end, and plotted on squared paper, he would have discovered the leak shortly after it had occurred, and had it rectified before it became so expensive. Fig. 178* represents the quarterly water consumption by a large engineering firm for a period of four years, and, the account being regularly posted up in this diagrammatic form, attention was drawn to the great rise in the second and third quarters of the year 1895, with the result that leaks due to two broken pipes were found and repaired.

For the power-house the figures relating to coal and water should be plotted week by week, together with the number of hours per week the boiler is under steam, to show the total costs of power production. If the factory be driven electrically, the total kilo-watt hours should also be plotted. For purposes of comparison and economic steaming, it is advisable to use some

* From a paper by Mr John Jamieson, *Inst. Mech. Engineers*, 1897.

standard of evaporation which takes into account the cost of the fuel used, such as the cost of evaporating 1000 gallons of water, or the cost of raising 1 ton of steam. The former can be found from the following simple formula:—

$$C = \frac{4.464 \times F}{E},$$

where C = cost (in shillings) of evaporating 100 gallons.

F = average cost of fuel per ton, in shillings.

E = evaporation in lbs. of water per lb. of fuel.

Quarterly Water Consumption.

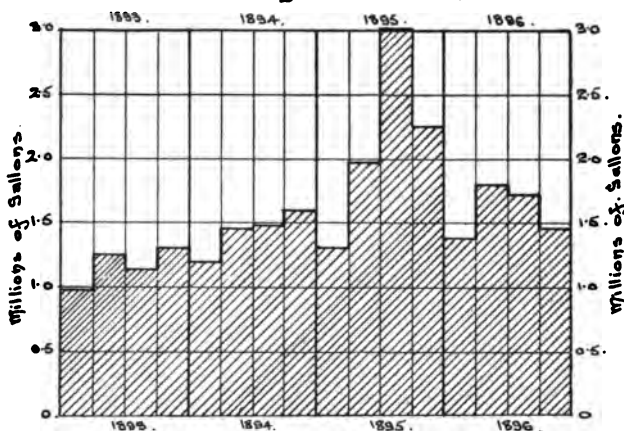


FIG. 178.

In this way the effects of burning lower-grade fuels, such as coke breeze, coal dust, etc., can be estimated.

A typical case of a Lancashire boiler steaming some 80 to 90 hours per week on low-grade fuel is shown in fig. 179, where all the weekly returns for the first four months are plotted as follows:—

Curve A = total cost of fuel per week, in shillings.

„ B = total number of hours steamed per week.

„ C = mean cost of evaporating 1000 gallons, in shillings.

„ D = evaporation in lbs. of water per lb. coal (from cold feed).

„ E = total weight of fuel burned per week, in tons.

„ F = average cost of fuel per ton, in shillings.

In the case under consideration, the fuel used was a mixture of coke breeze and small coal, the former costing 5s. per ton delivered, and the latter varying from 10s. 6d. to 14s. 6d. per ton. In the first two months the fuels were used in about equal proportions, but during March and April the proportion of breeze

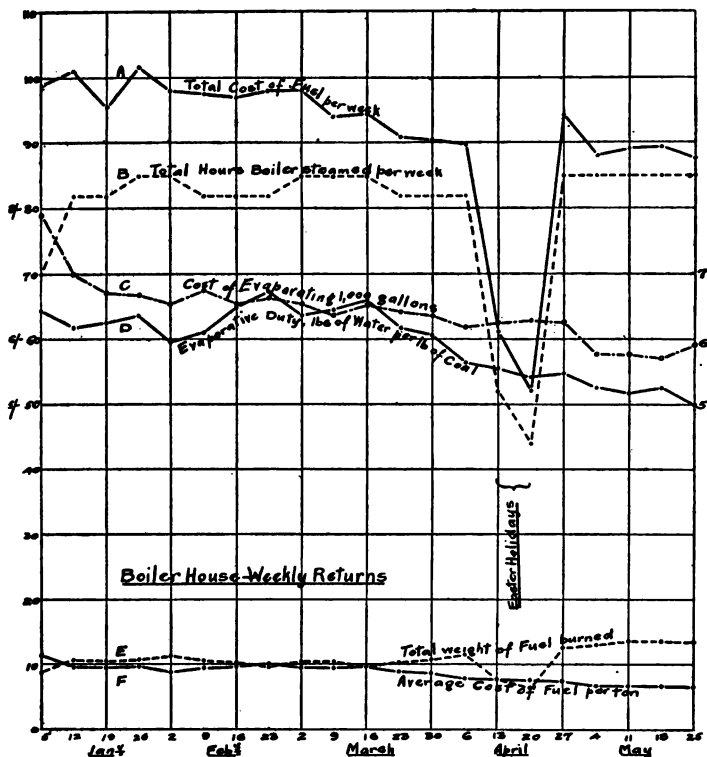


FIG. 179.

was gradually increased until, during May, the ratio of breeze to coal was about 2 to 1. A reference to curve C shows immediately the rise and fall in the cost of fuel for evaporating 1000 gallons of water, and the two curves D and F indicate the reasons for it. Thus, for week ending 2nd February the average cost of the fuel came out rather low, and although the evaporative duty fell, yet the cost on curve C fell to about 6s. 7d., and practically remained

there till the middle of March, when the influence of the increased consumption of breeze brought it lower still.

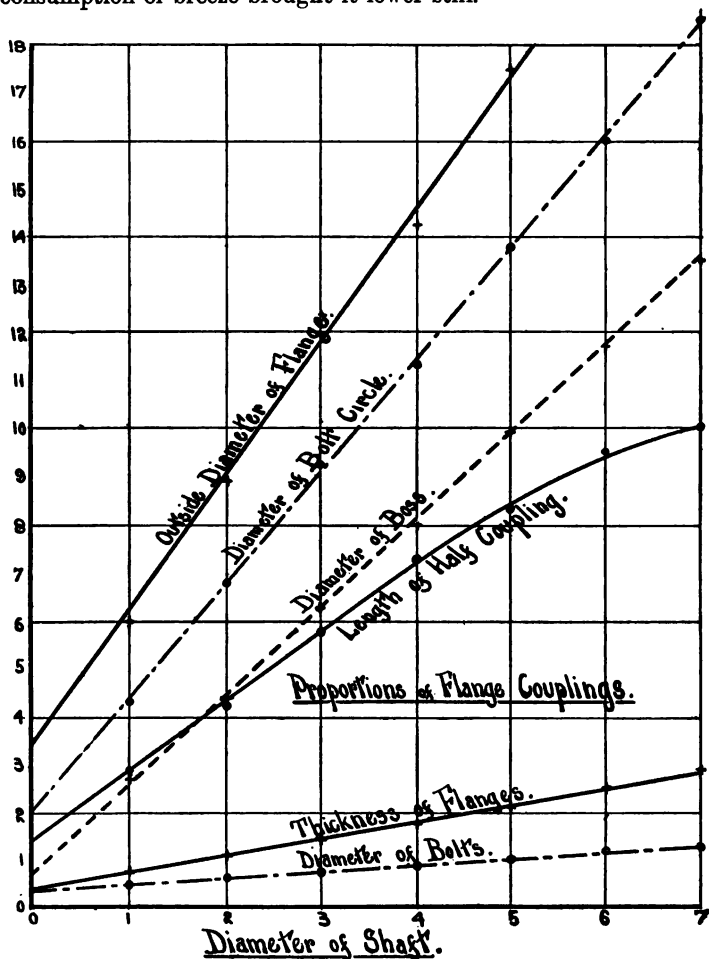


FIG. 180.

Similarly, if with all other departments of an engineering works the management keep a graphic record on squared paper of such things as gas-meter hours worked per week, weight of coal burned in boiler-house, total wages paid, cost of manufacturing specialities,

they are in a position to detect any irregularities as soon as they occur, and to put them right. Further, the comparison of the variation of one curve with another will throw an additional light upon causes and results, absolutely invaluable to an economic management.

The opportunities for using squared paper to advantage in the **drawing office** are innumerable. For designing standard articles, the principal dimensions and sizes should be plotted to a base of the size of the object. For example, a firm that makes a speciality of mill gearing should have standard curves representing the principal proportions of plummer blocks, wall brackets, hangers, couplings, etc., all plotted to a base of the size of shaft, when any **discrepancies or irregularities can be detected and remedied at once**. Fig. 180, p. 419, shows the over-all dimensions of flange couplings, all plotted to a base of size of shaft, when it is seen that all the proportions follow the straight-line law. Notice C, the diameter of the bolt circle, is seen to follow almost exactly the law,

$$C \text{ in.} = 2\frac{2}{3}d + 2,$$

and

$$B \text{ in.} = 1\frac{3}{4}d + \frac{1}{2},$$

where d = diameter of shaft in inches.

Similarly, the **fundamental sizes for steam engine design may be plotted on squared paper** to a base of diameter of cylinder or horse-power, when it will easily be seen where the various parts are deficient or in excess as compared with the general tendency of the series. Fig. 181, p. 421, shows the diameter of piston rod, crosshead pin, and crank-shaft respectively plotted to a base of diameter of cylinder for a series of single-cylinder steam engines, when it is seen that the first two curves follow a straight-line law, whilst the diameter of crank-shaft gets proportionately smaller in the larger sizes (due to its strength varying as the cube of its diameter). It is also seen at a glance that the piston rod and crosshead pin for the 9-in. and 10-in. cylinder engines are weaker than the average, whereas all the sizes for the 12-in. cylinder are above the average. This method of graphically representing the more important sizes and dimensions in designing is of great assistance in keeping a series uniform in strength, and in fixing the dimensions of a new size if required.

Additional curves should also be plotted for the sizes of the more important details, such as stuffing boxes (plotted to a base of size of spindle), connecting-rod ends (plotted to a base of diameter of pin), piston rings (plotted to a base of size of piston), plummer-block brasses (on a base of size of shaft), etc. With a **standard**

series of curves such as this for the guidance of the draughtsman, the work is kept more uniform in size and weight, and frequently an unnecessary multiplicity of patterns is avoided. The curves for flywheels should also include the calculated weight of the rim, and the finished weight of the complete wheel, in order that, when a new wheel is required of given size or weight, it can be seen at a glance whether or not an existing pattern can be used, by lining up or turning down the rim.

To the prime-cost clerk and the estimating department, curves showing the actual costs of all the standard sizes of the specialities made by the firm would be of the greatest possible

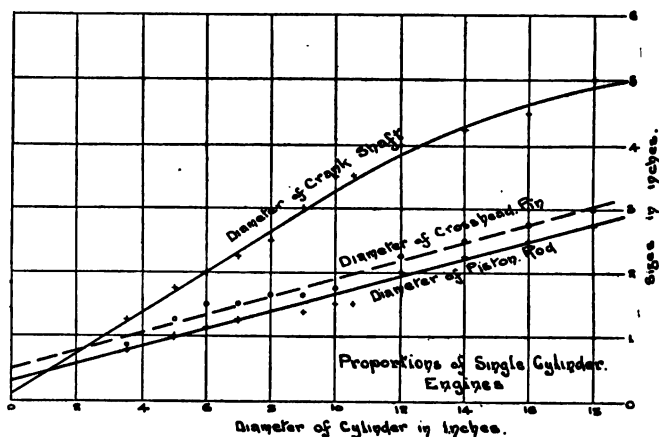


FIG. 181.

assistance. For example, the nett cost of some sizes of high-class plumber blocks is shown in fig. 182, plotted to a base of size of shaft, where it is seen that, with but one or two slight exceptions, the prices fall on a curve.

If, however, the prices be plotted to a base of $(\text{diameter})^2$, it is found that the points fall more nearly on a straight line, from which the law of the price can be approximately determined. It is seen to vary according to a power of the diameter which lies between the second and third powers. In fig. 183, p. 423, the costs of planing machines have been plotted to a base of $(\text{length} \times \text{width planed})$, when the total cost falls very nearly on a straight line, the law of which is approximately—

$$\text{Cost in £'s} = 3.35(\text{length in feet} \times \text{breadth in feet}) + £60.$$

Similarly, the cost of a standard type of valves, lubricators, spanners, governors, etc., should all be represented graphically.

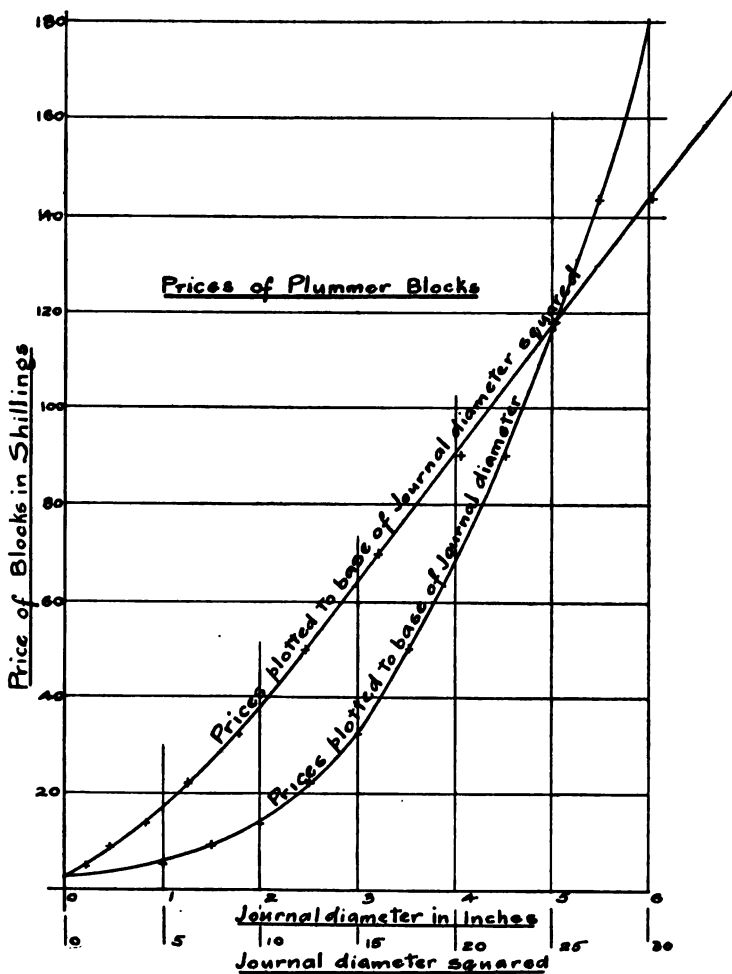


FIG. 182.

Fig. 184 shows the cost of compound tandem steam engines of high-class design, including rope, flywheel, and jet condenser,

plotted to a base of I.H.P. developed when working with steam at 100 lbs. pressure expanding 10 times, at a piston speed of 600 to 700 ft. per minute. In this case the law of the cost curve is approximately—

$$\text{Cost in £'s} = 2.75(\text{I.H.P.}) + \text{£}440.$$

The cost per ton of nett weight (exclusive of packing-cases) is also plotted to the same base, showing the uniformity of the

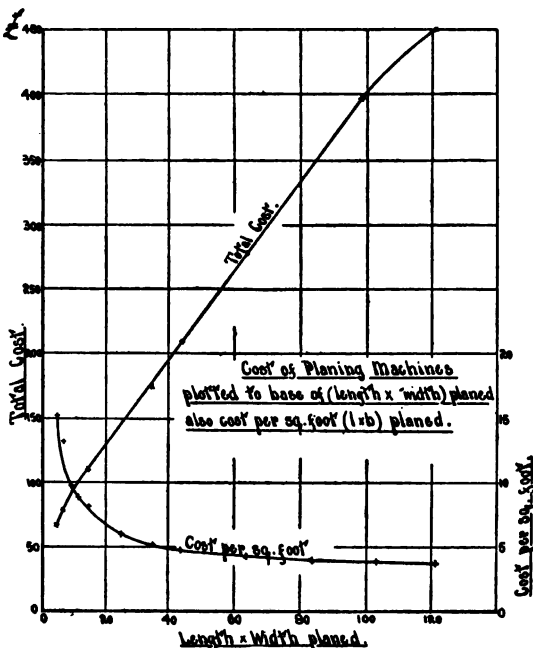


FIG. 183.

series, both as regards cost and weight. Useful exercises in curve-plotting, giving valuable information, would be given by plotting two additional curves to those given, showing (a) weight of engine complete (in tons) to a base of I.H.P., and (b) cost of engine per I.H.P. to a base of I.H.P.

Frequently, in plotting the costs of different sizes of articles, one is able to discover departures from the law which seems to govern the cost, and often, by tracing the reason for such departure, the anomaly can be remedied. For example, on plotting the selling

price for the size of ordinary steam stop valves to a base of (diameter)², as shown in fig. 185, it was seen that the price of the 4-in. valve, as shown at A, was too high, and on examination it was found that the cost had been based on some specially heavy

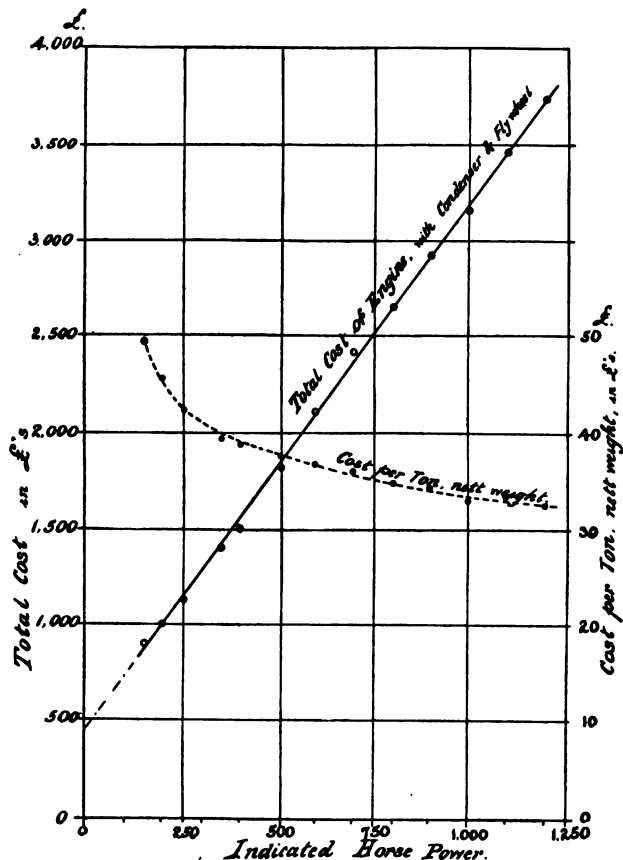


FIG. 184.

valves that had been made to suit particular requirements as to pressure. The list was therefore corrected by bringing the price for this size of valve down from A to B, resulting in an increased sale of this particular size.

The weights of some ordinary Lancashire boilers constructed

for a working pressure of 100 lbs. per sq. in., complete with all fittings and mountings, are shown in fig. 186, plotted to a base

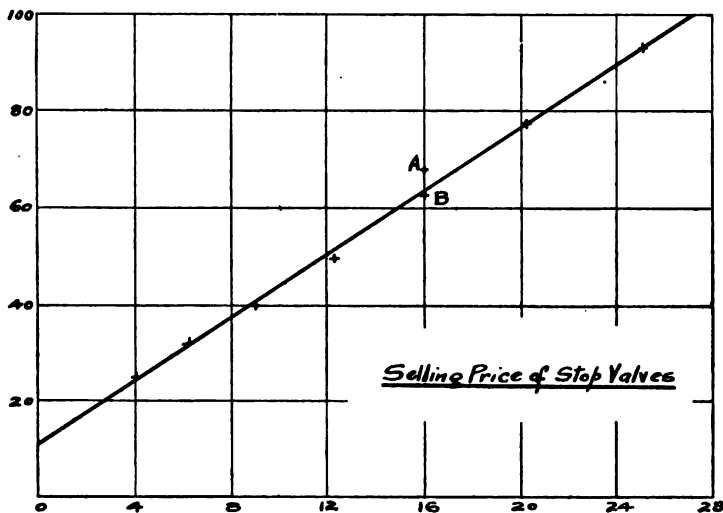


FIG. 185.

of (diameter² × length) in feet, when the weight was seen to follow the straight-line law—

$$W \text{ tons} = \left(\frac{D^2 \times L}{100} \right) + 3.5 \text{ tons.}$$

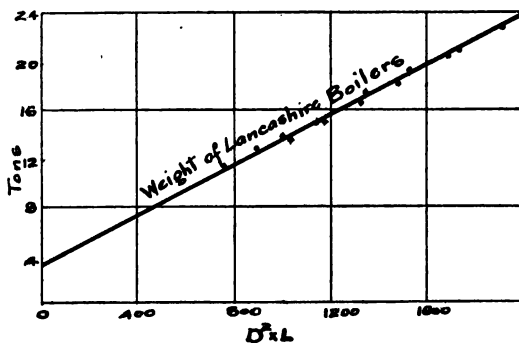


FIG. 186.

Fig. 187 shows (A) the total cost, (B) the B.H.P., and (C) the cost per ton for a series of small high-speed vertical engines, plotted to a base of diameter of cylinder $\times$ stroke, from which the cost follows very nearly the straight-line law—

$$\text{Cost in £'s} = 1.05(d \text{ in.} \times l \text{ in.}) + \text{£}27, 10\text{s. } 0\text{d.}$$

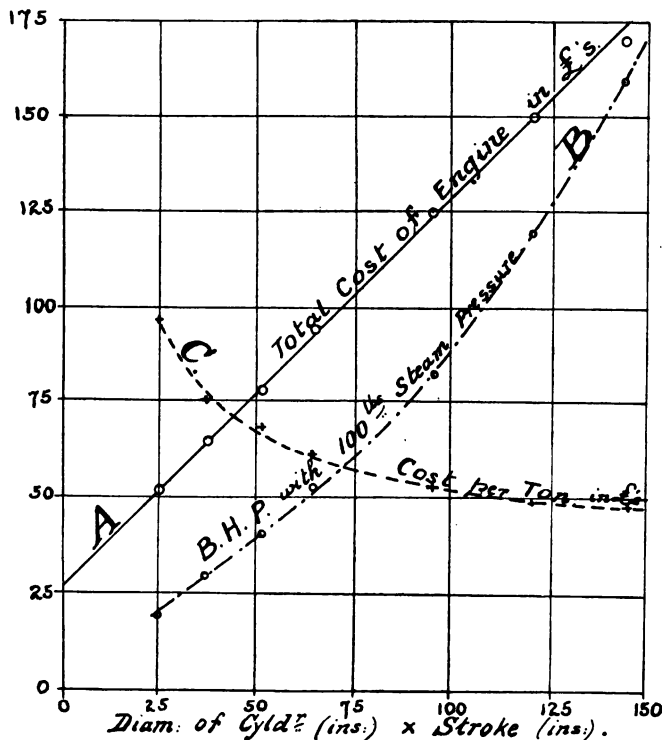


FIG. 187.—Total Cost, B.H.P., and Cost per Ton of Small High-speed Vertical Engines.

Value of Squared Paper to the Foreman and Works Manager.—

A large field exists in the workshop for plotting various quantities to ascertain their relation to each other. In the foundry, one of the most useful exercises is to try the effect of varying proportions of different kinds of pig-iron, to see in what way it influences the breaking stress of the iron. For instance, the proportion of Hæmatite, Graysbrook, Carron, or any other kind of pig-iron should

be varied in different runs, and two or three test bars cast in each run tested, and the mean breaking load plotted on squared paper to a base of percentage of the particular kind of pig-iron being used.

IRON FOUNDRY WEEKLY RETURNS FOR QUARTER ENDING MARCH 30, 1902.

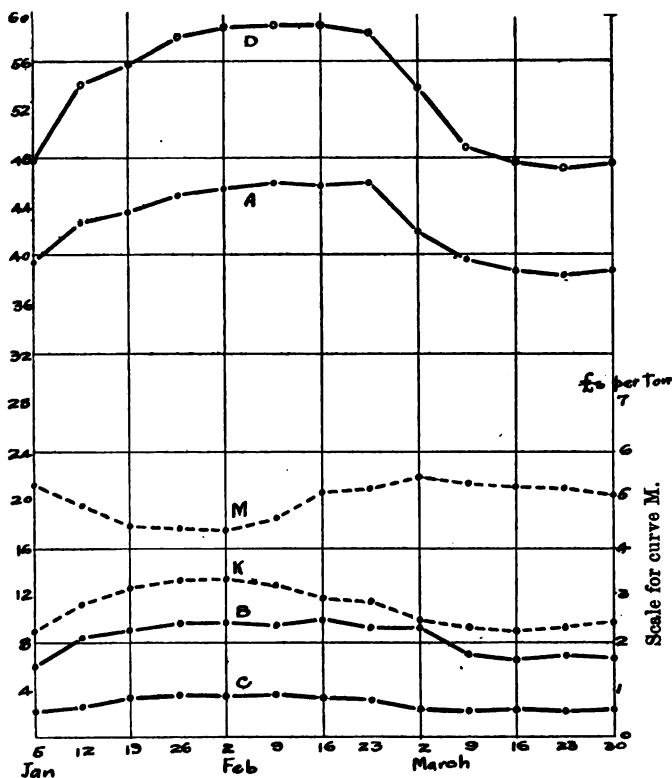


FIG. 188.

- A = Wages of moulders (sand and loam) in £'s per week.
 B = Wages of labourers.
 C = Wages of trimmers.
 D = Total wages (A + B + C).
 K = Weight cast in tons per week.
 M = Average cost (in wages) per ton.

In this way a practical knowledge of the influence of different kinds of pig-iron in various proportions can be obtained and compared one against the other, to obtain the most economical mixture to

give a required strength to the iron. varying the proportion of scrap-iron upon

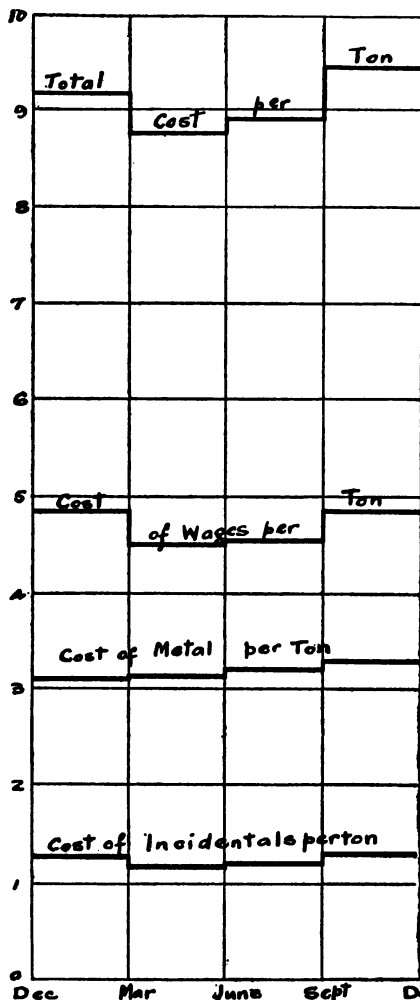


Fig. 189.—Quarterly Summary for Iron Foundry.

the various totals of wages paid and castings made are plotted for thirteen consecutive weeks, the curve A representing total

Similarly, the effect of the strength of the metal will show the foundry foreman what is the best proportion of scrap to use for a given strength. The cost of all the various incidentals used in the foundry should also be totalled up each quarter and plotted together with a curve representing the total output of castings for the quarter, to show up any undue expenditure in what one may term the "small things" of the foundry, but which, if allowed to pass unnoticed, will mount up to a very considerable item at the end of the quarter. Similarly, the total wages paid should be plotted week by week, dividing up the total sum into different classes according to the occupations of the workmen employed, such as sand moulders, loam moulders, core-makers, trimmers, labourers, furnacemen, etc., when any undue rise in the wages for any one or more departments can be immediately detected and rectified.

A typical example of such curves is shown in fig. 188, p. 427, where

wages per week of all moulders (sand and loam); B shows wages of labourers and core-makers, and C the wages for trimmers. The total weight of castings made weekly is also shown by the curve K, and the average cost (in wages) per ton of castings by the curve M. A careful consideration of such curves shows far more to the foundry foreman or manager than any amount of tabulated figures, as he can see at a glance the relation of one to the other, and the relative proportion of the various amounts. Thus, at the end of January and the beginning of February the weight cast reaches a maximum of $13\frac{1}{2}$ tons per week, and the wages cost per ton falls to its minimum value of about £4, 8s. 0d.; whereas at the end of February, when the output had been considerably diminished, the cost per ton mounted up to over £5. Why is this? Similarly,

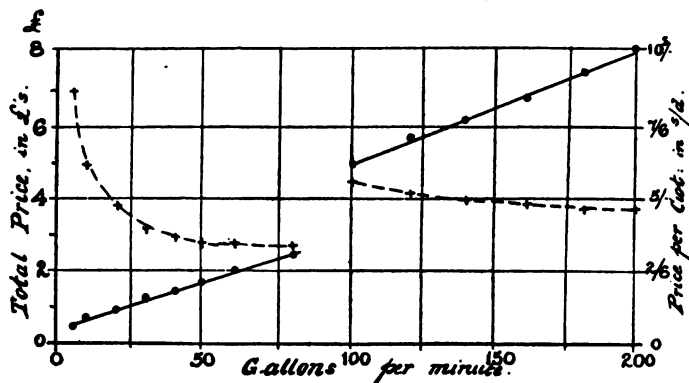


FIG. 190.

the total cost of pig-iron and scrap, sand, chalk, blacking, etc., should be plotted (say) every quarter, and compared with the total output of castings. Table XLI., p. 430, shows a typical case of total costs for a small foundry of about 600 tons per year, divided up into four quarters, the average cost per ton of good castings being plotted on squared paper as in fig. 189.

If such costs as these be plotted from year to year, it forms a complete record of the cost of the foundry under varying conditions, not only showing the actual rise or fall in the costs, but also the rate, or relative amount, and the connection between the various amounts. To afford a ready means of converting costs into cost per ton, a table of weight conversions is given (see Table XLII., p. 440), showing all weights in cwts., qrs., and lbs. as decimals of a ton.

Where the bonus or premium systems are in use, the foundry

foreman or manager will find squared paper of the utmost value to him, affording a ready means of checking the prices for the various sizes of standard articles made. As an example of this, fig. 190, p. 429, gives the **bonus price paid for moulding small cylinders for rotary pumps**, plotted to a base of capacity of pump in gallons per minute. In this case the smaller sizes were moulded in the sand from iron patterns, but the larger sizes were struck up in loam, with strickle boards and plates. It will be seen that the prices paid for the various sizes fall almost exactly on two straight lines, so that the corresponding price for any intermediate sizes could easily be determined as desired. The curves (dotted) also show the price per cwt. to a separate scale, showing how expensive the smaller sizes are, based on a cost per cwt. Similarly,

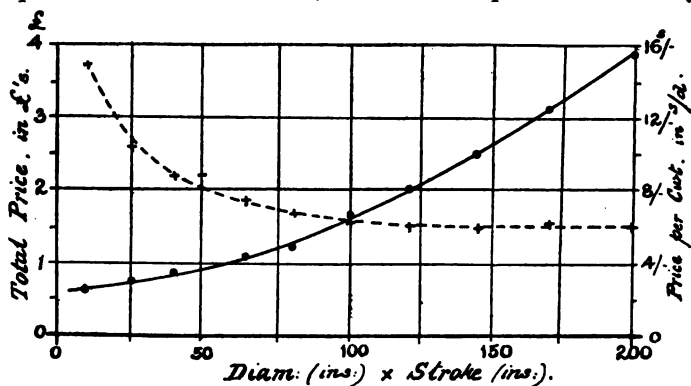


FIG. 191.

the bonus price paid for moulding a series of small steam cylinders is shown in fig. 191, plotted to a base of (diameter $\times$ stroke) in inches. The costs will be seen to be fairly well on the curve, the dotted curve showing the cost per cwt. as before. For the other departments of an engineering works the same use can be made of squared paper for plotting various prices and weights. Quarterly or monthly **summaries** should be plotted for the smithy, similar to those given for the foundry, showing the **weight of finished forgings** and **total wages paid**, also cost of coal for forges, etc., as shown in fig. 193, p. 433.

In the **machine shop** there is an unlimited field for the use of squared paper. The diagram,* fig. 193, shows, for example, the extent to which three classes of machines were used in some large engineering works for the two quarters January to July

* From a paper by Mr John Jamieson, *Inst. Mech. Engineers*, 1897.

1897. It will be seen that the lathes were almost fully employed running on an average about 95 per cent. of the full time. Any

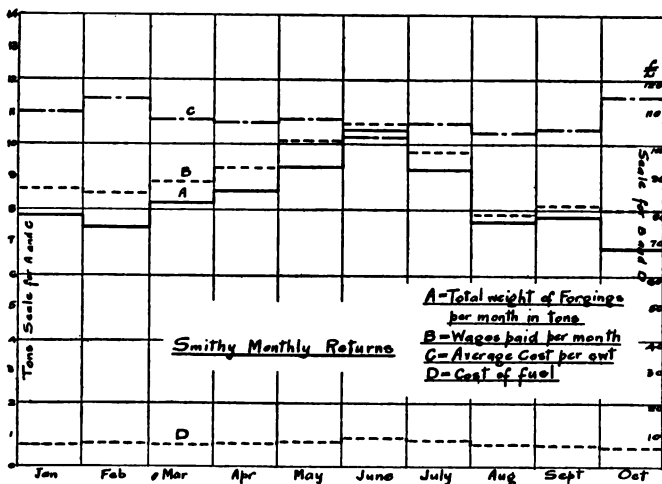


FIG. 192.

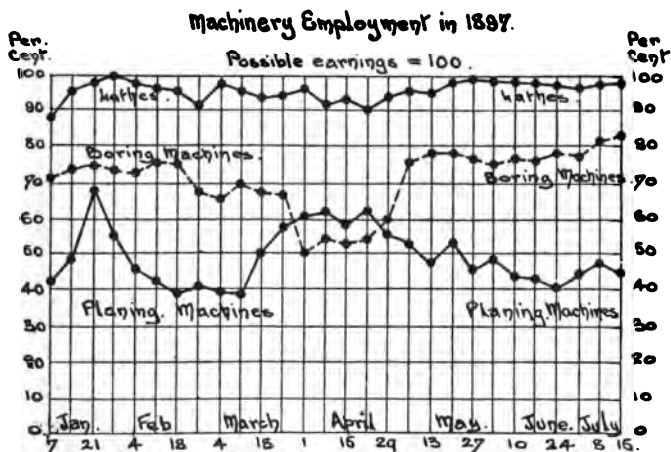


FIG. 193.

marked increase in the output would mean that additional lathes would be required. The boring and planing machines, on the

other hand, are not nearly fully employed—the former running on an average 70 per cent., the latter about 50 per cent. of the possible available hours.

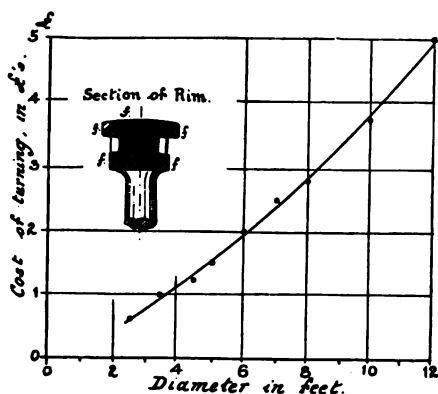


FIG. 194.

Where repetition work is done, diagrams showing the costs of machining precisely similar articles at different times, or of machining different sizes from time to time, are very valuable.

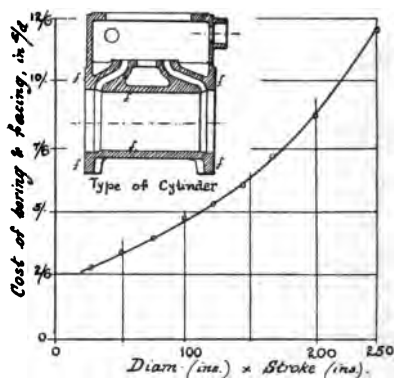


FIG. 195.

For example, the cost of turning and boring various sizes of flywheels is shown in fig. 194, plotted to a base of diameter of wheel; and the cost of boring and facing small steam cylinders, plotted to a base of (diameter $\times$ stroke) in inches, is shown in fig. 195.

The law of the curve for the latter is found to be (approximately)—

$$\text{Cost in shillings} = \left(\frac{d \times l}{50} \right)^{1.4} + 2.5,$$

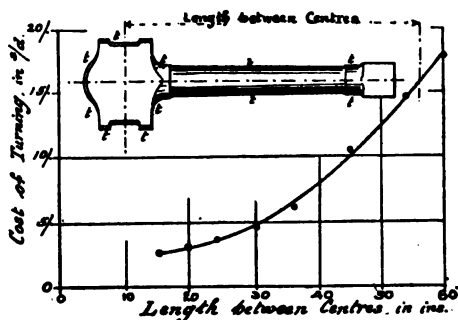


FIG. 196.

where d =diameter in inches, l =stroke in inches. Again, the cost of turning a number of different sizes of connecting rods for a series of high-speed steam engines, when plotted to a base of length of rod from centre to centre, as in fig. 196, gives a

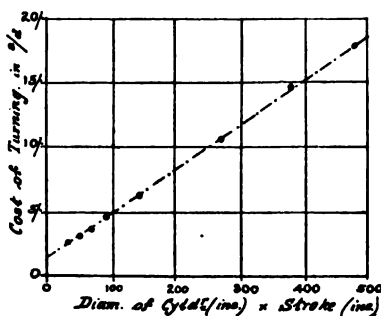


FIG. 197.

curve; but if plotted as shown in fig. 197 to a base of diameter of cylinder $\times$ length of stroke (in inches) it follows a straight-line law, given by the formula—

$$\text{Cost (in shillings)} = 0.035(d \times l) + 1.25,$$

where d and l are the diameter of cylinder and length of stroke respectively, in inches. The total wages and cost of machining

(complete) a number of different sizes of small single-cylinder horizontal engines is also found to follow a similar law when plotted to a base of diameter of cylinder $\times$ length of stroke. As seen in fig. 198, the law for such a series worked out to—

$$\text{Total cost in £'s} = 0.11(d - l) + £2.5.$$

The above costs are nett wages only paid for machining, exclusive of percentages for tool, foreman, or establishment charges. Fig. 199 shows the nett wages cost of fitting up a similar series of small engines.

To the **manager or general management** of an engineering establishment, the plotting of the results of working will be

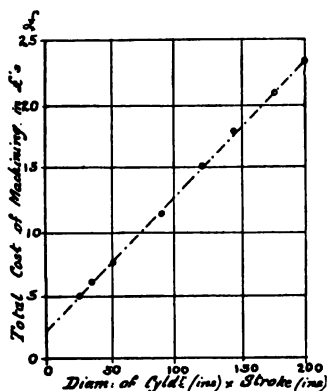


FIG. 198.

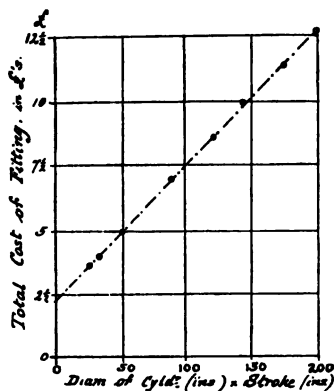


FIG. 199.

found to bring into prominence many important points which would otherwise be overlooked. For example, where the class of work is of a fairly uniform description, the **total wages per month** should be plotted to a base of output in tons. The weights per month will vary somewhat according to the despatch of large contracts and the work done for stock, but it will often show up some little discrepancy, such as non-delivery of certain work which ought to have been finished. Further, the **total annual output** as invoiced to customers should be plotted from year to year, with the **several costs**—material, wages, shop charges, and establishment charges—separately shown, and finally each item should be expressed as a percentage of the total output.

Fig. 200* shows graphically the **output costs and profits** for nine years' work by a large engineering firm. A close study of

* From a paper by Mr John Jamieson, *Inst. Mech. Engineers*, 1897.

this diagram will prove most instructive. Let the student ask himself (a) why the percentage wages increases with diminished output; (b) why the percentage establishment charges increases with the diminished output; and (c) the reason that the percentage stores issued is less with diminished output.

The application of squared paper to making more clear the state of business can be indefinitely extended, but we must finish with the following three illustrations, believing that, if the engineer or student

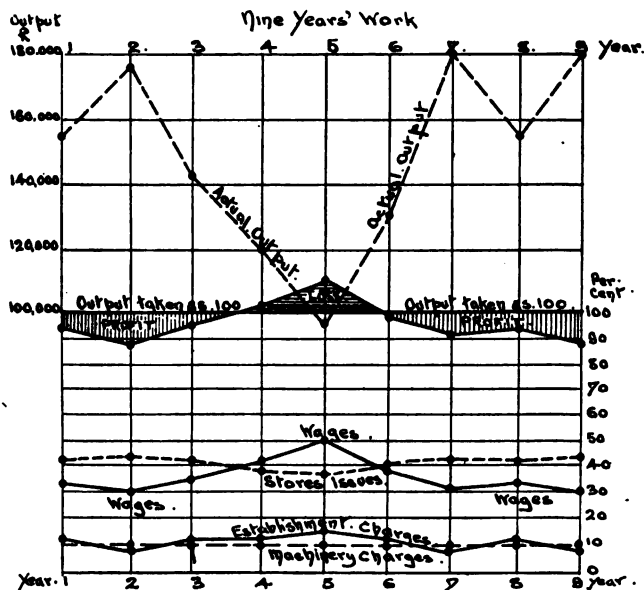


FIG. 200.

has sufficient interest in this kind of work, he will find, after reading and working out the results given in this chapter, that he will have no difficulty in using the diagrammatic method and in extending its uses as occasion and his own common sense suggest.

Fig. 201 * shows twenty-nine out of a hundred consecutive foundry orders arranged in the diagram in the order of invoice price, so as to get a steadily falling step-by-step diagram. The costs, without taking establishment charges into account, are shown by the height to the inner series of steps in the lower diagram, sometimes rising and sometimes falling, and the

* From a paper by Mr John Jamieson, *Inst. Mech. Engineers*, 1897.

difference in height between the two steps (shown shaded) represents what may be called the apparent profit. The ordinates

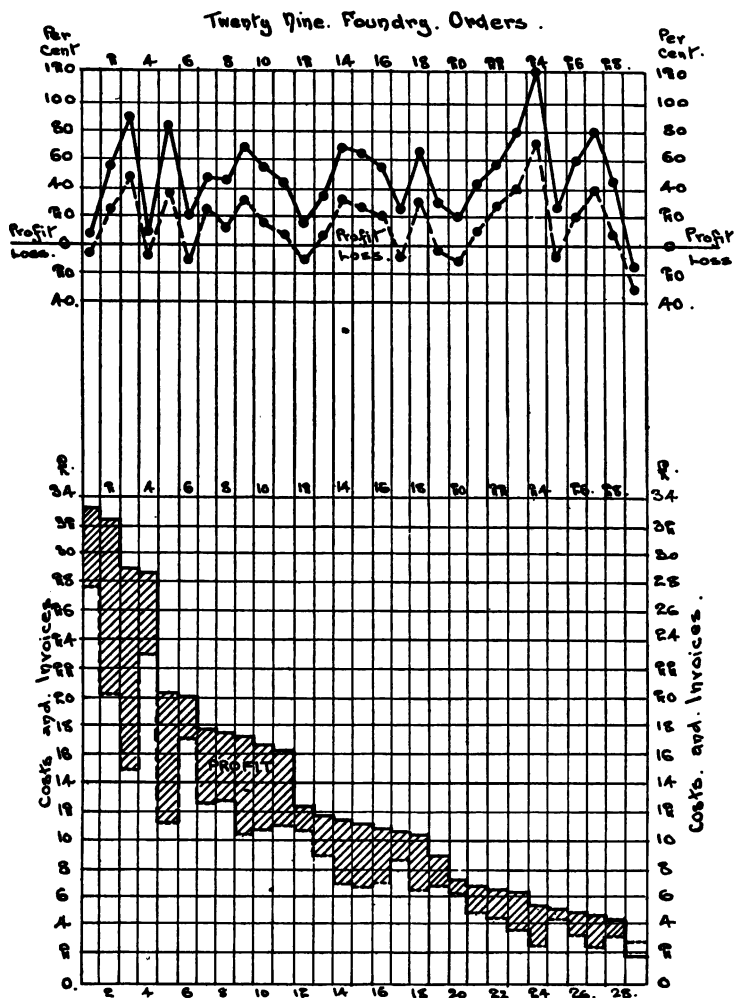
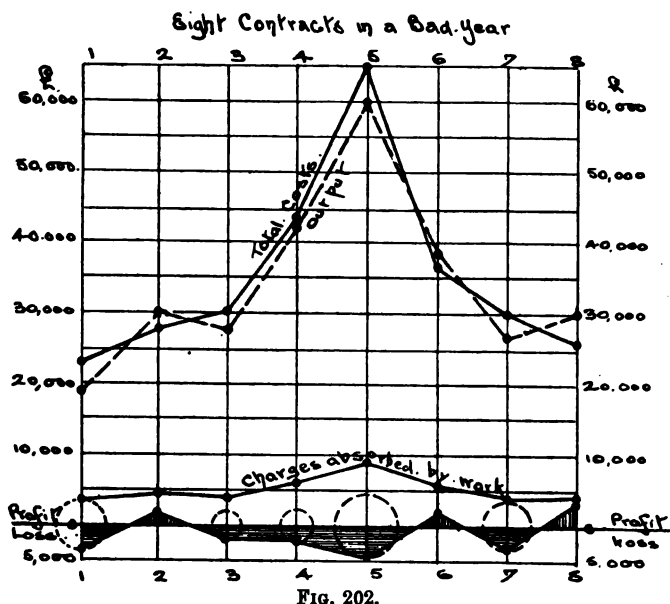


FIG. 201.

to the full thick line in the upper diagram represent the percentage which the apparent profit bears to the costs, not including

the establishment charges, while the lower dotted line defines the ordinates representing the percentage that the actual profit is of the costs, the establishment charges being distributed in proportion to the costs. A glance at the diagram shows that some of the foundry orders executed, while apparently yielding a profit, yet sustain a nett loss when the charges of the establishment are taken into account. We pointed out in Chapter XVII. that occasionally during periods of bad trade engineering firms, like other firms, have to do work at less than cost price in order



to help to keep the establishment going. This point is well illustrated in fig. 202,* representing in diagrammatic form eight large contracts during a bad year. Only three of these contracts, 2, 6, and 8, yielded actual profit, while the others were unremunerative. An inspection of these figures will reveal the fact that, although the contracts 3, 4, 5, and 7 show a loss, they yet cover a proportion of the establishment charges over and above material, wages, and shop charges, and help to keep the concern going over the bad year. Contract No. 1, however, shows an absolute loss; for not only does it totally fail to cover even a small proportion

* From a paper by Mr John Jamieson, *Inst. Mech. Engineers*, 1897.

of the establishment charges, but sustains a loss on the cost of material and wages. It would have been better if this contract had never been made.

TABLE XLII.—WEIGHT CONVERSIONS. CWTs., QRS., AND LBS. IN DECIMALS OF A TON.

Cwts.		Lbs.		1 Qr.		2 Qrs.		3 Qrs.	
1	0.06	1	0.00045	Lbs. 1	0.01296	Lbs. 1	0.02545	Lbs. 1	0.03795
2	0.1	2	0.00089	2	0.01839	2	0.02589	2	0.03839
3	0.15	3	0.00134	3	0.01884	3	0.02634	3	0.03884
4	0.2	4	0.00178	4	0.01428	4	0.02678	4	0.03928
5	0.25	5	0.00223	5	0.01473	5	0.02723	5	0.03973
6	0.3	6	0.00268	6	0.01518	6	0.02768	6	0.04018
7	0.35	7	0.00312	7	0.01562	7	0.02812	7	0.04062
8	0.4	8	0.00357	8	0.01607	8	0.02857	8	0.04107
9	0.45	9	0.00402	9	0.01652	9	0.02902	9	0.04152
10	0.5	10	0.00446	10	0.01696	10	0.02946	10	0.04196
11	0.55	11	0.00491	11	0.01741	11	0.02991	11	0.04241
12	0.6	12	0.00536	12	0.01786	12	0.03036	12	0.04286
13	0.65	13	0.00580	13	0.01830	13	0.03080	13	0.04331
14	0.7	14	0.00625	14	0.01875	14	0.03125	14	0.04375
15	0.75	15	0.00670	15	0.01920	15	0.03170	15	0.04420
16	0.8	16	0.00714	16	0.01964	16	0.03214	16	0.04464
17	0.85	17	0.00759	17	0.02009	17	0.03259	17	0.04509
18	0.9	18	0.00803	18	0.02053	18	0.03303	18	0.04553
19	0.95	19	0.00848	19	0.02098	19	0.03348	19	0.04598
20	1.0	20	0.00893	20	0.02143	20	0.03393	20	0.04643
Qrs.		21	0.00937	21	0.02187	21	0.03437	21	0.04687
		22	0.00982	22	0.02232	22	0.03482	22	0.04732
1	0.0125	23	0.01027	23	0.02277	23	0.03527	23	0.04776
2	0.025	24	0.01071	24	0.02321	24	0.03571	24	0.04821
3	0.0375	25	0.01116	25	0.02366	25	0.03616	25	0.04866
4	0.05	26	0.01161	26	0.02411	26	0.03660	26	0.04910
		27	0.01205	27	0.02455	27	0.03705	27	0.04955
		1 Qr. = 0.01250		2 Qrs. = 0.025		3 Qrs. = 0.0375		1 Cwt. = 0.05	

EXERCISES ON SECTION IV.

1. Find the weight of the cast-iron deadplate shown in fig. 203.

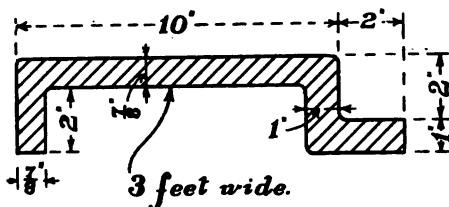


FIG. 203.

2. Find the weight of the wrought-iron crank shown in fig. 205 to finished sizes.

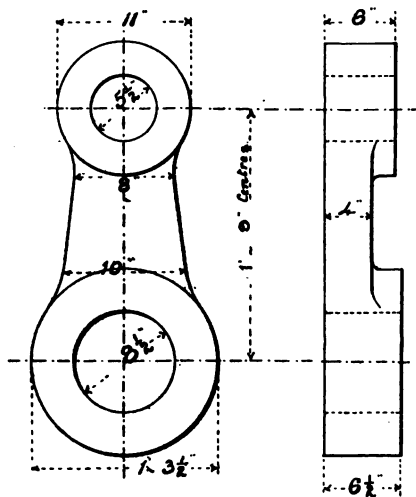


FIG. 204.

3. Calculate the approximate forged weight of the crank shown in fig. 204, allowing for excess metal for machining, as in p. 383, and find the cost of the forging at 18s. 6d. per cwt.

4. Calculate the weight of 500 cast-iron fire bars shown in fig. 205, and find the cost of same at 9s. 6d. per cwt.

5. Calculate the weight of the mild steel shaft shown in fig. 206.

tons cwt. qr. lbs.
 Ans. Act. wt. was 2 7 1 7

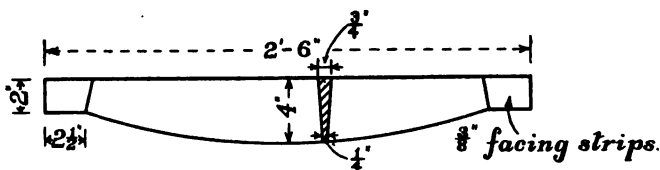


FIG. 205.

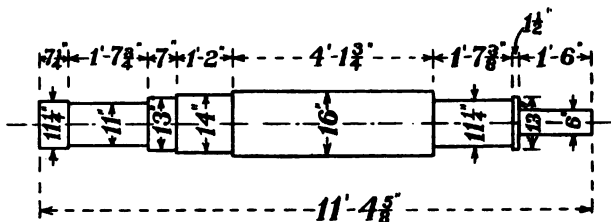


FIG. 206.

6. Calculate the weight of the forging for the above, allowing $\frac{3}{8}$ in. all over for machining, and find the cost of same at 16s. 6d. per cwt.

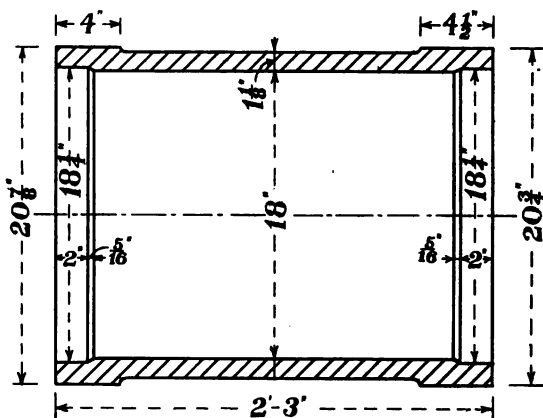


FIG. 207.

7. Calculate the finished weight of the C.I. cylinder-liner shown in fig. 207.

8. Find the weight of the 3-in. plummer block brass shown in fig. 208.

(a) Finished sizes.

(b) Allowing $\frac{1}{16}$ extra metal for turning where marked.

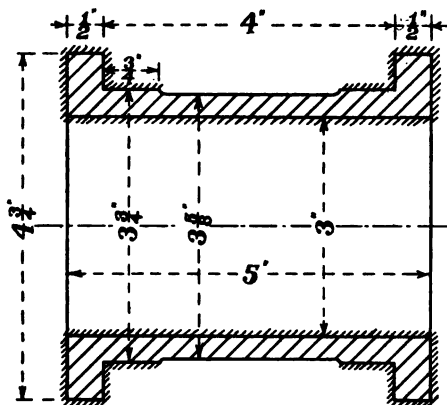


FIG. 208.

9. Calculate the finished weight of the G.M. feed-pump plunger shown in fig. 209.

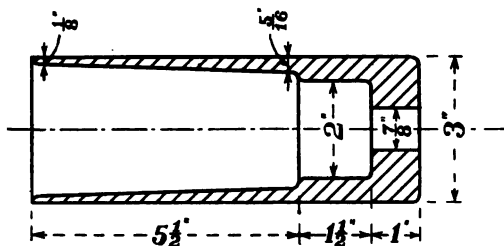


FIG. 209.

10. What is the finished weight of the C.I. coupling disc shown in fig. 210?

If the cast weight be 10 per cent. more than the finished weight, find the cost of the casting at 12s. 6d. per cwt.

11. Calculate the cast weight of the small fly-wheel shown in fig. 211—allowing for extra metal for turning where marked.

15. Find the weight of a Lancashire boiler 7 ft. diameter $\times$ 28 ft. long, with two flues each 2 ft. 9 in. diameter, the shell and flue plates $\frac{1}{2}$ in. thick, ends $\frac{5}{8}$ in. thick, allowing 30 per cent. for laps, rivets, stays, and mountings.

16. Calculate the weight of water which each of the above boilers would hold, making a proper allowance for the height of the water-level above the centre of the boiler shell.

17. Find the weight of a rectangular water tank 6 ft. long $\times$ 5 ft. wide $\times$ 3 ft. 6 in. deep, with cast-iron plates $\frac{1}{2}$ in. thick. Take the weight of flanges and ribs at 40 per cent. of the weight of the sides and bottom plates.

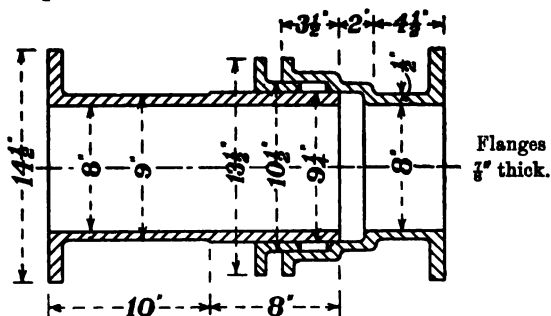


FIG. 212.

18. The prices of some large triple-expansion engines of four sizes are as follows :—

B.H.P.,	200	350	650	900
Price,	£1325	£2120	£3550	£4650

Estimate the approximate prices of engines of 300, 500, and 750 B.H.P.

19. Similar prices for some large compound engines are :—

B.H.P.,	280	400	600	800	1000
Price,	£1320	£1880	£2730	£3520	£4200

Find the approximate prices for engines of 300, 500, 700, and 900 B.H.P.

20. The prices of some small gas engines are :—

B.H.P.,	4	8	12	16	20
Price,	£50	£72	£90	£110	£128

Find the approximate prices of engines of 10, 15, 18, and 25 B.H.P.

21. During the month of November the price of copper varied as follows :—

November,	1	4	7	10	14
Price per ton,	£62 10s.	£63	£63	£64 10s.	£64 10s.
			17	21	24
			£66	£66 15s.	£66 10s.
					28
					£68

Plot a curve showing the fluctuation in price during the month, and find the average selling price for the month.

22. The selling price of a series of small steam engines is as follows :—

B.H.P.,	2	4	6	10	15
Price,	£24	£28	£32	£40	£50

Find the selling prices of engines of 5, $7\frac{1}{2}$, and $12\frac{1}{2}$ B.H.P.

23. The prices of small vertical boilers for the above engines were :—

B.H.P.,	2	4	6	10	15
Price,	£30	£36	£40	£54	£72

Find the selling prices of boilers for 5, $7\frac{1}{2}$, and $12\frac{1}{2}$ B.H.P.

24. The price of some water tube boilers is given as :—

Square feet heating surface,	160	215	270	320	375
Price,	£100	£112	£125	£137	£150

Find the probable prices of boilers of 180, 250, 450 sq. ft. heating surface.

25. The weights of some water tube boilers are given as :—

Square feet heating surface,	430	540	650	750	1075
Weight (tons),	6·5	7·8	8·8	10·8	13·2

Find the probable weights of boilers with 350, 600, and 900 sq. ft. heating surface.

26. During the month of November the price of hæmatite pig-iron varied thus :—

November,	1	4	7	10	14	16
Price per ton,	52s. 6d.	53s. 52s. 3d.	52s. 9d.	52s. 6d.	53s. 6d.	
		18	21	24	28	30
		53s. 3d.	54s. 3d.	54s. 3d.	55s.	56s.

Plot a curve showing the fluctuations in price during the month, and find

- (1) The average price for the month.
- (2) The saving by buying 100 tons on November 1st, as compared with buying it on November 29th.

INDEX.

- ACCELERATION, 219, 260.
- Addition (mechanical), 102.
- Adiabatic expansion, 103.
 - of air, 132.
 - work done by, 199.
- Air compressor, 214.
 - expansion of, 132.
 - pump, 68, 89.
 - quantity required for combustion, 307.
 - resistance of, 272.
- Amalsen planimeter, 323.
- Analysis of flue gases, 305, 311.
 - of fuel, 305, 311.
- Angles in triangle, 24.
 - circular measure of, 62.
- Annulus, area of, 43.
- Approximations, 1, 7.
- Arch, 65.
- Area, annulus, 43.
 - of circle, 41.
 - of sector, 42.
 - of segment, 42.
 - of cone, 45.
 - of cylinder, 47.
 - of ellipse, 43.
 - of parabolic segment, 45.
 - of parallelogram, 33.
 - of polygon, 41.
 - of prism, 47.
 - of pyramid, 45.
 - of rectangular hyperbola, 44.
 - of sphere, 48.
 - of segment, 48.
 - units of, 32.
- Areas, calculation of, 18, 31.
- Arithmetic, 22.
 - shortened forms of, 1, 6.
- Ash and clinker, percentage of, 20.
- Average, 22.
- Axis of curve, 59.
- B.T.U. (Board of Trade Unit), 195, 351.
- B.Th.U. (British Thermal Unit), 277.
- Babbitt's metal, 20.
- Back-gear of lathes, 179.
 - pressure of steam, 201.
- Balance, 22.
 - sheet (heat), 341.
 - weights, 250.
- Ball, weight of, 68, 69.
- Balloon, weight in, 271.
- Bays (tumbling), 124.
- Beams, pressure on supports of, 184, 188.
 - strength of, 18, 65.
- Belts, drives, 163.
 - power transmitted by, 205, 242.
 - ratio of tensions in, 126.
 - slip of, 162.
- Bending and twisting, combined, 130.
- Bevel wheels, 244.
- Blast fans, 159.
- Boiler, design, 274.
 - evaporation, 309.
 - heat losses in, 338.
 - heating surface, 308.
 - plating in, 67.
 - quantities for, 408, 406.
 - shop charges, 401.
 - size required, 307.
 - weekly returns, 418.
 - weight of, 424.
- Bonus system of payment, 407.
 - price, 432.
- Boring machines, per cent. employed, 438.
- Bottle jacks (see *Jacks*).
- Boyle's law, 197.
- Brake (for engine), 315.
 - H.P., 146, 159, 312, 315, 350.

- Braking power, 259, 271.
 Breaking load, 151.
 British Association unit of force, 266.
 O.G.S. system, 191.
 Calculation of weights (see *Weights*).
 Calorie, 277.
 Calorific value of fuel, 283, 296.
 of steam, 300.
 Calorimeter (fuel), 279, 283.
 (steam), 300.
 Capacity (see *Volume*).
 Capstan, 226.
 Carbon in fuel, 306.
 Carnot cycle, 333, 351.
 heat engine, 333.
 Castings, weight of, 388.
 Cast-iron, cost of, 398.
 Centigrade scale of temperature, 276.
 Centre of gravity, 57, 249.
 Centrifugal force, 249.
 pump, 165, 214.
 Chain, work done in raising, 212.
 Change wheels for lathe, 171, 179.
 Characteristic of logarithm, 71.
 Circles, angles in, 25.
 similarity, 28.
 Circular measure, 62.
 saw, 178.
 Circumference of circle, 29.
 Clearance volume, 21.
 Clockwise moment, 183.
 Coal bunker, 68, 69.
 Coefficient of discharge, 149.
 of displacement, 20.
 of friction, 126, 202, 242, 273.
 Combined calorimeter for steam, 304.
 efficiency, 11, 275.
 multiplication and division, 96, 112.
 Combustion, products of, 281.
 Commensurate quantities, 10.
 Commercial efficiency, 344.
 engineering, 353.
 problems, 275.
 Comparison of premium systems, 413.
 Compound belt drives, 179.
 engines, 314, 315, 326.
 Compounding ratios, 11.
 Compression of springs, 138, 139, 142.
 Concentrated loads on beams, 184.
 Condensation during expansion, 104.
 of steam, 286, 288.
 Condenser, cooling surface, 66.
 Cone, area of, 45.
 area of frustum of, 46.
 volume of frustum of, 53, 369.
 volume of, 53, 54, 369.
 Cone pulleys, 166.
 Conic sections, 59.
 Connecting rods, cost of turning, 435.
 Contracted forms of arithmetic, 1.
 division, 4.
 multiplication, 2.
 Conversion, metric measurements, 64.
 weight, 393, 440.
 Cooling surface, 66.
 Cost, of engines, 144, 422, 423, 426.
 of evaporating 1000 gallons, 417.
 of fitting up, 436.
 of fuel, 417.
 of machining connecting rods, 435.
 cylinders, 434.
 engines, 436.
 flywheels, 434.
 of materials, 398.
 of plummer blocks, 422.
 of power production (various), 343, 347.
 per I.H.P. engines, 423.
 per ton, engines, 423.
 Costs, departmental, 400.
 division of, 396.
 establishment, 402.
 shop, 401.
 Countershaft, speed of, 165.
 Couplings, flange, 419.
 Covers, circular, weight of, 366.
 conical, weight of, 367.
 Crane, derrick, 187, 189.
 power of, 229, 237.
 steam, 204, 205.
 test of, 158.
 Crank arm, weight of, 383.
 (bent), weight of, 381.
 disc, weight of, 377.
 Crankshaft (marine), weight of, 385.
 weight of, 384.
 Crossley gas engine, 314.
 Cube root, 127.
 Cubic measurements, 49.
 Cubical contents (see *Volume*).
 Cuboid, volume of, 49.
 Cursor for slide rule, 92, 100.
 Curves, trains rounding, 253.
 Cutting speeds, 175.
 Cycle of engine, 333, 351.

- Cylinder, surface of, 47.
 volume of, 51, 54.
 (hollow), volume of, 51.
- Cylinders, cost of boring, 434.
 moulding, 432.
- DAY-WORK wages, 407.
- Deflection of rectangular beams, 19.
- Density (relative), 21.
- Departmental costs, 400.
- Depreciation of plant, 353.
- Derrick crane, 187, 189.
- Diagram factor, 323.
- Diagram, indicator (see *Indicator Diagrams*).
 (stress-strain), 150.
- Diameters of engine cylinders, 349.
- Differential pulley blocks, 223, 225, 238.
- Directrix of curve, 59.
- Disc, rotating, 255.
- Discharge of water over weirs, 125.
 through orifices, 149.
- Displacement, coefficient of, 20.
- Distributed loads on beams, 185.
- Distribution of heat losses, 338.
- Division (contracted), 4.
 by logs, 73.
 by slide rule, 92.
 on spiral slide rule, 116.
 on watch calculator, 109.
- Drawing office charges, 401.
 squared paper in, 420.
 wages, 400.
- Drill, radial, 178.
- Drop hammer, 207.
- Drum (corrugated), weight of, 390, 392.
- Dryness fraction of steam, 299, 305.
- Duties of staff, 355.
- Dynamo, size of pulley on, 178.
- Dynamometer, water, 136.
- "E," modulus of elasticity, 194.
- "e," speed ratio of trains, 163, 170.
- Economical expansion, 332.
- Economiser, pipes in, 67.
- Effective pressure of steam (see *Mean*).
- Efficiency of engine and dynamo, 11.
 (mechanical), 220, 313.
 (thermal), 21, 275, 288, 333, 342.
 of boiler, 296.
 ratio, 336.
- Effort (friction), 221.
 to raise load, 216.
- Elastic limit, 151.
- Electric lighting engines, 210.
- Electrical energy, cost of, 351.
 unit of power, 195, 341.
- Ellipse, 62.
 area of, 43.
 ring, volume of, 54.
- Elliptical cover, pressure on, 68.
- Emery wheel, 179.
- Ends for drum, weight of, 392.
- Energy, definition of, 205.
 of flywheel, 209.
- Engine, cost of, 144, 422, 423, 426.
 of machining, 436.
 driving dynamo, 178.
 flywheel, 208.
 gas, 210.
 heat losses in, 338.
 power of, 412.
 pumping, 133.
 quantities for, 403, 405.
 size of, 330.
 steam, design, 275, 420.
 thermal efficiency, 333.
 weight of, 158.
- Equation to the straight line, 152.
- Equivalent evaporation, 298.
- Erecting shop, charges, 401.
 wages, 400.
- Establishment, 401.
 charges, 357, 397.
- Estimates, 396, 403.
- Estimating department, 421.
- Evaporation, from and at 212° F., 299,
 of boiler, 295, 298.
- Evolution (see *Powers and Roots*).
 by logarithms, 73.
- Excess air used in combustion, 306, 311.
- Expansion of air, 132.
 in cylinder (steam), 329, 332.
 ratio, 12, 21.
- Experiment on Weston's blocks, 223.
- Exponent in equation, 132.
- Extension of springs, 138, 139, 141.
 of steel bar, 159, 193.
- "F," effort, force, or power to raise
 load, 216.
- Face lathe, 176.
- Fahrenheit scale of temperature, 276.
- Fan for ventilation, size of, 63.
 (blast), 159.
- Fathom (6 feet), 193.
- Fitting-shop charges, 401.
 wages, 400.

- Flange couplings, proportions of, 419.
 Flow of water over weirs, 124.
 Fluctuation of speed, 209.
 Flue gases, analysis of, 305, 311.
 heat carried away by, 280.
 Fly-press, 207, 211.
 Flywheel, 66, 208, 214.
 cost of turning, 434.
 centrifugal force in rim of, 251.
 weight of, 377, 388.
 Focus, 60.
 Force, varying, 192, 194.
 Foreman, squared paper for, 426.
 Forgings, weight of, 381.
 Foundry charges, 401.
 orders, cost of, 437.
 wages, 400, 427.
 weekly returns, 427.
 Friction, coefficient of, 126.
 effort, 221.
 of slide valve, 208.
 percentage of, 220.
 Froude's water dynamometer, 136.
 Fuel, analysis of, 305, 311.
 consumption, 274.
 cost of, 417.
 heating power of, 283.
 (lower grade), cost of, 417.
 Fuller's spiral slide rule, 115.
 division on, 116.
 multiplication on, 116.
 Funnel, plates in, 66.

 "G," modulus of rigidity, 137.
 "g," acceleration due to gravity, 219.
 Gas engines, 210, 314.
 Gases, specific heat of, 283.
 Gauge (locomotive), 254.
 Gear wheels, 134, 168.
 for lathe, 174.
 Geometric mean, 14, 22.
 Girder, 189 (see *Beams*).
 Gradient, 65.
 Graphic representation of work, 191.
 Gravet slide rule, 87, 92.
 Gun, recoil of, 271.
 Gyration (radius of), 273.
 Gyrostat, 273.

 "H," total heat of evaporation of dry steam, 289.
 "h," sensible heat, 289.
 Halsey (premium) system, 410.

 Heat and work, 282.
 distribution and losses, 338.
 engine calculations, 274.
 in flue gases, 280.
 latent, 147, 289.
 sensible, 289.
 of superheated steam, 295.
 (total) of evaporation, 289.
 units of, 277.
 of wet steam, 294.
 Heating surface, 66.
 Helical springs, 66, 137, 139, 141, 142.
 Hollow cylinder, volume of, 51.
 shafting, strength of, 18.
 weight of, 373.
 sphere, volume of, 55.
 Horizon, distance to, 66.
 Horse-power, measurement of, 195.
 Hot water pipes, area of, 66.
 Hyperbola, 59, 62.
 Hyperbolic curve, area under, 44.
 method of drawing, 198.
 logarithms, 81.
 Hypotenuse of triangle, 26.

 IDEAL engine, 351.
 Impulse of force, 258.
 Incommensurate quantities, 10.
 Index of logarithm, 70.
 Indiarubber, extension of, 158.
 Indicated horse-power, 146, 312, 318, 349.
 Indicator, 320.
 diagrams, 320.
 division of, 101.
 probable, 324, 326.
 theoretical, 201.
 Indices, 16.
 Indirect expenses, 357.
 Involution (see *Powers and Roots*).
 by logs, 73.
 Irregular quadrilateral figure, 33.
 rectilinear figure, 34.
 Isothermal expansion, 197.

 "J," Joule's mechanical equivalent of heat, 282.
 Jack (hydraulic), 233.
 (screw), 232.
 Jackets, effect of, 134, 139.
 Jamieson, *Proceedings Inst. of Mechanical Engineers*, 432, 436, 437, 439.
 Jet (water), 262.
 condenser, 236.

- Joists (rolled), weight of, 370.
 Joule of work, 191.
 Joule's mechanical equivalent of heat, 282.
- "k," coefficient of fluctuation of energy, 209, 210.
- Kilo-calorie, 277.
 Kilogramme, 32.
 Kilowatt, 195, 341.
 Kinetic energy, 205, 206.
 Knot, 22.
- "L," latent heat of steam, 147, 289.
- Labour, cost of, 397.
 remuneration of, 407.
- Lathes, back gear for, 179.
 (face), 176.
 percentage in employment, 433.
 screw-cutting, 171.
 speed of, 105, 167.
 treble-gear, 174.
 work done by, 222.
- Laws of motion, 267.
- Lever, bell-crank, 186.
 moments of, 182.
 for pump, 68.
 weighted valve, 181.
- Limit of elasticity, 141.
- Limiting speed, 253.
- Liner for pump, 68.
- Locomotive, power exerted by, 204, 213.
 pull exerted by, 192.
 rounding curves, 253.
 speed of driving wheels, 66.
- Logarithmic sectional paper, 157.
- Logarithms, calculation by, 72.
 exercises on, 85.
 explanation of, 70.
 hyperbolic, 81.
 negative characteristics of, 75.
 on Perry's log-log rule, 123.
 on slide rule, 99.
 on spiral slide rule, 118.
 on watch calculator, 113.
 table of, 77.
- Log-log scale, 119.
 logarithms on, 123.
 powers on, 120.
 roots on, 121.
- "MA," mechanical advantage, 216.
- Machine-shop charges, 401.
 wages, 400.
- Machinery, percentage employment of, 433.
- Major calorie, 277.
- Mandrel of lathe, 167.
- Manhole cover, 68.
- Mantissa of logarithm, 71, 113.
- Marine type crankshaft, 385.
- Mass, 256, 265.
- Material, cost of, 397.
- Matter, quantity of, 257.
- Mean effective pressure, 202, 313, 320, 324, 330, 350.
 (pressure) of steam, 200.
 proportion, 14.
- Mechanical advantage, 216.
 efficiency, 220, 213.
 equivalent of heat, 282.
 summation, 100.
- Mensuration, 24.
- Metric units, 32, 49, 63.
- Modulus of elasticity, 194.
 of rigidity, 137.
- Moisture in coal, 296.
 in steam, 294.
- Moment of beams, 184.
 of force, 181.
 of lever, 182.
- Momentum, 256, 261.
- Monthly returns, smithy, 433.
- Motion, in a circle, 249.
 destroyed, 264.
 laws of, 267.
- Motor car, speed of, 272.
- Moulders' (iron) wages, 400, 422, 430
 (see *Foundry*).
- Moulding (iron), price of, 432.
- Moving body, work in, 206.
- Multi-cylinder engines, 314, 329, 350.
- Multiple, 10.
- Multiplication (contracted), 2.
 by logarithms, 63.
 on slide rule, 92.
 on spiral slide rule, 116.
 on watch calculator, 107.
- NAPERIAN logarithms, 81.
- Negative acceleration, 260.
 characteristic of logarithms, 75.
 powers, 122.
- OFFICE salaries, 400.
- Oil engines, horse-power of, 314.
 weight of, 158.
- Opisometer, 31, 390.

- Ordinate method of calculating areas, 36, 322.
 Ordinates, addition of, 102.
 Orifices, discharge of water through, 149.
 Output, costs and profits, 436.

 PAN, weight of, 380.
 Parabola, 60.
 Parabolic segment, area of, 45.
 Paraboloid, volume of, 58.
 Parallelogram, area of, 32, 33.
 Pattern-shop charges, 401.
 wages, 400.
 Pedestal, weight of, 374.
 Percentages, 97.
 Performance (of engine), 275.
 Perimeter of circle, 29.
 of ellipse, 31.
 Perry, Prof., log-log scale, 119, 120, 121, 123.
 Piecework, 407.
 Pig iron, effect of, 426.
 Pile-driver, 207.
 Pipe flanges, equivalent weight of, 372.
 Pipes, weight of, 371.
 Piston, 272.
 area of, 313.
 speed, 349.
 Piston rod, 313.
 correction for, 203, 213.
 Pitch of propeller, 65.
 Plane forms, similarity of, 26.
 Planimeter, area by, 37.
 (Amsler's), 37, 323.
 Planing machine, cost of, 421.
 percentage employment of, 433.
 table, 257, 271.
 Plant, efficiency of, 11, 338.
 Plates, weight of, 67, 385, 387.
 Plummer blocks, cost of, 422.
 Polygon, area of, 41.
 Potential energy, 205.
 Poundal, 269.
 Pointed cursor for slide rule, 100.
 Power (work), calculation of, 202, 349.
 rate of, 195.
 (unit of), 195.
 of belts, 242.
 of engine, 312, 349.
 of gearing, 243.
 Power-house, costs of, 416.
 production, cost of, 343.
 of ropes, 239.
 of shafting, 18, 127, 245.

 Powers, 15, 98, 113, 117, 120.
 of ten, 16.
 Premium system, 409.
 (Rowan's), 409, 411.
 (Halsey's), 411, 412.
 (Weir's), 413.
 comparison of systems, 414.
 Pressure on supports, 184.
 in cylinder (see *Mean Effective Pressure*).
 Prices (selling), averages, 404.
 Principle of work, 219.
 Probable m.e.p. in cylinder, 324.
 Products of combustion, 281, 305, 306.
 Profit, 358.
 Prime-cost clerk, 421.
 Prism, surface of, 47.
 volume of, 50.
 truncated, volume of, 52.
 Proportion, 13, 97.
 of flange couplings, 419.
 of pig iron, 426.
 Pulley blocks, 218, 225, 227, 238.
 Pulleys, speed ratio of, 161.
 Pump air, 189.
 centrifugal, 165, 214.
 liner, weight of, 68.
 power required by, 204, 214.
 rotary, cost of moulding, 432.
 speed of, 165.
 work done by, 192.
 Pumping engine, 183.
 Punching press, 207, 211.
 Purchase (see *Mechanical Advantage*).

 QUADRILATERAL figure, area of, 33.
 Qualifications of staff, 355.
 Quantity lists, 403, 405, 406.
 Quarterly returns, foundry, 430.
 summary, foundry, 428.

 RADIAN, 63.
 Railway embankment, 68.
 Rankine cycle, 333, 351.
 Ratio, 10, 11, 97.
 of expansion, 12, 21, 197.
 Reactions of supports, 184.
 Rectangular hyperbola, area under, 44.
 Rectilinear figure, angles in, 24.
 area of, 34.
 Reduction in area, 151.
 Relative density, 21.

- Remuneration of labour, 407.
- Resilience of iron bar, 194.
- Resistance, or load, 216.
 - in lathe, 224.
 - to train, 272.
- Retardation, 260.
- Revolution, solid of, 56.
 - surface of, 37.
- Right angle, forms with, 26.
- Rigidity, modulus of, 137.
- Ring, volume of, 53.
- Roof truss, 65.
- Roots, 15, 98, 113, 117, 121.
- Ropes, power transmitted by, 239.
 - ratio of tensions in, 126.
- Rowan, premium system, 409.

- SAFETY valve, 181, 188.
- Saturated steam, 290.
- Saturation curve, 103.
- Saw (circular), 178.
- Screw-cutting, wheels for, 171, 179.
- Screw-jack, 213.
 - propeller, 65, 66.
 - and worm-gears, 231.
- Sea water, salt in, 20.
- Sector of circle, area of, 42.
- Segment of circle, angles in, 25.
 - area of, 42.
 - of cylinder, volume of, 52.
 - of flywheel, 254.
 - of sphere, area of, 48.
 - volume of, 55.
- Selling prices (average), 404.
- Semicircle, angles in, 25.
- Service pipe, water, 262.
- Shafts, torsion in, 19, 127, 129, 131.
 - weight of, 373.
- Shearing stress, 127.
- Ship, resistance to, 205.
- Ship's capstan, 226.
- Shop charges, 397, 400.
 - wages cost, 400, 401.
- Shortened forms of arithmetic, 6.
- Shot (gun), 271.
- Shrouded wheels, 135, 244.
- Similar plane forms, 26, 28.
- Simpson's rule, 34.
- Sines of angles, 99, 100, 114.
- Single-acting engines, 314.
- Slide rule, calculation by, 92.
 - construction of, 89.
 - explanation, 87.
 - (Fuller's), 115.
- Slide rule (Gravet), 92.
 - valve, friction of, 203.
- Slip of belts, 162.
 - of propeller, 66.
- Smithy charges, 401.
 - returns, 433.
 - wages, 400.
- Solid of revolution, 56.
- Specific gravity, 21.
 - heat, calculations, 278.
 - of gases, 280.
 - table, 278.
 - water, 277.
- Specification, 358.
- Speed cones, 166.
 - of countershaft, 168, 177.
 - (cutting), 175.
 - fluctuation, 209.
 - of pump, 165.
 - ratio of pulleys, 161.
- Sphere, area of surface, 48.
 - segment of, area of, 48.
 - volume of, 54, 378.
- Spiral slide rule, 115.
- Spur gears, horse-power of, 134, 243.
 - ratio of, 168.
- Spring (helical), 66, 137, 141.
- Square of numbers, 114.
 - metre, 32.
 - root, 5, 113.
 - springs, 143.
- Squared paper, 140, 416, 420, 426.
 - logarithmic, 157.
- Stamping press, 207.
- Standard prices (selling), 404.
- Steam, condensation of, 286, 288.
 - consumption, 159, 336.
 - (Willan's law), 336.
 - formation of, 291.
 - generation of, 274.
 - latent heat of, 147.
 - pipe, 66.
 - pressure and volume, 154, 290.
 - safety valve, 181, 188.
 - superheated, 295, 298.
 - efficiency of, 336.
 - tables, 289, 290.
 - temperature of, 290.
 - velocity of, 66.
 - volume of, 67, 290.
 - wet, 294.
- Steel, cost of, 397.
- Step pulleys, 165.
- Stiffness of beams, 65.
- Strength of belts, 205.
 - of cylindrical shafts (hollow), 18.

- Strength of cylindrical shafts (solid),
 18, 127, 129, 131, 245.
 of gearing, 243.
 of rectangular beams, 18, 65.
 of ropes, 239.
- Stress-strain diagrams, 150, 159.
- Stress in beams, 189.
 in iron bar, 194.
- Submultiple, 10.
- Superheated steam, 295, 298, 336.
 efficiency of, 336.
- Supports, pressure on, 184.
- Surds, 15.
- Surface, condenser, 286.
 of pipes, 66, 67.
 of revolution, 37.
- TABLES, anti-logarithms, 82.
 bonus system, 408.
 cost of materials, 398.
 foundry returns, 430.
 logarithms, 78, 80.
 mean effective pressure, 332.
 premium system (Halsey's), 411.
 (Rowan's), 409.
 (Weir's), 413.
 selling prices (average), 404.
 shop charges, 401.
 specific heat, 278, 280.
 steam, 290.
 wages, 399.
 weight conversions, 393, 440.
- Tangents of angles, 99, 100, 114.
- Tank, weight of, 387.
- Teeth, number of, 168.
- Temperature conversions, 287.
 limit of, 334.
 scales of, 276.
 of steam, 290.
- Tensile stress in flywheel rims, 251.
- Tensions in belts and ropes, 126.
- Thermal efficiency, 21, 275, 288, 295,
 333, 343, 345.
 of boiler, 297.
 unit, 277.
- Throttling calorimeter, 301.
 (of steam), 201.
- Thomson's fuel calorimeter, 283.
- Tooling, allowance for, 363.
- Torque, 239.
- Torsion of shafts, 19, 127.
- Total heat of evaporation of steam,
 290.
- Train of gear wheels, 168, 170.
 of pulleys, 161.
- Transmission of power, 239.
- Trapezium, area of, 34, 67.
- Triangles, angles in, 24.
 area of, 33.
 similar, 27.
- Triangular notch, on weir, 138.
- Trigonometrical functions, 114.
- Triple-gear lathe, 174, 180.
- Triplicate ratio, 11.
- Truncated cylinder, volume of, 52.
 prism, volume of, 52.
- Tube, 189.
 surface of, 67.
- Tumbling bays, 124.
- Twisting and bending combined, 130.
 of shafts (see *Shafts*).
 of wrought iron, 159.
- UNBALANCED weights, effect of, 250.
- Unwin, Prof., on throttling calorimeter, 303.
- Useful work, 219.
- "V_R," velocity ratio, 217.
- Valves (stop), price of, 424.
- Variation, 17.
 in speed, 208.
- Velocity ratio of block and fall,
 223.
 of cranes, 230.
 of hydraulic jacks, 233.
 of mechanical powers, 217.
 of pulley blocks, 227.
 of pulleys, 161.
 of screw gears, 232.
 of screw jack, 218, 232.
 of wheel and axle, 225.
 of wheels, 168.
- Ventilation of room, 68.
- Volume of cone, 53, 54, 55.
 of cuboid, 49.
 of cylinder, 51, 54.
 of frustrum of cone, 53.
 of frustrum of prism, 53.
 of hexagonal prism, 50.
 of hollow sphere, 55.
 of paraboloid, 58.
 of pyramid, 53.
 of ring, 53.
 of segment of cylinder, 52.
 of segment of sphere, 55.
 of sphere, 54, 55.
 of steam, 154, 289, 290.
 of truncated cone, 53.
 of truncated cylinder, 52.
 of truncated prism, 52.

- Volume of truncated pyramid, 53.
 of zone of sphere, 56.
 Volumes, 18, 48.
 Volumetric analysis of fine gases,
 305.
 "W," load or resistance, or weight,
 216.
 Wages cost, 397.
 Watch calculator, 104.
 Water cistern, 68, 387.
 dynamometer, 136.
 equivalent of calorimeter, 279.
 flow of, over weirs, 124, 138.
 jet, 262.
 power required to raise, 204.
 scoop, 260, 271.
 work done in raising, 193.
 Watt (electrical unit), 195, 341.
 Watt's estimation of horse-power,
 195.
 Weekly returns, boilers, 418.
 foundry, 427.
 Weighing (area by), 40.
 conversion tables, 393, 440.
 Weight of ball, 68, 378.
 of boilers, 424.
 of bent crank, 381.
 of castings, 388.
 of cast-iron pans, 380.
 of circular covers, 366.
 of corrugated drum, 390.
 of crank arm, 383.
 of crank disc, 377.
 of crankshaft, 384, 385.
 of ends for drum, 392.
 of engines per I.H.P., 423.
 of flywheel, 377, 389.
 of forgings, 381.
 of hollow shafts, 373.
 of materials, 362, 366.
 of pedestal, 374.
 Weight of pipes, 371.
 of plate tank, 387.
 of plates, 67, 365, 366, 387.
 of rolled joists, 370.
 of shafts, 373.
 of square plates, 362.
 Weight, estimating by, 403.
 and mass, 265.
 Weir's premium system, 412, 413.
 Weirs, flow of water over, 124, 138.
 Wet steam, 294, 299.
 Wheel and axle, 225.
 Wheels, cast steel, 244.
 ratio of, 168.
 screw-cutting, 171.
 shrouded, 135.
 spur, 134, 243.
 Willan's law, 336, 337.
 Winches (see *Cranes*).
 Wire-drawing calorimeter, 301.
 of steam, 201.
 Work done, by gas, 195.
 (adiabatically), 199.
 raising water, 193.
 stretching iron bar, 193.
 varying force, 192.
 in cylinder, 196.
 in cylinder (expansive), 195.
 equation of, 190.
 and heat, 282.
 measurement of, 190.
 and power, 194.
 principle of, 219.
 stored up, 206.
 unit of, 191.
 Works manager, squared paper for,
 426.
 YIELD point, 141.
 ZONE of sphere, volume of, 56.

